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Functional Model of Wind Direction Data in Kuching, Sarawak, Malaysia

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Abstract

Sarawak is the largest state in Malaysia and is located in the country's east. Wind direction is critical in understanding the state's weather pattern. Wind direction is angular and cannot be analysed in the same way that linear data can. As a result, in this research, we discuss a model created to analyse circular data to estimate wind direction in Kuching, Sarawak. The von Mises distribution is used to build this model, with the parameters estimated using maximum likelihood and the covariance produced using the Fisher Information matrix. *Covratio* approach is used to detect if any outlier exists in the data. In contrast to the usual regression model which only examines the error term in the *y*-variable, in this functional model, the existence of the error terms is examined for all variables. The rotation parameters of wind direction in Kuching is 0.20444; meanwhile, the concentration parameter of the error terms is 1.47066. The proposed model can be used to understand wind direction for safety purposes.

Keywords: von Mises distribution; functional model; circular data; maximum likelihood estimation; outlier detection

1 Introduction

Wind energy, a clean and limitless renewable energy source, is a crucial component of sustainable growth. Therefore, it is crucial to undertake accurate and thorough feasibility studies in nations with high levels of traditional energy resource usage; this strategy is promoted and supported by green money and climate change action. Additionally, it is essential to support the economic and long-term prosperity of these nations [1].

A thorough understanding of the wind characteristics is critical for estimating the efficiency of a wind energy project [2]. Malaysia faces wind energy production challenges as it is located at 5◦ on the north side of the equator. Also, the wind circulation system is affected by coastal and land air [3]. The wind direction is circular. The data is described in degree or in radian, in which the measurements are from 0° to 360°, or from 0 radian to 2π radians, respectively. Circular data is unable to be analysed ordinarily like linear data. Therefore, to suit the wind direction data, a functional relationship model specifically designed for circular variables is applied in this research.

Von Mises distribution is well-known in studying circular data [4]. The von Mises distribution is a circular analogue of the normal distribution [5]. The probability distribution function of the Von Mises distribution is given by

$$g(\theta;\mu,\kappa)=\frac{1}{2\pi I_0(\kappa)}\exp(\kappa\cos(\theta-\mu)) \quad (1)$$

where $I_0(\kappa)$ is the modified Bessel function of the first kind and order zero, defined by

$$I_0(\kappa)=\frac{1}{2\pi}\int_0^{2\pi}\exp(\kappa\cos\theta)\,d\theta \quad (2)$$

where μ is the mean direction and κ is the concentration parameter for $0\le\theta<2\pi$ and $\kappa>0$. The concentration parameter denotes the distribution's spread [6].

Section 2 of this paper describes the location of Kuching, Sarawak, in Malaysia. Section 3 discusses the functional model and the parameter estimation fitted to the wind direction data. The results of the study are explained in Section 4, and then the conclusion is made in Section 5.

2 Location

The state of Sarawak is located in Malaysia, and its capital is Kuching. Most developments in Kuching were concentrated in the vicinity of Sungai Sarawak since here is where civilization in the city first emerged. The area has a tropical rainforest environment with 4600 millimetres of yearly precipitation [7]. Season in the northeast runs from September through March, and the southeast runs from May through August. The primary rainy season in Kuching is the northeast season, whereas the southeast season is dry. Kuching has 570 407 residents as of 2018 [8]. Therefore, in this paper, we study the relationship of wind direction during the northeast season in Kuching, Sarawak. The data is obtained from Malaysia Meteorological Department. Figure 1 shows the location of Kuching in Malaysia.

Figure 1: Location of Kuching in Malaysia.
Courtesy:
http://www.world-guides.com/asia/malaysia/sarawak/kuching/kuching_maps.html

3 Functional Model and Parameter Estimation

In this paper, wind direction data is modelled by using functional relationship. The functional model is an error-in-variables model that studies the relationship between the variables. Error-in-variable model is different from the ordinary linear regression model, in which the variables are considered with error terms [6]. The existence of error terms may affect the desirable criteria of estimators if they are ignored [9].

The functional relationship model is given by

$$Y = \alpha + X \ (mod \ \ 2\pi) \tag{3}$$

where the X and Y variables are subjected to random errors δ_i and ε_i, respectively. Given that the variables $X_i = x_i + \delta_i$ and $Y_i = y_i + \varepsilon_i$. The errors terms in this model are distributed with von Mises of $\delta_i \sim VM(0, \kappa)$ and $\varepsilon_i \sim VM(0, \kappa)$.

The method of maximum likelihood is used for parameter estimation. Maximum likelihood techniques are essential for parameter estimation and system modelling since they are based on probability and statistics theory [10]. The maximum likelihood estimation is derived through the least square method.

The log-likelihood function of von Mises is given by

$$\log L(\alpha,\kappa,X;x,y) = -2n\log 2\pi - 2n\log I_0(\kappa) + \kappa\sum_{i=1}^{n}\cos(x_i - X_i) + \kappa\sum_{i=1}^{n}\cos(y_i - \alpha - X_i) \quad (4)$$

for $0 \le x < 2\pi$, $0 \le y < 2\pi$ and $\kappa > 0$ where $I_0(\kappa)$ is the modified Bessel function of the first kind and order zero, which can be defined by :

$$I_0(\kappa) = \frac{1}{2\pi}\int_0^{2\pi} e^{\kappa\cos\theta}\,d\theta \quad (5)$$

where κ is the concentration of the error.

The estimation of the variable X_i is obtained by

$$\hat{X}_{i1} \approx \hat{X}_{10} + \frac{\sin(x_i - \hat{X}_{i0}) + \sin(y_i - \hat{\alpha} - \hat{X}_{i0})}{\cos(x_i - \hat{X}_{i0}) + \cos(y_i - \hat{\alpha} - \hat{X}_{i0})} \quad (6)$$

and rotation parameter is estimated by

$$\hat{\alpha} = \begin{cases} \tan^{-1}\left\{\frac{S}{C}\right\} & \text{when } S > 0, C > 0 \\ \tan^{-1}\left\{\frac{S}{C}\right\} + \pi & \text{when } C < 0 \\ \tan^{-1}\left\{\frac{S}{C}\right\} + 2\pi & \text{when } S < 0, C > 0 \end{cases} \quad (7)$$

where $S = \sum_{i=1}^{n}\sin(y_i - \hat{X}_i)$ and $C = \sum_{i=1}^{n}\cos(y_i - \hat{X}_i)$.

In estimating the concentration, κ for equal error concentration case, Fisher approximation is applied [22]. The approximation is given by:

$$A_1^{-1}(w) = \begin{cases} 2w + w^3 + \frac{5}{6}w^3 & \text{when } w < 0.53 \\ -0.4 + 1.39w + \frac{0.43}{(1-w)} & \text{when } 0.53 \le w < 0.85 \\ \frac{1}{w^3 - 4w^2 + 3w} & \text{when } w \ge 0.85 \end{cases} \quad (8)$$

Hence, the estimation becomes

$\hat{\kappa} = A_1^{-1}(w)$ where $w = \frac{1}{n}\{\sum_{i=1}^{n}\cos(x_i - \hat{X}_i) + \sum_{i=1}^{n}\cos(y_i - \hat{\alpha} - \hat{X}_i)\}$. (9)

where

$$\hat{\kappa} = A_1{}^{-1}\left(\frac{1}{n}\left\{\sum_{i=1}^{n} cos\left(x_i - \hat{X}_i\right) + \sum_{i=1}^{n} cos\left(y_i - \alpha - \hat{X}_i\right)\right\}\right) \quad (10)$$

In circular data, an adjustment is made in estimating the concentration parameter with dividing it by 2 [21]. Therefore, the estimate becomes $\tilde{\kappa} = \frac{\hat{\kappa}}{2}$.

4 Results

Before applying the data to the functional model, univariate analysis is carried out on the wind direction data of Kuching during the Northeast monsoon. We described the data graphically with the rose diagrams. Figures 2 and 3 show the rose diagrams of the wind direction data of Kuching in 2019 and 2020, respectively. The diagrams are plotted without any relationship between the wind direction for both years. Therefore, the relationship between the wind direction for both years will be studied using the functional relationship model for circular variables.

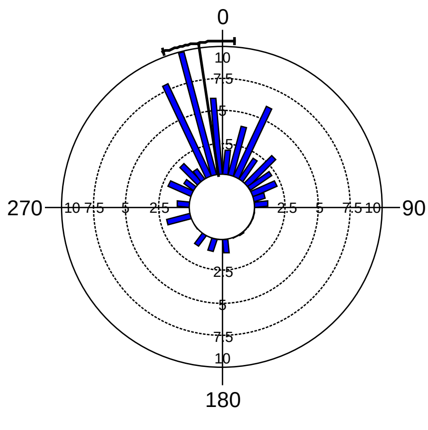


Figure 2. Rose diagram of Kuching wind direction in 2019 during northeast monsoon.

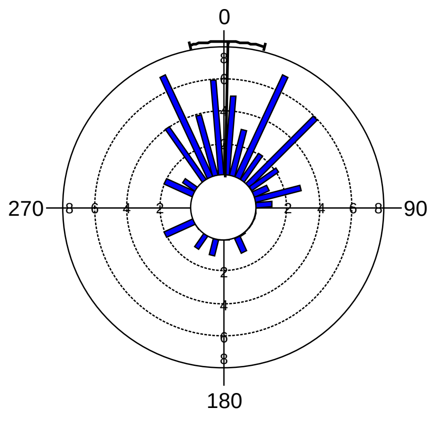
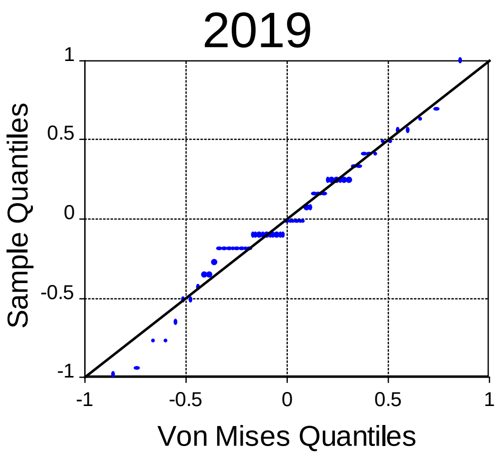


Figure 3. Rose diagram of Kuching wind direction in 2020 during northeast monsoon.

To demonstrate how well the data fit the von Mises distribution, Q-Q plots are created for the wind direction data for both years. Q-Q plots are utilised to determine if two data sets are from the same distribution. [11]. Figures 4 and 5 shows the plots for wind direction in 2019 and 2020, respectively.



Figures 4. QQ-plot for wind direction in 2019

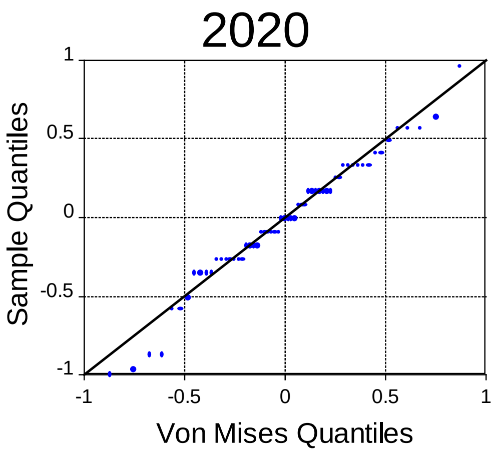


Figure 5. QQ-plot for wind direction in 2020

Next, the wind direction data is examined for any outliers using the *covratio* approach. The purpose of this *covratio* approach is to find outliers in circular variables. This *covratio* statistics is initially obtained through the Fisher Information matrix. Given that the formula of *covratio* is

$$COVRATIO_{(-i)} = \frac{|COV|}{|COV_{(-i)}|} \tag{10}$$

where $|COV|$ is the determinant of the covariance matrix of the parameter estimation. It is given by

$$|COV| = \frac{1}{n^2\tilde{\kappa}[A'_1(\tilde{\kappa})]^2}$$

and $|COV_{(-i)}|$ is the determinant of the covariance matrix of the reduced data set by omitting the i-th row.

Mokhtar et al. developed the cut-off equation for outlier detection in this functional model for circular data with $y = 3.7586n^{-0.71}$ [12]. Since the sample size used for this study is n =59, thus the cut-off point becomes $y = 3.7586n^{-0.71} = 0.20784$. This cut-off algorithm is employed to find outliers in the wind direction data. Figure 6 shows the values $COVRATIO_{(-i)}$ when each observation is omitted.

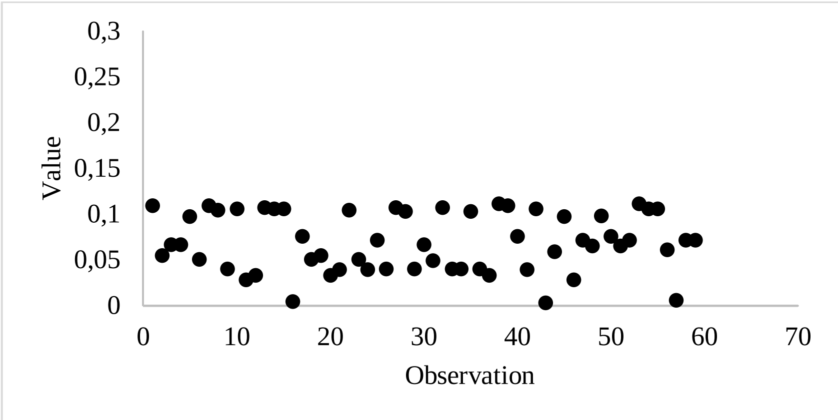


Figure 6. Values of $COVRATIO_{(-i)}$ for outlier detection

We fit the wind direction data to the functional connection model for circular variables after checking that there are no outliers in the data. Table 1 displays the parameter estimate values obtained using the maximum likelihood technique.

Table 1. Parameter estimates of Kuching, Sarawak wind direction data.

Detail	Value
Rotation parameter, $\hat{\boldsymbol{\alpha}}$	0.20444
Variance of $\hat{\boldsymbol{\alpha}}$	0.03915
Concentration parameter, $\tilde{\boldsymbol{\kappa}}$	1.47066
Variance of $\tilde{\boldsymbol{\kappa}}$	0.03351

Table 1 presents the parameter estimates for wind direction data collected in Kuching, Sarawak, during the northeast monsoons of 2019 and 2020 when fitted with a functional relationship model for circular variables. The rotation parameter is estimated to be 0.20444, which is extremely near to 0 radians; the concentration parameter is estimated to be 1.47066.The variance of $\hat{\alpha}$ and $\tilde{\kappa}$ are small given by 0.03915 and 0.03351, respectively, indicating that the values estimated are consistent and less dispersed.

5 Conclusion

In conclusion, during the northeast monsoon season in 2019 and 2020, this paper provides a functional model of wind direction data in Kuching, Sarawak. The benefit of this model is that it describes the data by including an error term for each variable. *Covratio* statistics is used to check for the presence of outliers in data, and it reveals that there are none. Maximum likelihood estimation is used in conjunction with the von Mises distribution to estimate wind direction parameters. The rotation estimate for the data is small and close to 0 radian, which is 0.20444. In this model, the concentration parameter is estimated to be 1.47066. The variance of the parameter estimates $\hat{\alpha}$ and $\tilde{\kappa}$ are 0.03915 and 0.03351, respectively which indicate that the values estimated are consistent and less dispersed. This model could be implemented in future study to investigate wind direction in a variety of additional sites.

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