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Random Walk with Non-recursive Functions

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Abstract

In this article, we propose a modelling of two kinds of random walks A and B without calling on recursive processes. This modelling is essentially based on two functions: a non-recursive noise function and the staircase function. The two random walks come from the same system of equations; they differ only by the values of the constants of the system of equations.

I. Introduction

We can consider the random walk as an erratic curve. There are several methods to model erratic curves [3] [4].
We will use some procedures presented in [1]. These can be summarized in three points:
(a) A noise source,
(b) Sample the data from the source with different sampling frequencies,
(c) Interpolate the different points chosen during the sampling step.
In this article we will use the three steps stated above, but in our case the noise source is generated by a non-iterative function.

You could choose any noise function among the different functions discussed in [2]. To sample the noise function, we will use the staircase function.
The system of equations which models the random walk (A) and (B) is the same. The change from (A) to (B) is only controlled by a few parameters.

Parameters used in system of equations

According to the values of the parameters C0, C1, C2, C3, d1, d2 and d3 we model either the curve A (Fig.10A) or the curve B (Fig.10B).
The parameter n was introduced to express the sensitivity to the initial conditions of the system of equations. The interval of the values of 'x' has been well chosen.

In a future article, we will try to make some modifications to enlarge the domain of values of x.

II. Modeling of random walks A and B

Modelling includes several steps .

COMMON PARAMETERS

$$r_1 := 2 : r_2 := 25 : r_3 := 150 : t := 36 : n := t \cdot 10^{-8} :$$

$$N2 := 2500 : N1 := 10^3 :$$

#PARAMETRES COURVE A

$$C_0 := 0.75 : C_1 := 1 : C_2 := 4 : C_3 := 10 :$$

$$d_1 := 1 : d_2 := 1 : d_3 := 1 :$$

#PARAMETRES COURVE B

$$\#C_0 := 1.5 : C_1 := 1.5 : C_2 := 0.035 : C_3 := 3 :$$

$$\#d_1 := 0.35 : d_2 := 2.5 : d_3 := 2 :$$

STAIRCASE FUNCTION OF DIFFERENT WITHS

$$Xh := x \rightarrow x - \frac{r_1}{\pi} \operatorname{arccot}\left(\cot\left(\frac{\pi \cdot x}{r_1}\right)\right) :$$

$$Xm := x \rightarrow x - \frac{r_2}{\pi} \operatorname{arccot}\left(\cot\left(\frac{\pi \cdot x}{r_2}\right)\right) :$$

$$Xb := x \rightarrow x - \frac{r_3}{\pi} \operatorname{arccot}\left(\cot\left(\frac{\pi \cdot x}{r_3}\right)\right) :$$

#NOISE FUNCTION ON TWO AXES

$$Sx := x \rightarrow C_0 \cdot \arccos\left(\cos\left((x)^{55.3 + n}\right)\right) :$$

$$Sy := x \rightarrow C_0 \cdot \arcsin\left(\sin\left((x)^{54.7 + n}\right)\right) :$$

plot(Sx(x), x = N1 ..N2, adaptive = false, numpoints = N2 − N1)

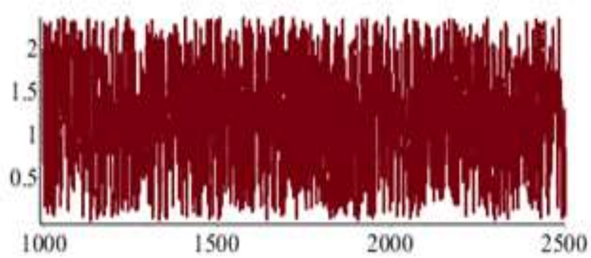

Fig.1

```
#NOISE SAMPLED WITH PERIOD r1 ON BOTH AXES
Ehx := x→ arccos(cos((Xh(x))^(3.1 + n))) :
Ehy := x→ arcsin(sin((Xh(x))^(3.3 + n))) :
plot(Ehy(x), x = N1 ..N2, adaptive = false, numpoints = N2 − N1)
```

Fig.2

```
#NOISE SAMPLED WITH PERIOD r2 ON BOTH AXES
Emx := x→ arccos(cos((Xm(x))^(2.3 + n))) :
Emy := x→ arcsin(sin((Xm(x))^(2.7 + n))) :
plot([Emx(x), Emy(x)], x = N1 ..N2, adaptive = false, numpoints = N2 − N1)
```

Fig.3

```
#NOISE SAMPLED WITH PERIOD r2 ON BOTH AXES
Emx := x→ arccos(cos((Xm(x))^(2.3 + n))) :
Emy := x→ arcsin(sin((Xm(x))^(2.7 + n))) :
plot([Emx(x), Emy(x)], x = N1 ..N2, adaptive = false, numpoints = N2 − N1)
```

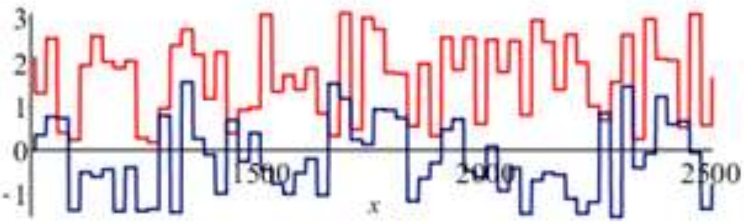

Fig.4

```
#NOISE SAMPLED WITH PERIOD r3 ON BOTH AXES
Ebx := x→ arccos(cos((Xb(x))^(2.36 + n))) :
Eby := x→ arcsin(sin((Xb(x))^(2.2 + n))) :
plot([Ebx(x), Eby(x)], x = N1 ..N2, adaptive = false, numpoints = N2 − N1)
```

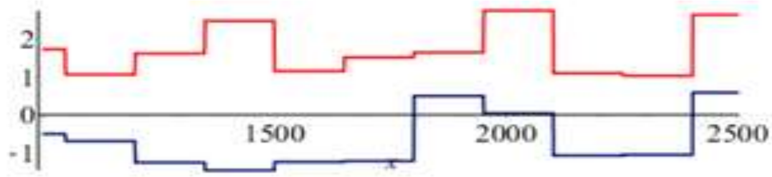

Fig.5

#LINEAR INTERPOLATION WITH A PERIOD r1 ON BOTH AXES

$$Ihx := x \to C_1 \cdot \left(\left(\frac{Ehx(x + r_1) - Ehx(x)}{r_1} \right) \cdot (x - Xh(x)) + Ehx(x) \right) :$$

$$Ihy := x \to C_1 \cdot \left(\left(\frac{Ehy(x + r_1) - Ehy(x)}{r_1} \right) \cdot (x - Xh(x)) + Ehy(x) \right) :$$

plot([Ihx(x), Ihy(x)], x = N1 ..N2, adaptive = false, numpoints = N2 − N1)

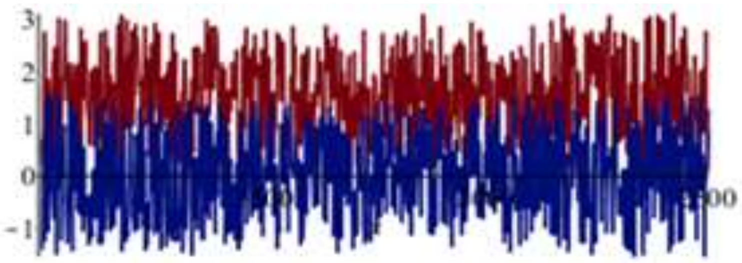

Fig.6

#LINEAR INTERPOLATION WITH A PERIOD r2 ON BOTH AXES

$$Imx := x \to C_2 \cdot \left(\left(\frac{Emx(x + r_2) - Emx(x)}{r_2} \right) \cdot (d_3 \cdot x - Xm(x)) + Emx(x) \right) :$$

$$Imy := x \to C_2 \cdot \left(\left(\frac{Emy(x + r_2) - Emy(x)}{r_2} \right) \cdot (d_3 \cdot x - Xm(x)) + Emy(x) \right) :$$

plot([Imx(x), Imy(x)], x = N1 ..N2, adaptive = false, numpoints = N2 − N1)

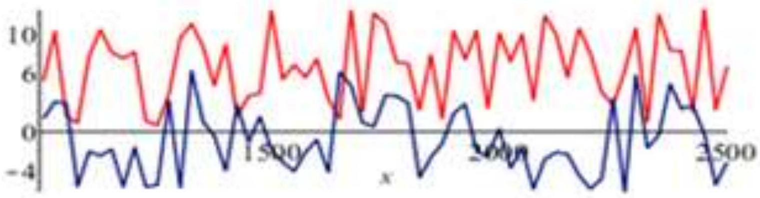

Fig.7

 # LINEAR INTERPOLATION WITH A PERIOD r3 ON BOTH AXES

$$Ibx := x \to C_3 \cdot \left(\left(\frac{Ebx(x + r_3) - Ebx(x)}{r_3} \right) \cdot (d_2 \cdot x - Xb(x)) + Ebx(x) \right) :$$

$$Iby := x \to C_3 \cdot \left(\left(\frac{Eby(x + r_3) - Eby(x)}{r_3} \right) \cdot (d_2 \cdot x - Xb(x)) + Eby(x) \right) :$$

plot([Ibx(x), Iby(x)], x = N1 .. N2, adaptive = false, numpoints = N2-N1)

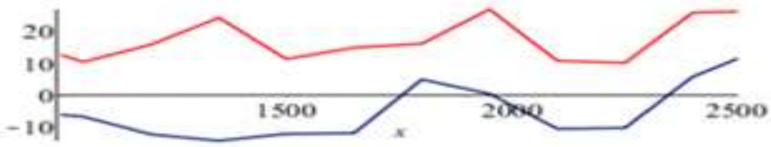

Fig.8

```
#FINAL EQUATION
Cx := x→ (Sx(x) + Ihx(x) + Imx(x) + d1·Ibx(x)) :
Cy := x→ (Sy(x) + Ihy(x) + Imy(x) + d1·Iby(x)) :
plot([Cx(x), Cy(x)], x = N1 ..N2, adaptive = false, numpoints = N2 − N1)
```

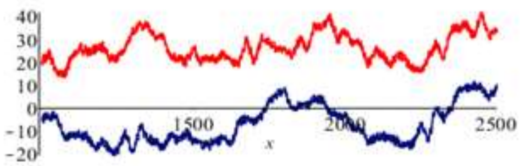

Fig.9

```
plot([Cx(x), Cy(x), x = N1 ..N2], adaptive = false, numpoints = N2 − N1)
```

Fig10A. Curve A

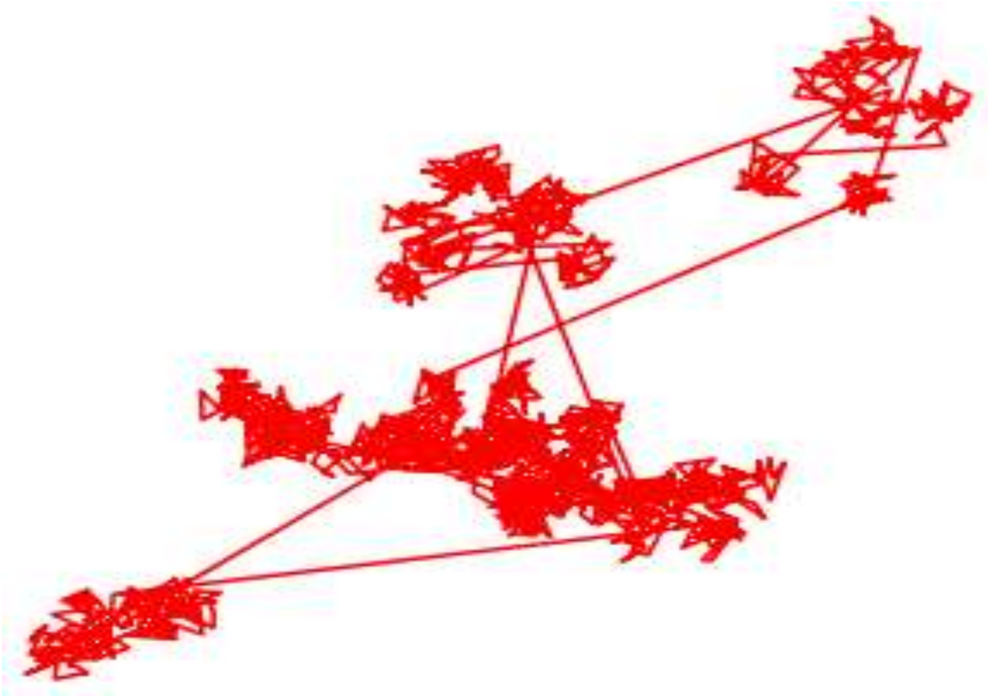

Fig.10B. Curve B

Conclusion

We have produced two random walk models with non-recursive functions. We will try all the modelled noise functions described in [2] and not just a uniform distribution noise. Also in the future we will try to model the random walk with avoidance.

References

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