Computational Mathematical Techniques Model for Investment Strategies

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Abstract

Various investment strategies were modeled, and the tradeoffs between risk and return were examined in each strategy. Two investment types were considered: stocks (higher return, higher risk), and bonds (lower return, no risk). We simulated various strategies that combined these two investment types, including strategies that suddenly switch between investment types, strategies that transition between investment types, and strategies that change investment types based on stock market performance. The trends of the various strategies were observed and were used to construct an optimal strategy.

Keywords: Computational Mathematical Techniques, Investment strategies, stock market, bonds, risk and return

1 Introduction

Investment is an important part of many people's lives in saving for retirement and increasing financial security. Different investment types provide various advantages and disadvantages.

Stocks, which are based off of the performance of particular companies, provide a relatively high investment risk, and a relatively high expected gain. Monthly returns of Dow Jones data from 1902 to 2019 is close to a normal distribution with a mean of 0.5%, with a standard deviation of 4.5% [4].
Whether the stock market follows a true random walk is debated. The random walk hypothesis for stock returns has been supported by some studies \[1\], \[6\] and denied by others \[2\], \[3\] but since the stock market is close to random, we shall assume a normal random distribution for the model. We will base our model's stock investment type on the Dow Jones industrial average, so this investment type is actually a diverse portfolio of stocks.

Bonds are widely considered to be a stable investment type. United States government bonds have an average return rate of around 0.2-0.3% per month, and are based on a fixed percentage of 0.2% per month, plus a variable percentage to account for inflation \[5\]. In this model, we will assume that bonds have a constant return rate of 0.3% per month.

2 The Model

We established constants for all strategies:

- $100 is added to the total money each month.
- Monthly bond growth is 0.3%.
- Monthly stock growth is a normal random variable, with a mean of 0.5%, and a standard deviation of 4.5%.
- Each month, the percentage of total money in each investment type is changed based on the particular strategy.
- Most simulations run over 20 years.

Each strategy was simulated for 10,000 trials. Using this model, we attempted to answer the following questions:

- How does investment time affect the strategy used?
- What is an optimal method of combining investment types that achieves a good combination of risk and return?
- Can a successful strategy take the stock market’s performance into consideration?
3 Simulations and Analysis

![Histogram of stocks](image1)

**Figure 2**: Histogram of stocks

This histogram of the final values of investments solely into stock is a right-tailed distribution, rather than a normal distribution. This appears to be an artifact from the regular addition of funds each month. Because any losses when the stock returns dip negative are offset by this influx of funds, the left tail of the distribution is shortened. Most strategies that combine stocks and bonds also have a right-tailed distribution, so the median is an important data point to consider when comparing strategies.

### 3.1 Varying Times

![Varying time](image2)

**Figure 3**: Varying time
In figure 3, we examined the performance of stock investments against bonds for various time spans. We looked at durations from four years to twenty years in four year steps. In each case, we see that the mean and median of stocks outperforms the investment in bonds, but the standard deviations of stocks extend into a region that performs worse than bonds. By the final run for twenty years, there is even a small, but not insignificant chance, that the stocks will have a return lower than not investing the money at all over the twenty years and simply saving it.

### 3.2 Fixed Percentages in Stock

In figure 4, we then examined the behavior of static percentages of stocks and bonds over twenty years. In these strategies, money is moved between investment types each month, so a constant percent of the total money is invested in stocks at the beginning of each month. The most significant effect of mixing stocks and bonds is that the addition of bonds can shorten the standard deviation in the final returns from the investments. Additionally, the mixture of bonds and stocks brings the median closer to the mean of the returns. These two factors help produce a range of choices for investors to balance their goals between higher returns and stable returns. 40% stock, 60% bonds seems to show a strong balance of stability and higher return without a significant chance that the returns will fall below simple saving the money.

![Figure 4: Fixed percentage in stock](image)

### 3.3 Sudden Change of Investment Types

The figure 5 shows the range of returns when the balance between stocks and bonds is suddenly changed at some time, where the time of the switch is varied.
Strategies that start with investment only in stocks and transfer to only bonds have a more stable returns, and comparable medians and means to the reverse strategies.

**Figure 5**: Sudden change of investment type

### 3.4 Linear Transition between Investment Types

Here in figure 6, we examined the results of a linear transition between stocks and bonds over the entire duration rather than a sudden, one time change in investing. In each of these strategies, the investment is half bonds/half stocks halfway through the investment time, and each month has an initial percent of money in stocks that is a constant amount greater or less than the previous month’s...
initial percentage. The graph shows a steady increase in the mean and standard deviation, and a comparably negligible increase in the median return as we increase the percentage of starting bonds and ending stocks.

4 Dynamic Strategies

4.1 Exchange Investment Types Based on Stock Performance

Figure 7 shows the results of a simple decision making process being applied to the investment percentages. In the intuitive model, when stocks over perform, more of the investments are changed to stock, and when stocks underperform, more of the investments are transitioned to bonds. For the counterintuitive system, the reverse holds, with less money being placed in stocks when stocks perform well and vice versa. Each exchange is based on the previous month’s return and is a constant percentage of the total money, and initially, 50% of the total money is in each investment type. In each case, the counterintuitive system results in a comparable mean to the intuitive system, but the counter intuitive system consistently has a smaller variance and higher median return than the intuitive system.

![Figure 7: Exchange investment types based on stock performance](image)

5 A Final Look at Various Strategies

Here in figure 8, we examine a number of the more effective strategies for our model, and add dynamic step strategies, which combine the dynamic model with the linear transition model. In the dynamic step model from stocks to bonds, initially all of the money is in stocks, and a fixed percent of the total money is taken out of stocks and placed into bonds each month (separate from the money that transfers
based on the stock market performance). The dynamic step from bonds to stock similarly starts out with all money in bonds, and transitions to stocks. We see that the strategies of the dynamic step from bonds to stocks and the counterintuitive strategy at 5% change have a higher chance to reach our goal than simple investing in stocks for the duration. Additionally, both show a significantly smaller variance than simple stock investment. Most other strategies usually result in a lower average and median return with a higher variance than these strategies.

Table 1: Numerical results

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Mean ($)</th>
<th>Median ($)</th>
<th>Standard Deviation</th>
<th>% time $40,000 reached</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonds to stocks step</td>
<td>41,939</td>
<td>39,608</td>
<td>14,211</td>
<td>48.87 %</td>
</tr>
<tr>
<td>Stocks to bonds step</td>
<td>39,026</td>
<td>38,182</td>
<td>6,456</td>
<td>38.81 %</td>
</tr>
<tr>
<td>Stocks to bonds sudden</td>
<td>38,133</td>
<td>36,919</td>
<td>7,571</td>
<td>33.36 %</td>
</tr>
<tr>
<td>Dynamic Step: Bonds to stock</td>
<td>41,675</td>
<td>41,397</td>
<td>12,174</td>
<td>54.59 %</td>
</tr>
<tr>
<td>Dynamic Step: Stock to bonds</td>
<td>38,662</td>
<td>39,450</td>
<td>5,822</td>
<td>44.78 %</td>
</tr>
<tr>
<td>Dynamic Strategy</td>
<td>40,120</td>
<td>40,796</td>
<td>8,913</td>
<td>54.54 %</td>
</tr>
<tr>
<td>Bonds</td>
<td>35,279</td>
<td>35,279</td>
<td>0</td>
<td>0 %</td>
</tr>
<tr>
<td>Stocks</td>
<td>46,979</td>
<td>41,613</td>
<td>23,467</td>
<td>53.68 %</td>
</tr>
</tbody>
</table>
6 Conclusion

The various models examined give insight as to what strategies give more desirable risks and returns, and allow an optimal strategy to be selected.

Based on the simulation with various investment lengths, any investment strategy with only stocks will have a non-negligible potential to make very little money. Therefore, for long term investment periods, strategies that combine bond investments with stock investments should be pursued.

Various strategies that combine stock and bond investments have their own advantages and disadvantages. Strategies that make a sudden change from stocks to bonds are more desirable than strategies that make a sudden change from bonds to stock, as the variance of the latter grows quickly, while the means and medians of both classes of strategies have comparable growth out of the pure bonds strategy.

When transitioning from stocks to bonds, a strategy that transitions into bonds has a lower variance, mean, and median, than a strategy that transitions into stocks. For this class of strategies, there is a tradeoff between risk and return, and a decision between these strategies depends on an individual investor’s goal.

For the strategies that shift stock investment percentage based on the past stock market performance, the counter-intuitive strategy seems to be more desirable. The counter-intuitive strategy’s medians are consistently higher, and its standard deviations are consistently lower than the intuitive strategy’s (the means of each strategy are comparable).

When comparing the various strategies, transitioning between investment types is more desirable than making a sudden switch; transitioning from stocks to bonds gives a lower standard deviation and a higher mean and median than suddenly changing from stocks to bonds. Combining a transition strategy with the counter-intuitive strategy for changing investments based on stock performance seems to yield desirable results; these strategies yield higher means and lower standard deviations than their counterparts that do not adjust for stock performance.

An individual’s return goals and willingness to take risk determines which transition strategy to combine with the dynamic strategy. If an individual does not want to take very much risk, but not gain as much money, they should pursue a stocks-to-bonds transition strategy, while an individual who wants to take more risk, but potentially gain more, should pursue a bonds-to-stocks transition strategy. If an investor only cares about reaching a certain investment goal, say $40,000, the dynamic strategy that transitions from bonds to stocks should be pursued since it reaches the goal more times than other strategies in this model.

In future research, the optimal strategy can be refined by further examining additional transition strategies, and by modifying the dynamic strategy. Nonlinear transitions between investment types, and non-constant exchanges in the dynamic strategy can be explored. Additional factors can also be added to the model, such as additional investment types, non-constant bond returns, and stock broker’s fees. These additional factors could make the model more true to reality.
References


Received: January 16, 2021; Published: January 30, 2021