Applied Mathematical Sciences, Vol. 14, 2020, no. 14, 669 - 677
HIKARI Ltd, / www.m-hikari.com
https://doi.org/10.12988/ams.2020.912157

Optimal Predictive Algorithm for the Pickup and Delivery Problem with Time Windows and Scheduled Lines: Innovative Routing Transport Systems

F. Nahayo [1] **and M. Bigirimana** [2]

[1] Research University Laboratory in Applied Statistics Modeling and Engineering "LURMISTA", Institute of Applied Statistics "ISTA"
University of Burundi, Burundi

[2] Research University Laboratory in Applied Statistics Modeling and Engineering "LURMISTA",University of Burundi, Burundi

Abstract

This paper concerns the Pickup and Delivery Problem with Time Windows and Scheduled Lines "PDPTW-SL" by modeling it mathematically on the one hand with the help of dynamic programming, on the other hand, it presents two optimal predictive methods of combinatorial optimization intended to solve it and which must provide stable and high accuracy solutions. The Pickup and Delivery Problem with Time Windows and Scheduled lines aims at routing a given set of vehicles to transport freight requests from their origins to their corresponding destinations, where the requests can use scheduled passenger transportation services during a stump of their journeys. We describe the PDPTW-SL as a mixed-integer program. Computational results on a set of small-size instances provide a clear understanding of the benefits of preprocessing step and valid inequalities using scheduled line services as a part of freights journey.

Keywords: Dynamic programming, Pickup and Delivery Problem, Routing and Scheduling, Transport Systems, Combinatorial optimization

1 Introduction

Vehicle routing problems are complex combinatorial optimization problems with important economic and environmental issues within the supply chain. The problem of public transport (pickup and delivery) with time windows and planned service lines (PDPTW- SL) consists in routing and programming a set of vehicles in the grant to scheduled public transport lines, to meet a set requests for goods in a timely manner on time windows. [4]. This paper presents, on the one hand, a mathematical formulation of the problem of vehicle routing built into the public transport with time windows and some existing resolution approaches. On the other hand, it focuses on the possibilities to use public transport available as part of the journey of goods from a route and a schedule. It is assumed that each vehicle of public transit considered operating according to routes and schedules predetermined, has a finite load capacity for requests for goods other than those available spaces for passengers. Therefore, the transfer of demand for goods to the available regular service lines (SLs) can benefit the entire transportation system. With the possibility of using SLs services, there can be two options for delivery to process transport requests. These include direct deliveries and indirect deliveries via SLs. The first implies that the origin and destination of points of an application are visited using a single pickup and delivery (PD) vehicle. The second type of delivery, which will be mentioned in this work, implies that a request is supported by a (PD) vehicle and transported to a transfer node. From there, demand continues course on regular lines. Then the request is resumed by another (PD) vehicle and must be delivered at the destination point [3]. A schematic overview of the example of this network is shown in Figure (1) showing three demands that have their pickup and delivery point close to two different transfer nodes [3].

Due to the fact that transport systems have a some rural or urban coverage, some PD vehicle routes can overlap with SLs services. So, use the transport public as a part of freight transport can entail costs and environmental benefits for the entire transportation system. For example, because of the reduced driving time of PD, logistic service providers may experience savings substantial operating costs. As a result, less travel time vehicles also leads to less emissions of equivalent carbon dioxide (CO_2e) worldwide for society as a whole. In addition, the use of regular lines for freight transport provide cost benefits for providers of public transport services as the use of the services SLs increases [3]. So, we proposed to use the scheduled line service that connects two transfer nodes instead of using a PD vehicle as in the systems of conventional pickup and delivery because scheduled line service contributes to improve travel time savings for vehicles with reduced mobility and we can expect reductions in operational costs with the system integrated presented. In

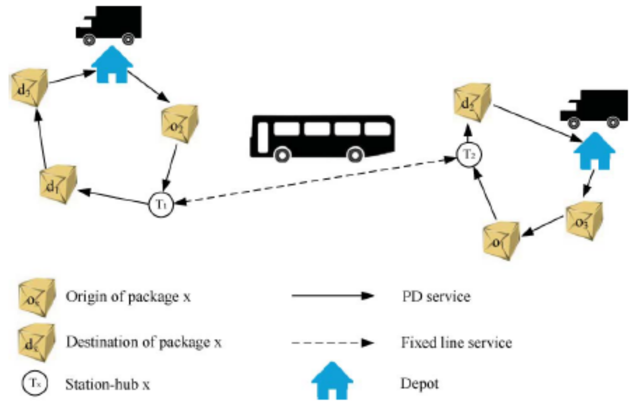

Figure 1: An illustration of the integrated public transport system.

this work, we arrive at a triple contribution: (i) formulating the PDPTW-SL as a mixed-integer program; (ii) describe the preprocessing step and present a family of effective inequalities to the proposed formulation; (iii) analyze the impact of preprocessing step and valid inequalities to the formulation with the data of instances of PDPTW-SL and interpret the experimental results.

2 Mathematical Modeling of the problem

In this section, we give a formal description of the PDPTW-SL, that is, we introduce the assumptions, definitions, and notations of the parameters, sets, and decision variables used in the formulation of the problem. The information about requests, travel time and slots hours are considered to be known in advance, and a possible transport plan for the entire planning horizon must be generated before carrying out transport activities. A solution problem is a routing plan and schedules for requests and PD vehicles [1]-[2]-[3].

Sets of the model:

$\mathcal{V}$: Set of PD vehicles and $\mathcal{O}$: Set of depots, $\mathcal{O} \equiv [1, \dots, \delta](i.e, o_v \in \mathcal{O}, v \in \mathcal{V})$.

$\mathcal{P}$: Set of pickup nodes, $\mathcal{P} \equiv [\delta+1, \dots, \delta+n]$ and $\mathcal{D}$: Set of delivery nodes, $\mathcal{D} \equiv [\delta+n+1, \dots, \delta+2n]$.

$\mathcal{T}$: Set of replicated transfer nodes, $\mathcal{T} \equiv [\delta+2n+1, \dots, \delta+2n+\tau]$ and $\mathcal{T}^k$: Set of replicated transfer nodes that represent the same physical transfer node as k.

$\mathcal{N}$: Set of all nodes in graph $\mathcal{G}$, $\mathcal{N} \equiv \mathcal{P} \cup \mathcal{D} \cup \mathcal{O} \cup \mathcal{T}$, $\mathcal{N}_1$: Set of request nodes $(\mathcal{P} \cup \mathcal{D})$ and $\mathcal{N}_2$: Set of request and replicated transfer nodes and $(\mathcal{P} \cup \mathcal{D} \cup \mathcal{T})$.

$\mathcal{E}$: Set of physical fixed lines which is defined as (i, j) with associated $\mathcal{K}^{ij}$ and k_{ij} and $\mathcal{K}^{ij}$: Set of indices for the departure times from the physical transfer

node i on fixed SL$(i,j) \in \mathcal{E}$.
$\mathcal{F}$: Set of replicated fixed lines which is defined as (i,j) with associated $\mathcal{K}^{\psi^i \psi^j}$, $\mathcal{F}^r$: Set of replicated fixed lines associated with request r and $\mathcal{F}^k$: Set of replicated fixed lines connected to the replicated transfer node k.
$\mathcal{F}^{ij}$: Set of replicated fixed lines associated with a physical fixed line SL$(i,j) \in \mathcal{E}$, $\mathcal{A}$: Set of arcs in $\mathcal{G}$ defined by $\mathcal{N} \times \mathcal{N}$ and $\mathcal{A}_1 = \mathcal{N}_2 \times \mathcal{N}_2 \backslash \mathcal{F}$

Parameters:
δ: Number of depots (i.e $|\mathcal{O}|$)
d_r: demand associated with request r
t_{ij}: Travelling time from node i to node j
s_i: Service time at node i
k_{ij}: Package carrying capacity on the fixed line (i,j)
Q_v: Freight carrying capacity of vehicle v
$[l_i, u_i]$: Time window at node i
p_{ij}^w: Departure time from node i, on the fixed line (i,j), indexed by w.
ψ^k: physical transfer node represented by replicated transfer node k
τ: The amount of replicated transfer nodes (i.e $|\mathcal{T}|$)

$$f_i^r = \begin{cases} 1 & \text{if node } i \text{ is the origin of request } r \\ 0 & \text{if node } i \text{ is an intermediate node }, \forall i \in \mathcal{N}_2 \backslash \{r, r+n\} \\ -1 & \text{if } i \text{ is the delivery node of } r \text{ (i.e,} r+n) \end{cases}$$

c_v: The routing cost of one time unit for vehicle v
η_{ij}: The cost of shipping one unit of parcel on the SL(i,j)
Decision variables:

$$x_{ij}^v = \begin{cases} 1 & \text{if arc } (i,j) \text{ s traversed by PD vehicle v} \\ 0 & \text{otherwise} \end{cases}$$

α_v: A continuous variable that indicates the time vehicle v returns to its depot, $\forall v \in \mathcal{V}$
β_i: A continuous variable that indicates the departure time of a PD vehicle from node i, $\forall i \in \mathcal{N}$ $y_{ij}^r = \begin{cases} 1 & \text{if arc } (i,j) \text{ is traversed by request r} \\ 0 & \text{otherwise} \end{cases}$
γ_i^r: A continuous variable that indicates the departure time of request r from node i, $\forall i \in \mathcal{N}_2$, $r \in \mathcal{P}$

$$q_{ij}^{rw} = \begin{cases} 1 & \text{if replicated } (i,j) \text{ is used by request } r \text{ that departs from } i \text{ at time } p_{ij}^w \\ 0 & \text{otherwise}, \forall r \in \mathcal{P},\ (i,j) \in \mathcal{F}^r,\ w \in \mathcal{K}^{\psi^i \psi^j} \end{cases}$$

3 The Predictive Optimal PDPTW-SL Modeling

After reviewing and ranking the quality of service criteria, The minimization of all the expenditures is applied. The PDPTW-SL is expressed in following mixed integer variable program "MIP":

$$\min \sum_{(i,j)\in\mathcal{A}}\sum_{v\in\mathcal{V}} c_v t_{ij} x_{ij}^v + \sum_{r\in\mathcal{P}}\sum_{(i,j)\in\mathcal{F}^r}\sum_{w\in\mathcal{K}^{\psi^i\psi^j}} \eta_{ij} d_r q_{ij}^{rw}; \tag{1}$$

$$\sum_{i\in\mathcal{N}}\sum_{v\in\mathcal{V}} x_{ij}^v = 1 \quad \forall j\in\mathcal{N}_1; \sum_{i\in\mathcal{N}_2} x_{o_v,i}^v \leq 1 \quad \forall v\in\mathcal{V}; \tag{2}$$

$$\sum_{i\in\mathcal{N}}\sum_{v\in\mathcal{V}} x_{ik}^v \leq 1 \quad \forall k\in\mathcal{T}; \sum_{j\in\mathcal{N}} x_{ij}^v - \sum_{j\in\mathcal{N}} x_{ji}^v = 0 \quad \forall i\in\mathcal{N}, v\in\mathcal{V}; \tag{3}$$

$$\sum_{j\in\mathcal{N}_2} y_{ij}^r - \sum_{j\in\mathcal{N}_2} y_{ji}^r = f_i^r \quad \forall r\in\mathcal{P}, i\in\mathcal{N}_2; \sum_{i\in\mathcal{N}}\sum_{v\in\mathcal{V}} x_{ik}^v \leq \sum_{r\in\mathcal{P}}\sum_{(i,j)\in\mathcal{F}^k} y_{ij}^r \quad \forall k\in\mathcal{T}; \tag{4}$$

$$\sum_{r\in\mathcal{P}} d_r y_{ij}^r \leq \sum_{v\in\mathcal{V}} Q_v x_{ij}^v \quad \forall (i,j)\in\mathcal{A}_1; \gamma_j^r \geq \gamma_i^r + t_{ij} + s_j \quad \forall r\in\mathcal{P}, (i,j)\in\mathcal{N}_2; \tag{5}$$

$$\beta_j \geq \beta_i + t_{ij} + s_j \quad \forall i\in\mathcal{N}, j\in\mathcal{N}_2; \tag{6}$$

$$\alpha_v \geq \beta_i + t_{i,o_v} + s_{o_v} \quad \forall i\in\mathcal{N}_2, v\in\mathcal{V}; \tag{7}$$

$$\beta_{r+n} \geq \beta_r + t_{r,r+n} + s_{r+n} \quad \forall r\in\mathcal{P}; l_i \leq \beta_i - s_i \leq u_i \quad \forall i\in\mathcal{N}_1; \tag{8}$$

$$l_{o_v} \leq \alpha_v \leq u_{o_v} \quad \forall v\in\mathcal{V}; \sum_{w\in\mathcal{K}^{\psi^i\psi^j}} q_{ij}^{rw} = y_{ij}^r \quad \forall r\in\mathcal{P}, (i,j)\in\mathcal{F}^r; \tag{9}$$

$$\gamma_i^r = p_{ij}^{rw} \quad \forall r\in\mathcal{P}, (i,j)\in\mathcal{F}^r, w\in\mathcal{K}^{\psi^i\psi^j}; \sum_{r\in\mathcal{P}}\sum_{(a,b)\in\mathcal{F}^{ij}} d_r q_{ab}^{rw} \leq k_{ij} \quad \forall (i,j)\in\mathcal{E}, w\in\mathcal{K}^{ij}; \tag{10}$$

$$\gamma_i^r = \beta_i \quad \forall r\in\mathcal{P}, i\in\mathcal{T}; \gamma_i^r = \beta_i \quad \forall r\in\mathcal{P}, i\in\mathcal{N}_1; \tag{11}$$

$$\gamma_{r+n}^r = \beta_{r+n} \quad \forall r\in\mathcal{P}; \gamma_k^r = \beta_k \quad \forall r\in\mathcal{P}, k\in\mathcal{T}, j\in\mathcal{T}^k; \tag{12}$$

$$x_{ij}^v \in \{0,1\} \quad \forall (i,j)\in\mathcal{A}, v\in\mathcal{V}; y_{ij}^r \in\{0,1\} \quad \forall (i,j)\in\mathcal{N}_2, r\in\mathcal{P}; \tag{13}$$

$$\alpha_v \in \mathbb{R}^+ \quad \forall v\in\mathcal{V}; \gamma_i^r \in \mathbb{R}^+ \quad \forall i\in\mathcal{N}_2, r\in\mathcal{P}; \tag{14}$$

$$\beta_i \in \mathbb{R}^+ \quad \forall i\in\mathcal{N}; q_{ij}^{rw} \in \{0,1\} \quad \forall r\in\mathcal{P}, (i,j)\in\mathcal{F}^r, w\in\mathcal{K}^{\psi^i\psi^j} \tag{15}$$

The equation (1) is the objective function to be minimized which encompasses the total cost of transporting PD vehicles and the cost of using SLs for transferring requests. The relations (2) - (5) are routing and flow constraints, the relations (6)-(10) are planning constraints. Relations (11) - (12) ensure synchronization, that is, coordination and consolidation of transportation. The coordination ensures the accuracy and timing of each branch of movement while the consolidation implies that the amount of goods transferred to the SLs matches the capabilities of the PDs and SLs vehicles and the relations (13) - (15) are integrity constraints and form the domain of decision variables.

4 Preprocessing step

A preprocessing is set up to reduce the number of decision variables in our model. As a result, some of the infeasible arcs of the graph shown in the figure 1 are removed. We are considering the following techniques for removing certain arcs[3].

- A vehicle can not leave and return to a depot other than its own.
- No vehicle may travel from the destination at the origin of the same

request.

- No PD vehicle can travel between the i and j nodes, if $(i, j) \in \mathcal{F}$.
- No request r can travel between the origin and destination of a fixed line other than an arc $(i, j) \in \mathcal{F}^r$.
- No request is allowed to travel to or from a depot.
- No flow is allowed between the nodes i and j if $l_i + s_i + t_{ij} > u_j$.
- No request can travel from its delivery node to another node.
- No request can go to its pickup node from another node.

In addition, some removal rules related to the transfer nodes can be applied. Arc (k_1, k_2) is infeasible if $l_r + s_r + t_{r,k_1} + s_{k_1} + t_{k_1,k_2} + s_{k_2} + t_{k_2,r+n} > u_{r+n}, \quad \forall r \in \mathcal{P}, (k_1, k_2) \in \mathcal{F}^r$. The number of departure times on the SLs for each request can be reduced by considering the time windows. For example, no request r can leave earlier than $(l_r + s_r + t_{r,k_1} + s_{k_1})$ on the SL(k_1, k_2), $\forall r \in \mathcal{P}, (k_1, k_2) \in \mathcal{F}^r$. Similarly, no request r can leave later than $u_{r+n} - s_{r+n} - t_{k_1,r+n} - s_{k_1} - t_{k_2,k_1} - s_{k_2}$ on the SL(k_2, k_1), $\forall r \in \mathcal{P}, (k_2, k_1) \in \mathcal{F}^r$

5 Tightening the model with valid cuts or inequalities [1]-[3]

As the proposed MIP formulation grows very quickly in size with the number of request, number of SLs and the number of start times on each line, some additional constraints (valid inequalities or slices) can be added to improve it. This section introduces three types of inequalities: *tighter flow, tighter vehicle maintenance, and repository reinforcement* that are valid for the PDPTW-SL.

$$\sum_{v \in \mathcal{V}} x_{ij}^v \geq y_{ij}^r \quad \forall (i, j) \in \mathcal{A}_1, \quad r \in \mathcal{P}. \tag{16}$$

$$\sum_{i \in \mathcal{N}} x_{ij}^v - \sum_{i \in \mathcal{N}} x_{r+n,i}^v \leq \sum_{(i,j) \in \mathcal{F}^r} y_{ij}^r \quad \forall r \in \mathcal{P}, \quad v \in \mathcal{V}. \tag{17}$$

$$\sum_{i \in \mathcal{N}} x_{ij}^v - \sum_{i \in \mathcal{N}} x_{r+n,i}^v \geq - \sum_{(i,j) \in \mathcal{F}^r} y_{ij}^r \quad \forall r \in \mathcal{P}, \quad v \in \mathcal{V}. \tag{18}$$

$$m \sum_{j \in \mathcal{N}_2} x_{o_v,j}^v - \sum_{i \in \mathcal{N}_2} \sum_{j \in \mathcal{N}_2} x_{ij}^v \geq 0 \quad \forall \quad v \in \mathcal{V}. \tag{19}$$

6 Numerical results

This section presents the results obtained by solving the MIP formulation of PDPTW-SL presented using the OPL Modeling Language combined with the optimization software CPLEX. The model is implemented on **Presario CQ57, Intel (R) Pentium (R) (2.20GHz-2.20GHz, 6.00GB RAM, 64-bit)** using the corresponding CPLEX 12.5 library. So we will show numerically, the preprocessing impacts on the model and valid inequalities proposed in previous section on operational costs. All of these instances gave optimal results with optimal variability (**GAP of 0.00%**).

Instance description: We have used two sets of instances namely R and RC each containing up to 12 requests, 4 heterogenous PD vehicles and 1 SL and differ in the distribution of the request nodes. In all cases, 2 depots are took into consideration. These instance sets are available on www.smartlogisticslab.nl [1-[3]. To simulate the model, we used 14 instances which are named in $G\_n\_sl\_v$ format, where G is the feature customer nodes, n is the number of requests to be served, sl is the quantity of SLs available and v is the number of PD vehicles available. We note that the cost driving time per unit time of all PD vehicles is assumed to be 0.5 units. The cost of each package request shipped on each SL is set at 1.5 unit, which includes the handling, storage and transportation costs. Figure 2 shows the evolution of the value of the objective

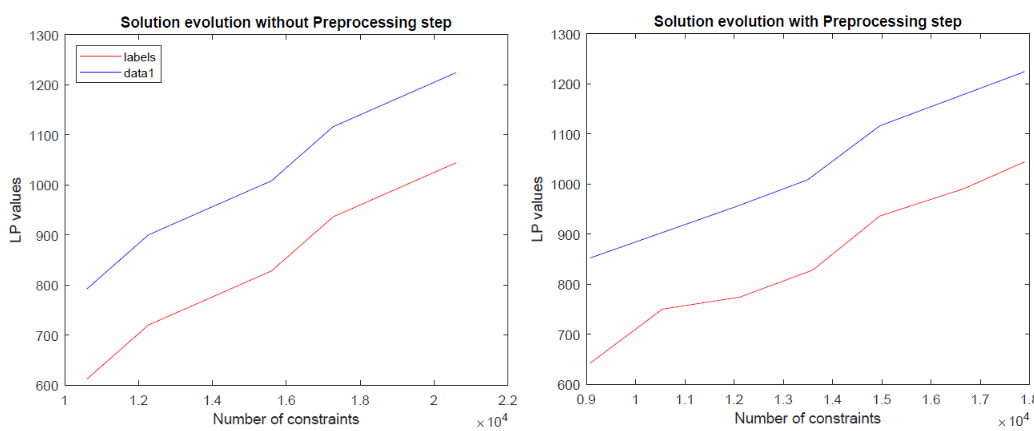

Figure 2: effect of preprocessing on the solution

function (LP values) according to the number of constraints and decision variables. The corresponding values are calculated with (right figure) and without the preprocessing (left figure) presented in the previous section. As shown at the same Figure , the results show significant reductions in terms of number of constraints and decision variables due to preprocessing. The average percentage of reductions for the number of constraints and variables is $13,37\%$ and 2% respectively. This result is consistent, the preprocessing is set up to reduce the number of constraints and decision variables of the model. It should also be noted that the use of the valid inequalities described by relations (16), (17), (18) and (19) does not modify or improve the value of the objective function. All the three inequalities are valid inequalities to the proposed formulation of the PDPTW-SL.

7 Conlusion

In this paper, the mathematical modelization of the problem of vehicle routing integrated with public transport with time windows and the possibility of using regular public transport had been developped. A mixed integer variables formulation was presented when programming the solving problem. The goal here is to minimize the total cost of transport of PD vehicles and the cost of using SLs for the transfer of requests, under routing and flow, planning, synchronization and integrity constraints. Two types of approaches from the existing literature are presented for the resolution of this problem. This is mainly the preprocessing step used to reduce the number of decision variables and constraints. The average percentages of reductions for the number of constraints and variables are $13,37\%$ and 2% respectively and the valid inequalities used to constrain the model that have no influence on the solution of the mathematical formulation of the proposed model. The model presented is however essential for further research. Future perceptives of this model concern the pursuit of our research for the methodologies of resolution, the development of specialized meta-heuristic algorithms that can generate good quality solutions with large instances to compare the effectiveness of these approaches in terms of quality of the optimal solution obtained and the computation time. The compromises between the number of SLs, the number of SLs vehicles rectified, the capacity and the frequency of SLs and their impact on the solution of the problem must be taken in perspective research.

References

[1] Ghilas V, Jean-Franois Cordeau, Demir Emrah, Van Woensel Tom, Branch-and-Price for the pickup and delivery problem with time windows and scheduled lines. *Transportation Science,* **52** (2018), 1191-1210. https://doi.org/10.1287/trsc.2017.0798

[2] Ghilas V, Demir E, Van Woensel T., An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows and scheduled lines, *Comput. Oper. Res.,* **72** (2016a), 12-30. https://doi.org/10.1016/j.cor.2016.01.018

[3] Ghilas V, Demir E, Van Woensel T., The pickup and delivery problem with time windows and scheduled lines, *INFOR: Inform. Systems Oper. Res.*, **54** (2) (2016b), 147-167. https://doi.org/10.1080/03155986.2016.1166793

[4] Shaw, P., Using constraint programming and local search methods to solve vehicle routing problems, in: Proceedings of the 4th International Conference on Principles and Practice of Constraint Programming, (1998), Springer, New York, 417-431. https://doi.org/10.1007/3-540-49481-2_30

[5] Dumas, Y., J. Desrosiers, F. Soumis, The pickup and delivery problem with time windows, *Eur. J. Oper. Res.,* **54** (1) (1991), 7-22. https://doi.org/10.1016/0377-2217(91)90319-q

[6] Cordeau, J.-F., G. Laporte, J.-Y. Potvin, M. W. P. Savelsbergh, Transportation on demand, C. Barnhart, G. Laporte, eds., Transportation, Elsevier, Amsterdam, (2007), 429-466. https://doi.org/10.1016/s0927-0507(06)14007-4

[7] Baldacci R., Enrico Bartolini, Aristide Mingozzi, An Exact Algorithm for the Pickup and Delivery Problem with Time Windows, *Operations Research,* **59** (2) (2011) 414-426. https://doi.org/10.1287/opre.1100.0881

[8] Liaw C.-F., White C., Bander J., A decision support system for the bimodal dial-a-ride problem, *IEEE Trans. Syst. Man Cybern. A.,* **26** (1996), 552-565. https://doi.org/10.1109/3468.531903

[9] Berbeglia G., Cordeau J.-F., Gribkovskaia I., Laporte G., Static pickup and delivery problems: a classification scheme and survey, *TOP: an Official Journal of the Spanish Society of Statistics and Operations Research,* **15** (2007), 1-31.https://doi.org/10.1007/s11750-007-0009-0

[10] Masson R., Trentini A., Lehuede F., Malhene N., Peton O., Tlahig H., Optimization of a city logistics transportation system with mixed passengers and goods, *Eur. J. Transportation Logist.,* **6** (1) (2017), 81-109. https://doi.org/10.1007/s13676-015-0085-5

[11] Trentini A., Malhene N., Toward a shared urban transport system ensuring passengers and goods cohabitation, *Trimestrale del Laboratorio Territorio Mobilita e Ambiente - TeMALab*, **4** (2010), 37-44.

Received: December 19, 2019; Published: August 28, 2020