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Does Transition to Democracy Lead to Chaos: A Theorem

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Abstract

When a country transitions to democracy, at first, many political parties appear. A natural question is whether the number of such parties feasible and reasonable – or whether this is a complete chaos. In this paper, we formulate a simplified version of this question in precise terms and show that the number of parties will be feasible – i.e., that transition to democracy does not lead to chaos.

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1 What Happens During Transition to Democracy

Before the transition. Before the transition, only a relatively small number of people can affect what is happening in the country. The corresponding group of people may change, one group may overthrow and replace another one – but at any given moment of time, in any specific situation, only a small group of people decides.

Others have no power to change anything. To be more precise, each of them cannot change anything on his/her own. If many people were able to get together, they would be able to change the society – but they are prevented by the powers-to-be from acting together.

Then, the transition comes. Eventually, in spite of the oppression, people get together, overthrow the oppressive regime, and establish democracy.

Does it lead to chaos? After the transition to democracy, many different interest groups form their own political parties or factions. As a result, we have a huge number of different political players – many more than in well established democracies.

An important question is: does this transition lead to a somewhat large but still reasonable and feasible number of parties – or does it leads to a non-feasible, unreasonably large number, i.e., to chaos, and the only thing we can do is wait for a more feasible political structure to emerge?

What we do in this paper. In this paper, we formulate this question in precise terms; of course, as usual with formalizations, this becomes possible if we simplify the situation.

Then, by analyzing this precise (simplified) model, we produce an answer to this question. The answer is: no, transition to democracy does not lead to chaos.

2 Towards the Formulation of the Problem in Precise Terms: A Simplified Model

Let us simplify the situation. In real life, we have a whole spectrum of decisions. In general, each decision means selecting one of *many* possible alternatives. The simplest possible decisions are when we only have *two* alternatives to select from, e.g. when we either approve some change or not. We can describe each such a decision by a propositional (= boolean = yes-no = true-false) variable d.

This country-affecting decision is based on the opinions and actions of all the people of the country. Again, in general, each person can perform many different actions and can have many different opinions. The simplest possible case is when each person has only two choices of action – e.g., to act in a certain way or not. Thus, in this simplified model, the action of the i-th person can be also described by a propositional variable a_i.

The overall country-wide decision is determined by all these actions, i.e., by the values of all these variables $a_1, \ldots, a_n$, where n is the number of people in the country. Let us denote the decision corresponding to the tuple $(a_1, \ldots, a_n)$ by $f(a_1, \ldots, a_n)$. In mathematical terms, this means that we have a function that assigns a Boolean value to each such tuple.

What does absence of democracy means in these terms. As we mentioned earlier, the absence of democracy means that only a few people can affect the resulting decision when acting on their own. Let $m \ll n$ be the overall number of such decision-affecting people.

The ability of the i-th person to change a decision means that if this person changes his/her action a_i to the opposite one $\neg a_i$, then the decision will change, i.e., that we will have

$$f(a_1, \ldots, a_{i-1}, \neg a_i, a_{i+1}, \ldots, a_n) \neq f(a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_n). \quad (1)$$

So, the absence of democracy means that for each tuple $(a_1, \ldots, a_n)$, there are no more than m indices i for which the inequality (1) is satisfied.

What happens after the transition to democracy. After this transition, people start forming political parties. In precise terms, this means that in the set $\{1, \ldots, n\}$ of all the people, several mutually disjoint groups $g_1, \ldots, g_c$ are formed. Of course, for each situation $(a_1, \ldots, a_n)$, it makes sense to only consider parties that have some influence, i.e., parties g_j for which some change of the corresponding values a_i for $i \in g_j$ can change the decision $f(a_1, \ldots, a_n)$. The question is: will the number c of such parties feasible?

How to formalize what is feasible. In theoretical computer science, there is a formalization of feasible, which applies to feasible computation time (= number of computational steps), feasible length (= number of symbols) of the answer, feasible number of processors, etc. Namely, a number is called feasible if it is bounded by a polynomial of the input. This is not a perfect formalization, but it is the best one available; see, e.g., [1, 4, 5, 6].

Now, we can formulate the question in precise terms.

Formulation of the question in precise terms. Does there exist a polynomial $P(m)$ such that for every function $f(a_1, \ldots, a_n)$:

- if for each tuple $(a_1, \ldots, a_n)$, no more than m indices have the property (1),
- then for each tuple, we can have no more than $P(m)$ disjoint sets

  $$g_1, \ldots, g_j, \ldots \subseteq \{1, \ldots, n\}$$

  for which , for each j, some change of the values a_i for $i \in g_j$ can change the decision $f(a_1, \ldots, a_n)$.

Comment. In our formalization:

- If such a polynomial exists, this means that the situation is feasible.
- If no such polynomial exists, then transition to democracy can indeed lead to a non-feasible number of political parties, i.e., to chaos.

3 Solution of the Problem

The solution to this precisely formulated problem is known. Interestingly, the solution to the above question is known. To explain this solution, let us introduce a few definitions.

For any boolean function $f(a_1, \ldots, a_n)$ and for any tuple $a = (a_1, \ldots, a_n)$, the number of indices i for which the inequality (1) holds is known as the *sensitivity of* f *at* a. For each boolean function $f(a_1, \ldots, a_n)$, its *sensitivity* $s(f)$ is defined as the largest of the sensitivities at different tuples. In these terms, the condition of the question is that $s(f) \le m$.

For any boolean function $f(a_1, \ldots, a_n)$ and for any tuple $a = (a_1, \ldots, a_n)$, the largest number c of disjoint sets $g_1, \ldots, g_j, \ldots \subseteq \{1, \ldots, n\}$ for which, for each j, some change of the values a_i for $i \in g_j$ can change the decision $f(a_1, \ldots, a_n)$ is known as the *block sensitivity of* f *at* a. For each boolean function $f(a_1, \ldots, a_n)$, its *block sensitivity* $bs(f)$ is defined as the largest of the sensitivities at different tuples.

In these terms, the question is whether there exists a polynomial $P(m)$ for which, for all boolean function $f(a_1, \ldots, a_n)$, we have $bs(f) \le P(s(f))$. The hypothesis that such a polynomial exists is known as the *sensitivity hypothesis*. This hypothesis was an open problem for more than 30 years, but in 2019, it was proven to be true; see [2, 3].

Conclusion. Transition to democracy does not lead to chaos – at least within our simplified model.

Open problem. What will happen if we introduce a more complex – and more realistic – formalization of the question, e.g., by allowing each participant to have several possible actions?

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