Applied Mathematical Sciences, Vol. 14, 2020, no. 15, 711 - 715
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ams.2020.914261

The Similarity Between Earth's and Mars's Core-Mantle Boundary Seems to Be Statistically Significant

Laxman Bokati, Olga Kosheleva, and Vladik Kreinovich

University of Texas at El Paso
500 W. University
El Paso, TX 79968, USA

Abstract

Latest, most accurate measurements of the depth of the Mars's core-mantle boundary shows that the ratio between this depth and Mars's radius is the same as for the Earth – and with new measurements, this coincidence has become statistically significance. This coincidence seems to confirm a simple scale-invariant model in which for planets of Earth-Mars type, this depth is proportional to the planet's radius. Of course, we need more observations to confirm this model, but the fact that, for the first time, we got a statistically significant confirmation, is encouraging: it makes us believe that this coincidence is not accidental.

Mathematics Subject Classification: 85A99, 86A15, 86A22, 62P35

Keywords: scale-invariant dependencies, statistical significance, core-mantle boundary, Mars

1 Formulation of the Problem

What is the internal structure of the planets. Planets usually have a super-heated *core* surrounded by mostly solid *mantle*, above which lies a thin outer layer, the *crust*.

How the border between the layers is determined. Boundaries between different layers are usually very crisp, since they correspond to phase transitions. For example, the mantle is mostly solid, while the core – at least the outer core that borders the mantle – is mostly liquid.

Thus, seismic waves reflect from these borders, providing us with a good understanding of where this border is located.

For which celestial bodies do we know the core-mantle boundary. Of course, since we have been measuring the effect of many strong earthquakes for quite some time, we have a very good understanding of the location of the Earth's core-mantle boundary: it is located at the depth of approximately 2890 km. For reference, the Earth's radius is 6571 km.

Spaceships have brought seismometers to Moon and Mars. On the Moon, the core is at the depth of 3140±20 km out of the 3474 km radius. For Mars, previous data showed that this boundary is at the depth of 1591±65 km [4, 5]; latest data gives a more accurate depth of 1520±40 km [1]. Mars's radius is 3389.5 km.

What can we conclude from this data? OK, Moon is not a planet, it is different, but based on the information about Earth and Mars, what can we conclude?

Of course, based on only two example, it is difficult to make generalizations, but maybe some conclusions can still be made?

What we do in this paper. In this paper, we show that there is a similarity between the depths of the Earth's and Mars's core-mantle boundaries, and that these similarities seem to be statistically significant.

2 Simple Model of Core-Mantle Boundary Depth

What we want. We want to have a formula that, given a planet's radius r, would predict the depth d of its core-mantle boundary. Let us denote the desired expression by $d = f(r)$.

Scale-invariance: a natural idea. Numerical values of physical quantities such as radius and depth depend on the choice of a measuring unit: we can measure them in km, we can measure them in meters, we can measure them in miles, etc. If we replace the original measuring unit by a unit which is λ times smaller, then all numerical values are multiplied by λ: instead of the original value x, we have a new value $x' = \lambda \cdot x$.

The change of a measuring unit changes the scale, so this transformation is known as *scaling*; see, e.g., [2, 7].

In many physical situations, there is no reason to prefer one or another unit of length. In such situations, it is reasonable to require that the dependence between two physical quantities should retain the same form, no matter what units we use. Such dependencies are known as *scale-invariant*.

Towards the resulting simple model. So maybe the dependence of the core-mantle depth d on the planet's radius r is scale-invariant? That would mean that if we have $d = f(r)$ and we use a different measuring unit for measuring distance, i.e., replace d with $d' = \lambda \cdot d$ and r with $r' = \lambda \cdot r$, then, in the new units, we will also have $d' = f(r')$.

Substituting the expressions $d' = \lambda \cdot d$ and $r' = \lambda \cdot r$ into the formula $d' = f(r')$, we get $\lambda \cdot d = f(\lambda \cdot r)$. Since $d = f(r)$, we get

$$\lambda \cdot f(r) = f(\lambda \cdot r).$$

In particular, for $r = 1$ and $\lambda = z$, we get $f(z) = c \cdot z$, where we denoted $c \stackrel{\text{def}}{=} f(1)$. So, we arrive at the following model:

$$d = c \cdot z. \tag{1}$$

3 Observations Seem to Confirm This Model

Confirmation. From the Earth data, we get $c = 2890/6571 \approx 0.440$.

For Mars, the radius is $r = 3389.5$. Based on the latest Mars data, $d = 1520 \pm 40$. Here, $1520/3389.5 \approx 0.448$ and $40/3389.5 \approx 0.012$, so, based on the Mars data, $c = 0.448 \pm 0.012$.

The values are consistent with each other: the Earth's value 0.440 is in the interval $[0.448 - 0.012, 0.448 + 0.012] = [0.436, 0.460]$.

Is this coincidence statistically significant? Statistical significance usually means that the probability of random coincidence is smaller than 0.05, which corresponds to the two-sigma interval; see, e.g., [6]. For latest Mars measurements, $\sigma = 0.012$, so the two-sigma interval is

$$[0.448 - 2 \cdot 0.012, 0.448 + 2 \cdot 0.012] = [0.424, 0.472].$$

The width of this interval is 0.048.

A priori, the Mars ratio could be any value from the interval $[0, 1]$. There is no reason to assume that some of these values are more probable and some are less probable, so a reasonable idea is to have a prior distribution for which all the values have equal probability – i.e., the uniform distribution of the interval $[0, 1]$; see, e.g., [3]. For this uniform distribution, the probability to be within each interval equal to this interval's length. Thus, the probability that the Earth's value falls within this interval is $0.048 < 0.05$.

So, the coincidence is indeed statistically significant.

Only the latest most accurate data make this coincidence statistically significant. Could we make the same conclusion based on the previously known less accurate estimate for Mars's depth of the core-mantle boundary? Let us check.

The previous Mars data was $d = 1591 \pm 65$. Here, $1591/3389.5 \approx 0.469$ and $65/3389.5 \approx 0.019$, so $c = 0.472 \pm 0.019$. Here, the two-sigma interval

$$[0.469 - 2 \cdot 0.019, 0.469 + 2 \cdot 0.019] = [0.421, 0.507]$$

also contains the Earth value 0.440, but the width of this interval is 0.076, so the probability of a number accidentally falling into this interval is $0.076 > 0.05$ – not statistically significant.

Acknowledgments. This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), HRD-1834620 (CAHSI Includes), and HRD-1242122 (Cyber-ShARE Center of Excellence).

References

[1] S. Deng and A. Levander, Autocorrelation reflectivity of Mars, *Geophysical Research Letters*, **47** (2020), paper e2020GL089630.
https://doi.org/10.1029/2020gl089630

[2] R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Addison Wesley, Boston, Massachusetts, 2005.

[3] E. T. Jaynes and G. L. Bretthorst, *Probability Theory: The Logic of Science*, Cambridge University Press, Cambridge, UK, 2003.
https://doi.org/10.1017/cbo9780511790423

[4] F. Nimmo and K. Tanaka, Early Crustal Evolution of Mars, *Annual Review of Earth and Planetary Sciences*, **33** (1) (2005), 133–161.
https://doi.org/10.1146/annurev.earth.33.092203.122637

[5] A. Rivoldini, T. Van Hoolst, O. Verhoeven, A. Mocquet, and V. Dehant, Geodesy constraints on the interior structure and composition of Mars, *Icarus*, **213** (2) (2011), 451–472.
https://doi.org/10.1016/j.icarus.2011.03.024

[6] D. J. Sheskin, *Handbook of Parametric and Non-Parametric Statistical Procedures*, Chapman & Hall/CRC, London, UK, 2011.
https://doi.org/10.4324/9780203489536

[7] K. S. Thorne and R. D. Blandford, *Modern Classical Physics: Optics, Fluids, Plasmas, Elasticity, Relativity, and Statistical Physics*, Princeton University Press, Princeton, New Jersey, 2017.

Received: August 13, 2020; Published: September 3, 2020