

Under Limited Resources, Lottery-Based Tutoring Is the Most Efficient

Olga Kosheleva ¹, Christian Servin ², and
Vladik Kreinovich ³

¹ Department of Teacher Education

³ Department of Computer Science

University of Texas at El Paso

500 W. University

El Paso, TX 79968, USA

² Computer Science and

Information Technology Systems Department

El Paso Community College (EPCC)

919 Hunter Dr., El Paso, TX 79915-1908, USA

This article is distributed under the Creative Commons by-nc-nd Attribution License.
Copyright © 2020 Hikari Ltd.

Abstract

In the ideal world, every student who needs tutoring should receive intensive one-on-one tutoring. In practical, schools' resources are limited, so the students get only a portion of needed tutoring. It would have been not so bad if, e.g., half-time tutoring would be half as efficient as the intensive one. However, research shows that half-time tutoring is, on average, 15 times less efficient – and, e.g., for math tutoring 20 times less efficient. To increase the efficiency, we propose to randomly divide the students who need tutoring into equal-size groups, and each year (or each semester) provide intensive tutoring to only one of these groups. This will drastically increase the efficiency of tutoring. Even larger efficiency can be attained if we determine, for each student, the optimal number of tutoring per week – the formulas for this determinations are provided – and distribute the resources accordingly.

Mathematics Subject Classification: 97B10, 91C99, 90B30

Keywords: tutoring, limited resources, optimal schedule, randomization

1 Formulation of the Problem

Need for tutoring. In spite of all the teachers' efforts, not everyone is doing well in schools. Some students start lagging behind in different disciplines. A known way to help these students catch up is *tutoring*, when:

- in addition to regular classes, with one teacher for many students,
- these students also have additional classes which are taught either one-on-one or by one instructor to a small group of students.

Our resources are limited. In the ideal world, every student who lags behind should get as much individual or small-group tutoring as needed for them to catch up. However, most schools do not have enough instructors for this: the school's resources are limited.

How are limited resources distributed now. Schools try to be fair. So, the available resources are usually distributed equally between all students who need tutoring – with a special emphasis on students whose parents cannot afford to hire extra tutors.

The resulting limited tutoring does not work well. The idea of fair distribution of resources seems right at first glance, but empirical studies show that the results are not very good.

As we have mentioned, intensive tutoring – at least four times a week – is usually successful: the students' grades rise from the failing to acceptable level. One would expect that if we replace intensive with a lighter – e.g., two times a week – tutoring, we will get approximately half $0.5 \cdot g$ of the needed grade increase g .

We wish this was true. In reality, as a Harvard study [1] shows, if we replace intensive tutoring with the half-as-intensive one, the efficiency decreases 15 times, to an almost indistinguishable grade change of $\approx 0.07g$.

In math tutoring, the result is even worse – there is a 20 times decrease, to $\approx 0.05g$.

So what should we do? Of course, the best solution is to view this study as a battle cry, to make sure that a sufficient amount of resources is allocated to schools, so that every student who needs tutoring will get intensive tutoring.

But what should we do at present, when these resources are not yet available? Shall we continue the current inefficient practice – or shall we try to come up with a more efficient way of spending the available tutoring resources?

What we do in this paper. In this paper, we show that it is indeed possible to spend the limited tutoring resources much more efficiently – by replacing limited tutoring for all with intensive tutoring for randomly selected students. At first, this may sound as if we doom the non-selected students to failure, but, as we will explain, this is not the case: in the proposed new scheme, everyone will benefit.

2 Main Idea

Observation that led to our idea. Our idea comes from the fact that, in general, the same students need tutoring semester after semester, year after year.

In view of the above-mentioned Harvard study, this fact is quite understandable: the current light tutoring is not efficient. So, in spite of this (light) tutoring, students fall further and further behind.

Resulting idea: explanation. Light tutoring every year increases the student's grade by $\approx 0.07g$. So, two years of light tutoring increase the grade by $\approx 0.14g$.

Light tutoring takes half of the time needed for intensive tutoring. So:

- instead of performing light tutoring for two years,
- by using the same resources, we can provide intensive tutoring for one year.

If we do it, during this year, the student's grade will increase by the desired value g – so this student will catch up.

This is not a perfect solution: if we only tutor half of the students the first year, these students will, in general, progress, while the other half will do even worse than they are doing now. To be fair, we should therefore select students-to-be-tutored randomly, by a lottery – just like in some locations, a lottery decided which of many almost equally qualified kids gets accepted into the best schools.

This is not as bad as being placed for the whole school life into a not-so-good school: in our cases, students who are not selected for tutoring the first year will be tutored the next year anyway, so they will have a chance to catch up.

Resulting idea: details. If we only have half of the resources needed to provide full tutoring to all the students who are in need of tutoring, then we randomly divide these students into two equal groups;

- the students from the first group get full tutoring during the first year, and no tutoring the second year, while
- the students from the second group get no tutoring in the first year, but full tutoring in the second year.

If we only have one-third of the needed tutoring resources, then we randomly divide the students into three equal groups:

- the students from the first group get full tutoring during the first year, and no tutoring the second and third years;
- the students from the second group get no tutoring in the first year, full tutoring in the second year, and no tutoring in the third year;
- the students from the third group get no tutoring in the first two years, but full tutoring in the third year.

Similarly, if we have only one-fourth of the needed resources, we divide the students into four groups, etc.

3 What Is the Optimal Strategy?

Can we make tutoring even more efficient? In the previous section, we considered only two options: full tutoring and light tutoring that only uses one half (or less) of the full tutoring time. Maybe by using different amounts of tutoring, we can get even more efficient results? To answer this question, let us recall how efficiency of tutoring depends on the tutoring time.

Learning curve: reminder. The empirical dependence of the weekly grade increase g resulting from tutoring (of from other types of learning) on the time t spent on this learning, is known as the *learning curve*. Nowadays, this term is used in a qualitative sense – like when we say that some topics have a very steep learning curve – but originally, it means the quantitative dependence.

The most accurate description of empirical learning curves is provided by the logistics formula

$$g = \frac{g_0}{1 + \exp(-k \cdot (t - a))}, \quad (1)$$

where parameters g_0 , k , and a describe individual students; see, e.g., [2].

So what is the optimal strategy: analysis of the problem. Suppose that each year, we can denote time T to tutoring for each student who needs tutoring, and that we are considering schools in which education lasts for Y years – e.g., for US high schools, $Y = 4$. Thus, during the whole education period, we can allocate tutoring time $T \cdot Y$ to each student.

What is the best way to distribute this amount between different weeks? If we allocate t hours per week for tutoring, then we can afford to provide this tutoring for $\frac{T \cdot Y}{t}$ weeks. During each week, the grade increase is described by the formula (1). Thus, the overall grade increase based on all this tutoring will be equal to

$$\frac{T \cdot Y}{t} \cdot \frac{g_0}{1 + \exp(-k \cdot (t - a))} = \frac{T \cdot Y \cdot g_0}{t \cdot (1 + \exp(-k \cdot (t - a)))}. \quad (2)$$

To find the most efficient arrangement, we need to find the value t that maximizes this expression.

This expression attains its largest possible value if its denominator

$$t \cdot (1 + \exp(-k \cdot (t - a))) \quad (3)$$

is the smallest possible. Differentiating the expression (3) with respect to time and equating the derivative to 0, we conclude that

$$1 + \exp(-k \cdot (t - a)) + t \cdot (-k) \cdot \exp(-k \cdot (t - a)) = 0, \quad (4)$$

i.e.,

$$1 + \exp(-k \cdot (t - a)) = (k \cdot t) \cdot \exp(-k \cdot (t - a)). \quad (5)$$

Here, $\exp(-k \cdot (t - a)) = \exp(-(k \cdot t)) \cdot \exp(k \cdot a)$. Thus, the equation (5) can be simplified if we denote $z \stackrel{\text{def}}{=} k \cdot t$ and $C \stackrel{\text{def}}{=} \exp(k \cdot a)$. In these notations, the equation (5) has the form

$$1 + C \cdot \exp(-z) = z \cdot \exp(-z). \quad (6)$$

If we multiply both sides by $\exp(z)$, we get an even simpler equation

$$\exp(z) + C = z, \quad (7)$$

i.e., equivalently,

$$z - \exp(z) = C. \quad (8)$$

Thus, we arrive at the following solution.

Optimal strategy: description. For each student:

- First, we find the parameters a and k describing this student's reaction to tutoring.
- Then, we compute the value $C = \exp(k \cdot a)$ and find the solution z of the equation (8) corresponding to this value t .
- Finally, we compute $t = \frac{z}{a}$.

This is the optimal amount of tutoring per week for this particular student. When the amount of resources per student is $T \cdot Y$, we provide the tutoring for

$$\frac{T \cdot Y}{t}$$

weeks.

Acknowledgments. This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), HRD-1834620 (CAHSI Includes), and HRD-1242122 (Cyber-ShARE Center of Excellence).

References

- [1] R. G. Fryer, Jr., *The Production of Human Capital in Developed Countries: Evidence from 196 Randomized Field Experiments*, National Bureau of Economic Research (NBER) Working Paper No. 22130, March 2016. <https://doi.org/10.3386/w22130>
- [2] F. E. Ritter and L. J. Schooler, The learning curve, in: *International Encyclopedia of the Social and Behavioral Sciences*, Pergamon, Amsterdam, 2002, pp. 8602–8605. <https://doi.org/10.1016/b0-08-043076-7/01480-7>

Received: August 12, 2020; Published: September 3, 2020