Applied Mathematical Sciences, Vol. 14, 2020, no. 15, 725 - 736
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ams.2020.914265

# An Inventory Model for Deteriorating Items with a Generalised Exponential Increasing Demand, Constant Holding Cost and Constant Deterioration Rate

**I. Aliyu**

Department of Mathematics and Statistics
Kaduna Polytechnic, Kaduna, Nigeria

**B. Sani**

Department of Mathematics
Ahmadu Bello University, Zaria, Nigeria



**Abstract**

This paper proposes a deteriorating inventory model with a generalised exponential increasing demand where the deterioration rate is a time-varying linear function of time and the holding cost is constant. Shortages are not allowed. A numerical example is presented to demonstrate the application of the model and sensitivity analysis is carried out to see the effect of parameter changes on the solution.

**Keywords:** Constant Holding Cost, Constant Deterioration Rate, Generalised Exponential Increasing Demand, Inventory.

## 1.0 INTRODUCTION

Inventory is defined broadly as any stock of economic resources that is idle at a given point in time. This may include raw materials awaiting use in manufacturing operations, semi-finished goods temporarily stored during manufacturing process, finished goods awaiting distribution and finished goods stocked in wholesale or retail outlets for selling to customers. Inventory may also include such nonphysical assets as cash, inventories of accounts receivable, or inventories of human resources, lee and moore (1975).

Inventory analysis was one of the first areas of application of decision science models. An economic lot-size equation was first developed by F. E. Harris in 1915. Models of inventory systems were further developed in the early 1930's, (see Raymod, 1930).

Generally, deterioration items refer to the items that become damaged, decayed, spoiled, lost their utility or lost their marginal value, becomes invalid and so on through time, Wee (1993).

The proposed model is an inventory model for a deteriorating item which has a time-dependent generalised exponential increasing demand rate and time dependent, linear deterioration rate with a constant holding cost. In the case of increasing demand, the demand of a product may decrease with time due to the introduction of a new product which is either technically superior or otherwise more attractive and cheaper than the old one. The demand of the new product, in this case, will increase with time. Most food products, chemicals and electronic components will fall into this category.

Quite a good number of researchers are involved in developing inventory models for items which are deteriorating with time. Among them include Whitin (1957), who considered deterioration of fashion goods at the end of prescribed storage period. Ghare and Schrader (1963) extended the classical Economic order quantity (EOQ) formula to cover exponential decay of inventory due to deterioration and gave a Mathematical model for the inventory. Their model used a constant deterioration rate, while demand rate was taken to be constant. Shah and Jaiswal (1977) presented an order-level inventory model for deteriorating items with a constant rate of deterioration. Later, Aggarwal (1978) developed an order-level inventory model by correcting and modifying the error in Shah and Jarwal's analysis (1977) in calculating the average inventory holding cost. Dave and Patel (1981) developed the first deteriorating inventory model with linear trend in demand. They considered demand as a linear function of time. The model was then extended by Sachan (1984) to cover the backlogging option. Bahari-Kashani (1989) studied the inventory replenishment policy for items when demand is time-proportional and inventory deteriorates at a constant rate $\theta$ over time. Jalan and Chaudhuri (1999) developed their model taking exponentially time varying demand pattern. Bhunia and Maiti (1999) developed a deterministic inventory model over a finite planning horizon with constant rate of deterioration, linearly increasing demand, and complete backlogging of the excess demand. Teng et al (2005) presented an EOQ model for deteriorating items with power form stock-dependent demand in which they extended their EOQ model to allow for not only deteriorating items but also non-zero ending inventory. Mishra and Singh (2011) presented a related article in which they developed an inventory model for deteriorating items with uniform replenishment rate having power form demand and without shortages. They considered the rate of deterioration as a cubic polynomial function of time. A total cost function was constructed and a computing algorithm developed to find the solution of the developed model. Some of the recent models developed include Kumar and Saini (2012), who developed a two-ware house inventory model with partial backordering and Weibull distribution deterioration. They considered inflation and also used discounted cash flow in problem analysis. Singh and Pattnyak (2013) presented an EOQ model for a deteriorating item with time-dependent quadratic demand and variable deterioration, under permissible delay in payment. Some recent works also include Dash et al (2014), who established an inventory model for deteriorating items where they considered exponential declining demand and time-varying holding cost. Aliyu and Sani (2016) presented an inventory model for deteriorating items with generalised exponential decreasing demand and linear time-varying holding cost. The rate of deterioration was considered to be constant and shortages were not allowed. Again Aliyu and Sani (2018) developed an inventory model for deteriorating items with generalised exponential decreasing demand, constant holding cost and time-varying deterioration rate. Shortages were not considered.

The difference between this paper and that of Aliyu and Sani (2018) is that in this paper, we consider the demand to be exponentially increasing while in that of Aliyu and Sani (2018), the demand was considered to be exponentially decreasing and also the deterioration rate was considered to be constant.

## 2.0 ASSUMPTIONS AND NOTATION

In formulating the mathematical model, the following notation and assumptions are used.

### 2.1 Assumptions

i. The inventory system considers a single item only.

ii. The demand rate is deterministic and is a generalised exponential increasing function of time.

iii. The deterioration rate is considered to be constant.

iv. Lead time is zero.

v. Shortages are not allowed.

vi. The inventory system is considered over an infinite time horizon.

**Notation**

$A_0$: The fixed ordering cost per order

$I(t)$: The inventory at any time $t, 0 \leq t \leq T$

$D(t)$: The exponential demand rate, where $D(t) = ke^{h+\gamma t}$, with $k > 0, \gamma > 0, h > 0$, and are all constants.

The deterioration rate is a constant given as $\theta$

The holding cost which is a constant is given as iC where C is the unit cost of an item and i is the inventory carrying charge.

$A_c$: The cost of each deteriorated unit.

$T$: The length of the ordering cycle.

$I_O$: Initial stock.

$TC$: The total cost per unit time.
$T^*$: The optimal length of the cycle.
$I_o^*$: The economic order quantity
$TC^*$: The minimum total cost per unit time.

The difference between the work in Aliyu and Sani (2018) and this work is that in Aliyu and Sani (2018) the demand was considered to be exponentially decreasing while in this work, we consider the demand to be exponentially increasing. Moreover in Aliyu and Sani (2018) the deterioration rate is linear while in this paper, the deterioration rate is a constant.

The figure below shows the demand levels with various values of h.

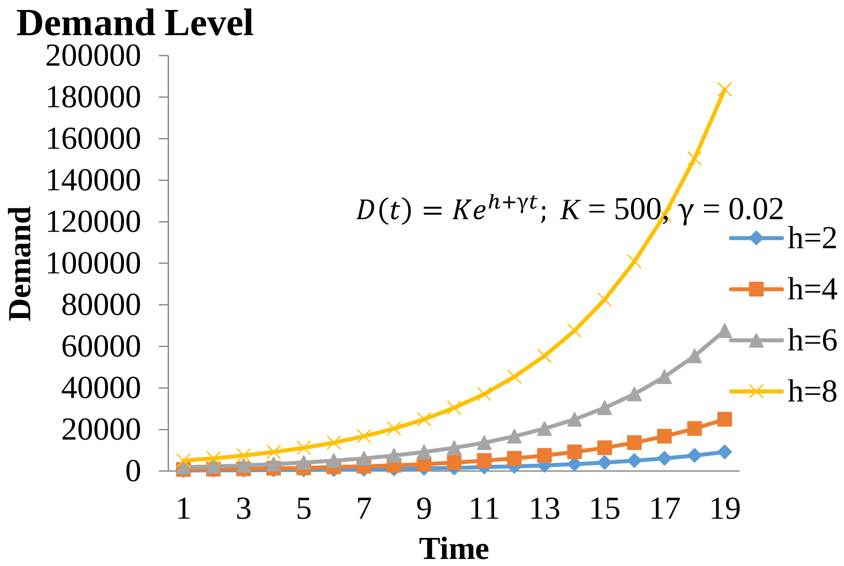


Figure 1: Graphical Representation of Various Demand Levels having Different h Values

As we can observe from Figure 1 above, as the values of h increases, the demand also increases.

## 3.0 MATHEMATICAL MODEL AND ANALYSIS

Using the assumptions above, a typical cycle for the variation of inventory level with time is shown in Figure 2 below.

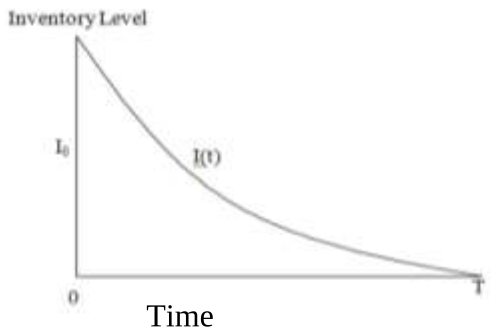


Figure 2: The Graphical Representation for the Inventory System

From the figure above, one can see the inventory level as it gradually decreases from initial stage due to both demand and deterioration. The differential equation which describes the state of inventory levels at any time, t, represented as I(t) in the interval $[0, T]$, is given by;

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t), \; 0 \le t \le T \qquad (1)$$

where $D(t) = Ke^{h+\gamma t}$

Evaluating equation (1) with boundary conditions $I(0) = I_0$ and $I(T) = 0$ gives the solution as follows:

$$\frac{dI(t)}{dt} + \theta I(t) = -Ke^{h+\gamma t}$$

The integrating factor $= e^{\int \theta dt} = e^{\theta t}$

Thus

$$I(t)e^{\theta t} = -K\int_0^T e^{h+\gamma t}.e^{\theta t}dt$$
$$= \frac{-K}{\gamma+\theta}e^{h+\gamma t+\theta t} + p$$
$$\therefore \quad I(t) = \frac{-K}{\gamma+\theta}\, e^{h+\gamma t} + pe^{-\theta t} \qquad (2)$$

Using the boundary condition $I(t) = 0$ when $t = T$ in equation (2), gives;

$$I(T) = 0 = \frac{-K}{\gamma+\theta}e^{h+\gamma T} + pe^{-\theta T}$$
$$\Longrightarrow \frac{K}{\gamma+\theta}e^{h+\gamma T} = pe^{-\theta T} \; or \; p = \frac{K}{\gamma+\theta}e^{h+\gamma T}.e^{\theta T} = \frac{K}{\gamma+\theta}e^{(\gamma+\theta)T+h}$$

We substitute $p$ in (2) to obtain

$$I(t) = \frac{-K}{\gamma+\theta}e^{h+\gamma t} + \frac{K}{\gamma+\theta}e^{(\gamma+\theta)T}.e^{h}.e^{-\theta t}$$
$$= \frac{Ke^h}{\gamma+\theta}\left[e^{(\gamma+\theta)T-\theta t} - e^{\gamma t}\right], \qquad 0 \le t \le T \qquad (3)$$

We can obtain the initial order quantity by putting the boundary condition $I(0) = I_0$ into equation (3) as follows:

$$I(0) = I_0 = \frac{Ke^h}{\gamma+\theta}\left[e^{(\gamma+\theta)T-\theta(0)} - e^{\gamma(0)}\right]$$
$$= \frac{Ke^h}{\gamma+\theta}\left[e^{(\gamma+\theta)T} - 1\right] \qquad (4)$$

Hence the total demand during the cycle period [0, T] is given as follows:

$$\int_0^T D(t)dt = \int_0^T Ke^{h+\gamma t}dt = \frac{K}{\gamma}\left[e^{h+\gamma t}\right]_0^T$$
$$= \frac{Ke^h}{\gamma}\left[e^{\gamma T} - 1\right] \quad (5)$$

Thus, the number of deteriorated units is
Initial order quantity minus the total demand in the cycle period, which is given by

$$I_0 - \int_0^T D(t)dt = \frac{Ke^h}{\gamma+\theta}\left[e^{(\gamma+\theta)T} - 1\right] - \frac{Ke^h}{\gamma}\left[e^{\gamma T} - 1\right]$$
$$= \frac{Ke^h}{(\gamma+\theta)\gamma}\left[\gamma e^{(\gamma+\theta)T} - \gamma e^{\gamma T} - \theta e^{\gamma T} + \theta\right] \quad (6)$$

Now, the deterioration cost (DC) for the cycle period [0, T] is

$$A_c \text{ x (the number of deteriorated units)}$$
$$= \frac{A_c Ke^h}{(\gamma+\theta)\gamma}\left[\gamma e^{(\gamma+\theta)T} - \gamma e^{\gamma T} - \theta e^{\gamma T} + \theta\right] \quad (7)$$

Hence, total inventory holding cost (IHC) for the cycle [0, T] is as follows:

$$= \int_0^T iCI(t)dt = \frac{iCKe^h}{\gamma+\theta}\left[\frac{1}{\theta}e^{(\gamma+\theta)T} - \frac{1}{\theta}e^{\gamma T} - \frac{1}{\gamma}e^{\gamma T} + \frac{1}{\gamma}\right] \quad (8)$$

Total Variable Cost = Ordering Cost (OC) + Deterioration Cost (DC) + Inventory Holding Cost (IHC).
The total Variable Cost per unit time $TC(T)$ is

$$TC(T) = \frac{\text{Total Variable Cost}}{\text{T}} = \frac{A_0}{T} + \frac{A_c Ke^h}{(\gamma+\theta)\gamma T}\left[\gamma e^{(\gamma+\theta)T} - \gamma e^{\gamma T} - \theta e^{\gamma T} + \theta\right] + \frac{iCKe^h}{(\gamma+\theta)T}\left[\frac{1}{\theta}e^{(\gamma+\theta)T} - \frac{1}{\theta}e^{\gamma T} - \frac{1}{\gamma}e^{\gamma T} + \frac{1}{\gamma}\right] \quad (9)$$

The main objective is to find the minimum variable cost per unit time. The necessary and sufficient conditions to minimize TC(T) are respectively

$$\frac{dTC(T)}{dT} = 0 \text{ and } \frac{d^2TC(T)}{dT^2} > 0.$$

Now we differentiate equation (9), with respect to T as follows:

$$\frac{dTC(T)}{dT} = -\frac{A_0}{T^2} + \frac{A_C ke^h}{\gamma(\gamma+\theta)}\left[\frac{\gamma(\gamma+\theta)e^{(\gamma+\theta)T}}{T} - \frac{\gamma e^{(\gamma+\theta)T}}{T^2} - \frac{\gamma^2 e^{\gamma T}}{T} + \frac{\gamma e^{\gamma T}}{T^2} - \frac{\theta\gamma e^{\gamma T}}{T} + \frac{\theta e^{\gamma T}}{T^2} - \frac{\theta}{T^2}\right] + \frac{iCke^h}{(\gamma+\theta)}\left[\frac{(\gamma+\theta)e^{(\gamma+\theta)T}}{\theta T} - \frac{e^{(\gamma+\theta)T}}{\theta T^2} - \frac{\gamma e^{\gamma T}}{\theta T} + \frac{e^{\gamma T}}{\theta T^2} - \frac{e^{\gamma T}}{T} + \frac{e^{\gamma T}}{\gamma T^2} - \frac{1}{\gamma T^2}\right] \quad (10)$$

We set equation (10) to zero and simplify by multiplying both sides with $[-T^2\theta\gamma(\gamma+\theta)]$ in order to determine the T that minimizes the variable cost per unit time as follows:

$$A_0\,\gamma\,\theta(\gamma+\theta) - A_C k e^{\,h} T\,\gamma\theta(\gamma+\theta)e^{(\gamma+\theta)T} + A_C k e^{\,h}\gamma\,\theta e^{(\gamma+\theta)T} + A_C k e^{\,h} T\,\gamma^2\theta e^{\gamma T} - A_C k e^{\,h}\,\gamma\theta e^{\gamma T} + A_C k e^{\,h}\gamma\,\theta^2 T e^{\gamma T} - A_C k e^{\,h}\,\theta^2 e^{\gamma T} + A_C k e^{\,h}\,\theta^2 - iCke^{\,h} T\gamma\,(\gamma+\theta)e^{(\gamma+\theta)T} + iCke^{\,h}\,\gamma\,e^{(\gamma+\theta)T} + iCke^{\,h} T\gamma^2 e^{\gamma T} - iCke^{\,h}\gamma e^{\gamma T} + iCke^{\,h}\gamma T\theta e^{\gamma T} - iCke^{\,h}\theta e^{\gamma T} + iCke^{\,h}\theta = 0 \quad (11)$$

The value of T obtained, gives the minimum cost provided it satisfies the following condition

$$\frac{d^2TC(T)}{dT^2} > 0.$$

Putting the values of $A_0$, K, $\theta, \gamma,\ i, C,\ A_c$, and h into equation (11), gives the $T$ value which provides the minimum cost, with the proviso above. The example below satisfies the condition and so it gives the T value providing the minimum cost.

## 4.0 NUMERICAL EXAMPLE

**Example 1**

Let $A_0$ = 5000 per order, $K$ = 500, $\theta$ = 0.2, $\gamma$ = 0.02, $i$ = 0.1 per Naira per unit time, $C$ = ₦200 per unit, $A_c$ = ₦250, and $h$ = 2. Dash et al (2014).

Substituting into equation (11) and solving, we obtain $T^*$ = 0.194521 (71 days). On substitution of the optimal value $T^*$in equations (9) and (4), we obtain the minimum total cost per unit time as $TC^*$ = 51253.06 and economic order quantity $I_0^*$ = 734.2607. The $T^*$ value satisfies $\frac{d^2TC(T)}{dT^2} > 0$, as already mentioned.

## 5.0 SENSITIVITY ANALYSIS

We have performed sensitivity analysis on example1 through changing each of the parameters $A_0$, K, $\theta,\ \gamma,\ i,\ C,\ A_c$ and h by 50%, 25%, 5%, 2%, -2% -5% -25% -50%, while keeping the remaining parameters at their original values. The corresponding changes in the cycle time, total cost per unit and the economic order quantity are shown in Table2.

**Example 2.** Using the same values as in example 1, with h changed to 3 the solutions are $T^*$= 0.120548(44 days), $TC^*$ = 84260.88 and $I_0^*$ = 1226.831.

**Example 3.** Using the same values as in example 1, with h changed to 4, the solutions, are $T^*$ = 0.073973 (27 days), $TC^*$ = 138691 and $I_0^*$ = 2035.905

A summary of the results for the above three examples is shown in table 1 below:

**Table 1: Summary of the Results of Examples 1, 2 &3**

| h | $T^*$ | $TC^*$ | $I_0^*$ |
|---|---|---|---|
| 2 | 0.194521 (71 days) | 51253.06 | 734.2607 |
| 3 | 0.120548(44 days) | 84260.88 | 1226.831 |
| 4 | 0.073973 (27 days) | 138691 | 2035.905 |

As we can observe from Table1, as the value of h increases $TC^*$ and $I_0^*$ increase while $T^*$ decreases as it is expected. This is due to the fact that whenever h increases, the demand increases and so the economic order quantity also increases, hence the total variable cost, $TC^*$ also increases. The cycle period decreases as a result of higher demand.

**Table 2: Sensitivity Analysis on example1 to see changes in the values of $T^*$, $TC^*$ and $I_0^*$ as other parameters change**

| Parameter | % change in parameter | $T^*$ | | $TC^*$ | $I_0^*$ |
|---|---|---|---|---|---|
| $A_0$ | 50 | 0.238356 | (87 days) | 62882.56 | 904.1114 |
| | 25 | 0.216438 | (79 days) | 57354.37 | 818.9813 |
| | 5 | 0.2 | (73 days) | 52530.11 | 755.4026 |
| | 2 | 0.19726 | (72 days) | 51768.56 | 744.8285 |
| | **0** | **0.194521** | **(71days)** | **51253.06** | **734.2607** |
| | -2 | 0.191781 | (70 days) | 27432.58 | 723.6994 |
| | -5 | 0.189041 | (69 days) | 49945.02 | 713.1443 |
| | -25 | 0.169863 | (62 days) | 44342.65 | 639.4371 |
| | -50 | 0.139726 | (51days) | 36163.45 | 524.2379 |
| $K$ | 50 | 0.158904 | (58 days) | 62682.28 | 896.1871 |
| | 25 | 0.175342 | (64 days) | 57258.66 | 825.5807 |
| | 5 | 0.189041 | (69 days) | 52508.39 | 748.8016 |
| | 2 | 0.191781 | (70 days) | 51758.54 | 738.1733 |
| | **0** | **0.194521** | **(71days)** | **51253.06** | **734.2607** |
| | -2 | 0.19726 | (72 days) | 50743.33 | 729.9319 |
| | -5 | 0.2 | (73 days) | 49966.1 | 717.6325 |
| | -25 | 0.224658 | (82 days) | 44440.19 | 638.1428 |
| | -50 | 0.273973 | (100 days) | 36357.7 | 521.6632 |
| $\theta$ | 50 | 0.167123 | (61 days) | 59809.55 | 634.2502 |
| | 25 | 0.178082 | (65 days) | 55689.42 | 674.0036 |
| | 5 | 0.191781 | (70 days) | 52171.07 | 724.3986 |
| | 2 | 0.194521 | (71 days) | 51624.81 | 731.5485 |
| | **0** | **0.194521** | **(71days)** | **51253.06** | **734.2607** |
| | -2 | 0.19726 | (72 days) | 50884.65 | 744.5326 |
| | -5 | 0.19726 | (72 days) | 50319.56 | 744.089 |
| | -25 | 0.216438 | (79days) | 46408.94 | 814.5308 |
| | -50 | 0.243836 | (89 days) | 41007.29 | 914.1667 |
| $\gamma$ | 50 | 0.194521 | (71 days) | 51286.28 | 734.9804 |
| | 25 | 0.194521 | (71 days) | 51269.67 | 734.6205 |
| | 5 | 0.194521 | (71 days) | 51256.38 | 734.3327 |
| | 2 | 0.194521 | (71 days) | 51254.39 | 734.2895 |
| | **0** | **0.194521** | **(71days)** | **51253.06** | **734.2607** |
| | -2 | 0.194521 | (71 days) | 51251.73 | 734.232 |
| | -5 | 0.194521 | (71 days) | 51249.74 | 734.1888 |
| | -25 | 0.194521 | (71days) | 51236.47 | 733.9012 |
| | -50 | 0.194521 | (71days) | 51219.89 | 733.542 |

**Table 2 (continued): Sensitivity Analysis on example1 to see changes in the values of $T^*$, $TC^*$ and $I_0^*$ as other parameters change**

| Parameter | % change in parameter | $T^*$ | | $TC^*$ | $I_0^*$ |
|---|---|---|---|---|---|
| $C$ | 50 | 0.183562 | (67 days) | 54768.19 | 692.0534 |
| | 25 | 0.189041 | (69 days) | 53040.21 | 713.1443 |
| | 5 | 0.194521 | (71 days) | 51618.05 | 734.2607 |
| | 2 | 0.194521 | (71days) | 51399.06 | 734.2607 |
| | **0** | **0.194521** | **(71days)** | **51253.06** | **734.2607** |
| | -2 | 0.194521 | (71 days) | 51107.07 | 734.2607 |
| | -5 | 0.19726 | (72days) | 50891.41 | 744.8285 |
| | -25 | 0.20274 | (74 days) | 49404.87 | 765.9831 |
| | -50 | 0.210959 | (77 days) | 47482.37 | 797.7629 |
| $i$ | 50 | 0.183562 | (67 days) | 54768.19 | 692.0534 |
| | 25 | 0.189041 | (69 days) | 53040.21 | 713.1443 |
| | 5 | 0.194521 | (71days) | 51618.05 | 734.2607 |
| | 2 | 0.194521 | (71 days) | 51399.06 | 734.2607 |
| | **0** | **0.194521** | **(71days)** | **51253.06** | **734.2607** |
| | -2 | 0.194521 | (71 days) | 51107.07 | 734.2607 |
| | -5 | 0.19726 | (72 days) | 50891.41 | 744.8285 |
| | -25 | 0.20274 | (74 days) | 49404.87 | 765.9831 |
| | -50 | 0.210959 | (77days) | 47482.37 | 797.7629 |
| $A_c$ | 50 | 0.167123 | (61days) | 59642.29 | 628.9329 |
| | 25 | 0.180822 | (66 days) | 55611.31 | 681.5174 |
| | 5 | 0.191781 | (70days) | 52154.28 | 723.6994 |
| | 2 | 0.194521 | (71 days) | 51618.05 | 734.2607 |
| | **0** | **0.194521** | **(71days)** | **51253.06** | **734.2607** |
| | -2 | 0.19726 | (72 days) | 50891.41 | 744.8285 |
| | -5 | 0.19726 | (72days) | 50336.1 | 744.8285 |
| | -25 | 0.213699 | (78 days) | 46488.7 | 808.3689 |
| | -50 | 0.241096 | (88 days) | 41171.96 | 914.7816 |
| $h$ | 50 | 0.120548 | (44 days) | 84260.88 | 1226.831 |
| | 25 | 0.153425 | (56 days) | 65703.36 | 950.4985 |
| | 5 | 0.186301 | (68 days) | 53863.12 | 776.4883 |
| | 2 | 0.191781 | (70 days) | 52282.63 | 753.2341 |
| | **0** | **0.194521** | **(71days)** | **51253.06** | **734.2607** |
| | -2 | 0.2 | (73days) | 50249.65 | 725.7828 |
| | -5 | 0.205479 | (75 days) | 48774.83 | 702.6696 |
| | -25 | 0.249315 | (91 days) | 40004.07 | 574.2818 |
| | -50 | 0.317808 | (116 days) | 31242.11 | 447.4047 |

## 6.0 DISCUSSION OF RESULTS

Observing Table 2 carefully, we can make the following deductions.

(i) With increase in the value of the parameter $A_0$, the values of $T^*, TC^*$ and $I_0^*$ all increase. This is expected since when ordering cost increases then the model will avoid more orders and so both $T^*$and $I_0^*$ increase. $TC^*$ will therefore increase due to increase in both ordering cost and stockholding cost. The increases in the values are moderate hence the decision variables $T^*$, $TC^*$ and $I_0^*$ are moderately sensitive to changes in $A_0$.

(ii) With increase in the value of parameter $K$, the values of $TC^*$ and $I_0^*$ increase while $T^*$ decreases. This is expected because when $K$

increases, the demand in that case will also increase which results in increase in the economic order quantity ( $I_0^*$ ) and the optimal total order ($TC^*$). To reduces the cost, the model probably forces the total cycle period($T^*$) to decrease. The increase/decrease in the values are moderate hence the decision variables are moderately sensitive to changes in *K*.

(iii) With increase in the value of parameter $\theta$, the values of $TC^*$ increases while $T^*$ and $I_0^*$ decrease. This is probably because when $\theta$ increases, deterioration increases and so the optimal total cost ($TC^*$ ) increases due to the cost of deterioration. To avoid much deterioration the model forces ( $I_0^*$) and the total cycle period ($T^*$ ) to decrease. The increases in the values are low hence the decision variables are lowly sensitive to changes in $\theta$.

(iv) With increase in the value of the parameter $\gamma$, the values of $I_0^*$ and $TC^*$ increase while $T^*$ remains constant. This is probably because when $\gamma$ increases, the demand increases and so $I_0^*$ and $TC^*$ increase. $T^*$ remains constant because the increases in $I_0^*$ and $TC^*$ are very low. The increase in the values are low hence the decision variables are not very sensitive to changes in $\gamma$.

(v) With increase in the value of parameter C, the value of $TC^*$ increases while $T^*$ and $I_0^*$ decrease. This is expected since when C increases the total cost increases. To avoid much cost, the model forces $T^*$ and $I_0^*$ to decrease. The increase/decreases in the values are moderate hence the variables are moderately sensitive to changes in *C*.

(vi) With increase in the value of parameter *i*, the value of $TC^*$ increases while $T^*$ and $I_0^*$ decrease. This is expected because when the inventory carrying charge, *i* is increased the total cost increases. To avoid much cost, the model forces $T^*$ and $I_0^*$ to decrease. The increase/decreases in the values are moderate hence the variables are moderately sensitive to changes in *i*.

(vii) With increase in the value of the parameter $A_c$, the values of $T^*$ and $I_0^*$ decrease while $TC^*$ increases. This is because when $A_c$ increases, deterioration cost increases and so the model forces a reduction in $T^*$and $I_0^*$ to avoid much deterioration cost. Increase in deterioration cost makes $TC^*$ to increase. The increase/decrease in the values are moderate hence the decision variables are moderately sensitive to changes in $A_c$

(viii) With increase in the value of parameter *h*, the values of $TC^*$ and $I_0^*$ increase while $T^*$ decreases. This is because as h increases, the demand increases, and so the economic order quantity also increases, hence the total variable cost, $TC^*$ also increases. On the other hand however the cycle period decreases as a result of higher demand. The increase/decrease in the values is high hence the decision variables are highly sensitive to changes in h.

## 7.0 CONCLUSION

In this paper, an inventory model is developed which determines the optimal order quantity of an on-hand inventory due to a generalised exponential decreasing demand rate. The deterioration rate is constant and the stockholding cost is also a constant. The model has been solved analytically by minimizing the total inventory cost. A numerical example has been given to show the application of the model. Later, a sensitivity analysis is carried out to see the effect of changes in the parameter values. The analysis shows that $T^*$, $TC^*$ and $I_0^*$ are sensitive to changes in the parameters, $A_0, K, A_c, C, i$ and h. However $T^*$, $TC^*$ and $I_0^*$ are not very sensitive to changes in the parameters $\theta$ and $\gamma$. Moreover, it has been shown that the values $T^*, TC^*$ and $I_0^*$ all increase with increase in the parameter $A_0$.

## REFERENCES

[1] Aliyu I. and Sani B., On an inventory model for deteriorating items with generalised exponential decreasing demand and time-varying holding cost, *Journal of Nigeria Association of Mathematical Physics*, **36** (1) (2016)**,** 193-202.

[2] Aliyu, I. and Sani, B., An inventory model for deteriorating items with generalised exponential decreasing demand, constant holding cost and time-varying deterioration rate, *American Journal of Operations Research*, **8** (2018), 1-16. https://doi.org/10.4236/ajor.2018.81001

[3] Aggarwal, S. P., A note on an order-level Model for a system with constant rate of deterioration, *Opsearch*, **15** (1978), 184-187.

[4] Bhunia A. K. and Maiti M., An inventory model of deteriorating items with lost-size dependent replenishment cost and a linear trend in demand, *Appl. Math. Model*, **23** (1999), 301-308. https://doi.org/10.1016/s0307-904x(98)10089-6

[5] Bahari-Kashani H., Replenishment schedule for deteriorating items with time -proportional demand, *J. Opl. Res. Soc.*, **40** (1989), 75-81. https://doi.org/10.1057/jors.1989.7

[6] Dash B. P, Singh. T. and Pattanyak, An inventory model for deteriorating items with exponential declining demand and time-varying holding cost, *American Journal of Operational Research*, **4** (2014), 1-7. https://doi.org/10.4236/ajor.2014.41001

[7] Dave, U. and Patel, L.K., (T, $S_i$), Policy inventory model for deteriorating items with time proportional demand, *Journal of Operational Research Society*, **32** (1981)**,** 137-142. https://doi.org/10.1057/jors.1981.27

[8] Ghare; P.N. and Schrader, G.P., A model for exponential decaying inventory, *Journal of Industrial Engineering*, **14** (1963), 238-243.

[9] Jalan A. K. and Chaudhuri K. S., An EOQ model for deteriorating items indeclining market, with SFI policy, *Korean Journal of Computational and Applied Mathematics*, **6** (2) (1999)**,** 437-449. https://doi.org/10.1007/bf03014386

[10] Kumar S. K., P. and Saini M., An order level for deteriorating items with quadratic demand rate and variable holding cost, *International Journal of Scientific Research Engineering and Technology (IJSRT)*, **1** (5) (2012), 253-263.

[11] Lee S. M. and Moore L. J., *Introduction to decision science*, First Edition, New York, 1975, p. 304-343.

[12] Mishra S. S. and Singh P. K., A computational approach to EOQ model with power-form stock-dependent demand and cubic deterioration, *American Journal of Operational Research*, **1** (1) (2011) , 5-13.

[13] Raymond F. E, *Introduction to Decision Science*, By Sang M. Lee and Laurence J. Moore, Polytechnic Institute and State University, First Edition (1930), 304.

[14] Sachan R. S., On (T, $S_i$ ) inventory policy model for deteriorating items with time proportional demand, *J. Opl. Res. Soc*., **35** (1984), 1013-1019. https://doi.org/10.1057/jors.1984.197

[15] Shah, Y. K. and Jaiswal, M. C., An order-level inventory model for a system with constant rate of deterioration, *Opsearch*, **14** (1977), 1744-1784.

[16] Singh T. and Pattanyak H., An EOQ model for a deteriorating item with time dependent quadratic demand and variable deterioration under permissible delay in payment, *Applied Mathematics Science*, **7** (59) (2013), 2939-2951. https://doi.org/10.12988/ams.2013.13261

[17] Teng M. T, Ouyang L. Y. and Cheng M. C., An EOQ model for deteriorating items with power-form stock-dependent demand, *Information and Management Sciences*, **16** (1) (2005), 1-6.

[18] Wee H.M., Economic order quantity model for deteriorating items with partial back ordering, *Computer and Industrial Engineering*, **24** (3) (1993), 449-458. https://doi.org/10.1016/0360-8352(93)90040-5

[19] Whitin, T, M., *The theory of inventory management*, 2nd ed., Princeton, NJ, Princeton University press, 1957.

**Received: August 25, 2020; Published: September 26, 2020**