The Problem of Obstacle for the First Eigenvalue for the p-Laplacian Operator

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Abstract

Let $\lambda_{1,p}(\Omega)$ the first eigenvalue of the p-Laplacian operator for a domain $\Omega = D/K$, with $K$ moves in $D$ without going out.

Using the tools of the derivatives with respect to the domain with some assumptions have given the necessary and sufficient conditions for the first eigenvalue of the p-Laplacian operator to be minimal for the obstacle problem.

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1 Introduction

In [16] Long-Jiang Gua, Xiaoyu Zengb, and Huan-Song Zhoub have studied the existence of asymptotic behavior of the base states for the eigenvalue problem of the following p-Laplacian equation:

$$\Delta_p u = V(x)|u|^{p-2}u = \mu|u|^{p-2}u + a|u|^{s-2}u, \quad x \in \mathbb{R}^N$$

with $p \in (1,n), s = p + \frac{p^2}{n}, a \geq 0$ and $\mu \in \mathbb{R}$ is a parameter and $V(x)$ is a field of vectors satisfying certain assumptions.
In [15] Leandro, Del Pezzo and Julio Studied the first eigenvalue for the p-Laplacian operator with the boundary conditions of Dirichlet and Neumann (mixed boundary conditions). They considered the following problem:

\[
\begin{align*}
\Delta_p u &= \lambda \alpha |u|^{\alpha - 2} u |v|^\beta & \text{on } \Omega \\
\Delta_q u &= \lambda \beta |u|^{\alpha} u |v|^{\beta - 2} v & \text{on } \Omega
\end{align*}
\]

(1)

with \( \frac{2}{p} + \frac{\beta}{q} = 1 \) and Next mixed boundary conditions:

\[ u = 0, \quad |\nabla v|^{\alpha - 2} \frac{\partial v}{\partial \nu} \text{ sur } \partial \Omega \]

In [7] Daniele Valtorta gave the estimate of the first non-trivial eigenvalue of the p-Laplacian on a compact Riemannian manifold with a non-negative Ricci curvature and characterize the case of equality. He studied the following problem:

\[
\begin{align*}
\Delta_p (u) &= \lambda_{1,p} |u|^{\alpha - 2} u & \text{on } \Omega \\
\langle \nabla u, n \rangle &= 0 & \text{on } \partial \Omega
\end{align*}
\]

(2)

Daniele Valtorta has proved the following strong estimate:

\[
\frac{\lambda_{1,p}}{p - 1} \geq \frac{\Pi_p}{d^p}
\]

With

\[
\Pi_p = \int_{-1}^{1} \frac{ds}{(1 - |s|^p)^{\frac{1}{p}}} = \frac{2\pi}{p \sin(\frac{\pi}{p})}
\]

In this article we will study the obstacle position problem for the p-Laplacian operator.

The obstacle locating problem for the fundamental eigenvalue is to locate the position of the obstacle placement so as to maximize or minimize the eigenvalue of the p-Laplacian operator. We were interested in the following problems:

Let \( \Omega \) be a bounded open set of \( \mathbb{R}^N \) and \( K \) An obstacle that moves The interior of \( D \). We consider the problem :

\[
\begin{align*}
\Delta_p (u) &= \lambda_{1,p} |u|^{\alpha - 2} u & \text{in } D \setminus K \\
u &= 0 & \text{on } \partial \Omega
\end{align*}
\]

(3)

Let \( \lambda_{1,p} \) The first eigenvalue of the p-Laplacian operator with certain hypotheses we want to give the necessary and sufficient conditions so that the first eigenvalue of the p-Laplacian operator is minimal or we want to determine the position of \( K \) in \( \Omega \) so that \( \lambda_{1,p} \) Is minimal where \( \lambda_{1,p} \) Represents the first eigenvalue of the p-Laplacian operator.
2 Position of the problem

Let $D$ a fixed open set of $\mathbb{R}^N$ and $K$ an obstacle that is a subset of $D$. In this work we study the minimization of the first eigenvalue of the operator $P$-Laplacian with conditions at the edges of Dirichlet null on the border of $\Omega = D/K$. More specifically, we place $K$ at the interior of $D$. With conditions at the edges of Dirichlet null on the border of $\Omega = D/K$.

The question is: We want to determine the position of $K$ in $D$ so that $\lambda_{1,p}$ minimal where $\lambda_{1,p}$ represents the first eigenvalue of the $P$-Laplacian Dirichlet

\[
\begin{cases}
\Delta_p(u) = \lambda_{1,p}|u|^{\alpha-2}u & \text{in } \Omega \\
u = 0 & \text{on } \partial \Omega
\end{cases}
\] (4)

With the $P$-Laplacian operator defined by the following relation:

\[
\Delta_p : W^{1,p}_0(\Omega) \rightarrow W^{-1,p}(\Omega)
\]
\[
\Delta_p u \rightarrow \text{div}(\nabla u)^{p-2}\nabla u)
\]

$W^{-1,p}(\Omega)$ la dual de $W^{1,p}_0(\Omega)$

Define a vector field

\[
V: \mathbb{R}^N \rightarrow \mathbb{R}^N \\
x \rightarrow (V_1(x), V_2(x), V_3(x), ... ,V_N(x))
\]

For all real $t$ small, we define the domains Disturbed:

$\Omega_t = (Id + tV)(\Omega) = \{x + tV(x), \ x \in \Omega\}$

The variation of $\Omega$ is explained by the fact that $K$ moves into $\Omega$ without going out. If $K$ is a hard obstacle, the movement of $K$, $\Omega$ is done either by translation or by rotation, or one combines these two types of motion.

If $K$ is considered a soft obstacle, $K$ may undergo a transformation by homothety.

After perturbation of the problem (4) becomes:

\[
\begin{cases}
\Delta_p(u_t) = \lambda_{1,p}|u_t|^{\alpha-2}u_t & \text{in } \Omega_t \\
u_t = 0 & \text{on } \partial \Omega_t
\end{cases}
\] (5)

By using the variational formulation the first eigenvalue of the operator $P$-Laplacian is defined by the following Rayleigh nonlinear quotient

\[
\lambda_1(\Omega_t) = \min_{u \in W^{1,p}_0(\Omega_t), u \neq 0} \frac{\int_\Omega |\nabla u|^p}{\int_\Omega |u|^p}
\]

$W^{1,p}_0$ The adherence of all functions $C^\infty$ has compact media contained in $W^{1,p}$, with $W^{1,p}(\Omega) = \{u \in L^p(\Omega); \frac{\partial u}{\partial x_i} \in L^p(\Omega), (i = 1, ..., N)\}$

We will give some definitions before we formulate more precisely the problem.
Definition 2.1 Let $\xi$ a unit vector of $\mathbb{R}^N$, $\varepsilon$ a strictly real number positive and $y$ belonging to $\mathbb{R}^N$, the summit cone $y$ and direction $\xi$, of angle at the top and height $\varepsilon$. The set defined by

$$C(y, \xi, \varepsilon, \varepsilon) = \{x \in \mathbb{R}^N : |x - y| \leq \varepsilon \text{ et } |(x - y) \cdot \xi| \geq |x - y| \cos \varepsilon\}$$

Definition 2.2 Let $\Omega$ be an open set of $\mathbb{R}^N$, $\Omega$ has the property of $\varepsilon$ - Cone if for any $x \in \partial \Omega$, There is a direction $\xi$ and a strictly positive number $\varepsilon$ such as

$$C(y, \xi, \varepsilon, \varepsilon) \subset \Omega \text{ pour tout } y \in B(x, \varepsilon) \cap \overline{\Omega}$$

In the case of a hard obstacle: Define for any real $t$ pretty small $T_t(B)$ Such as a translation, rotation, or face. Let $J_2(\Omega_t) = \int_{\Omega_t} dx - v_o$ with $v_o > 0$ and

$$\Theta_\varepsilon = \{\Omega_t = D \setminus T_t(B), \text{ open of } \mathbb{R}^N \text{ And verifying ownership of the } \varepsilon \text{ Cone and } \int_{\Omega_t} dx = v_o\}$$

So the problem becomes: determine the position of $B$ such that

$$\min_{\Omega_t \in \Theta_\varepsilon} \lambda_{1, p}(\Omega_t) \text{ Is reached}$$

or $\lambda_{1, p}(\Omega_t) = \min_{u \in W^{1, p}(\Omega)} \left\{ \int_{\Omega_t} |\nabla u|^p dx : \int_{\Omega_t} |u|^p dx = 1, \right\}$

3 The shape Critical of the first eigenvalue for the p-Laplace operator ($\lambda_{1, p}(\Omega_t)$)

Now we will start by studying the shape critical of the functional $\lambda_{1, p}(\Omega_t)$ (the first eigenvalue for the p-Laplacian operator) above. Indeed, the solution domain of the free boundary problem is not automatically a minimum for the function $\lambda_{1, p}(\Omega_t)$. this justifies the study of the shape critical of $\lambda_{1, p}(\Omega_t)$, followed by 1 The study of the quadratic form. The functional $\lambda_{1, p}(\Omega_t)$ is given by the following relation:

$$\lambda_{1, p}(\Omega_t) = \int_{\Omega_t} |\nabla u|^p dx$$

With $u$ solution of the following problem:

$$\begin{cases} 
\Delta_p(u_t) = \lambda_{1, p}|u_t|^{p-2}u_t & \text{in } \Omega_t \\
 u_t = 0 & \text{on } \partial \Omega_t 
\end{cases}$$
Using the hadamard formula we get

$$
\lambda'_{1,p}(\Omega_t) = \int_{\Omega_t} (|\nabla u|^p)' \, dx + \int_{\Omega_t} \text{div}(|\nabla u|^p V(0)) \, dx
$$

$$
\lambda'_{1,p}(\Omega_t) = \int_{\Omega_t} \nabla u \nabla ' u |\nabla u|^{p-2} \, dx + \int_{\partial K} |\nabla u|^p V(0) \, n \, dx
$$

With \( n \) is the unit exterior normal of \( K \)
with \( u' \) is the derivative of the form of \( u \), and \( u' \) Satisfying the following equation:

$$
\begin{cases}
- \text{div}(\nabla u' |\nabla u|^{p-2}) - (p - 2) \text{div}(|\nabla u|^{p-4} (\nabla u. \nabla u') \nabla u) = \lambda'_{1,p}|u_t|^p u' - (p - 1) \lambda_{1,p}|u_t|^{p-2} u' & \text{in } \Omega \setminus K \\
= (p - 1) \lambda_{1,p}|u_t|^{p-2} u' & \text{on } \partial K
\end{cases}
$$

(7)

We multiply the equation (7) by \( u \) and using Green formula we get

$$
-(p - 1) \int_{\Omega_t} - \text{div}(\nabla u' |\nabla u|^{p-2}) u' \, dx = \lambda'_{1,p}(\Omega, V) + \int_{\Omega_t} (p - 1) \lambda_{1,p}|u_t|^{p-2} u' u \, dx
$$

Using the Green’s formula then we get

$$
-(p - 1) \int_{\Omega_t} - \text{div}(\nabla u' |\nabla u|^{p-2}) u' \, dx + -(p - 1) \int_{\partial K} |\nabla u|^{p-2} \nabla u. u' \, dx = \lambda'_{1,p}(\Omega, V) + \int_{\Omega_t} (p - 1) \lambda_{1,p}|u_t|^{p-2} u' u \, dx
$$

(8)

We are getting

$$
-(p - 1) \int_{\Omega_t} ( - \text{div}(\nabla u' |\nabla u|^{p-2}) - \lambda_{1,p}|u_t|^{p-2} u') u' \, dx + -(p - 1) \int_{\partial K} |\nabla u|^{p-2} \nabla u. u' \, dx = \lambda'_{1,p}(\Omega, V)
$$

(9)

What gives after the simplification

$$
\lambda'_{1,p}(\Omega, V) = -(p - 1) \int_{\partial K} |\nabla u|^{p-2} \nabla u. n \, u' \, d\sigma
$$

since

$$
u' = - \frac{\partial u}{\partial n} V. n = \text{ on } \partial K
$$

and

$$
\nabla u. n = \frac{\partial u}{\partial n} = - |\nabla u|
$$
So we get

$$
\lambda'_{1,p}(\Omega, V) = -(p - 1) \int_{\partial K} |\nabla u|^p V(0) \cdot n d\sigma
$$

Since we have an optimization problem with equality constraint

$$
J_2(\Omega) = \int_{\Omega} dx - v_o = 0
$$

So there is a Lagrange multiplier $\beta < 0$ Depending on the domain $\Omega_t$ and verifying

$$
\lambda'_{1,p}(\Omega, V) = \beta dJ_2(\Omega, V) \quad (10)
$$

The derivative of $J_2(\Omega, V)$ Is given by

$$
dJ_2(\Omega, V) = \int_{\partial K} V \cdot n \ d\sigma \quad (11)
$$

By replacing in (10)

$$
-(p - 1) \int_{\partial K} |\nabla u|^p V(0) \cdot n d\sigma = \beta \int_{\partial K} V(0) \cdot n d\sigma \quad (12)
$$

Which give

$$
-(p - 1)|\nabla u|^p = \beta \quad \text{on} \quad \partial K \quad (13)
$$

So we get the following relation:

$$
|\nabla u| = \left(\frac{-\beta}{p - 1}\right) \quad \text{on} \quad \partial K \quad (14)
$$

Let $\Omega_t = \Omega \setminus K$ with $K$ An obstacle that moves inside of $\Omega$ So $\Omega_0 = \Omega \setminus K$ Is a shape critical of functional $\lambda_{1,p}(\Omega)$ If and if there exists a multiplier of lagrange $\beta < 0$ Depends on domain $\Omega_t$ Verifying the following relation:

$$
|\nabla u| = \left(\frac{-\beta}{p - 1}\right) \quad \text{on} \quad \partial K \quad (15)
$$

4 Quadratic form associated with the first eigenvalue of the p-Laplace operator ($\lambda_{1,p}(\Omega_t)$)

We have just proved that $\Omega_t = \Omega \setminus K$ is a shape critical of the functional $\lambda_{1,p}(\Omega)$. And our goal is to know if $\Omega_t$ can be the minimum of $\lambda_{1,p}(\Omega)$ under certain assumptions. This leads us to the study of the positivity of a quadratic form
that we will denote by \( Q \). This quadratic form is obtained by calculating the second derivative of \( \lambda_{1,p}(\Omega) \) with respect to the domain. So before we go on, we need some assumptions. Suppose that:

(i) \( \Omega \) is open of class \( C^2 \)-regular.

(ii) \( V(x; t) = v(x)n(x), \ v \in H^{1/2}(\partial \Omega), \ \forall \ t \in [0, \epsilon[. \)

**Proposition 4.1 (Hard obstacle case)**

Suppose that \( \Omega_0 \) is a shape critical, then quadratic form associated with the first eigenvalue of the p-Laplace operator is given by:

\[
Q(v) = d^2 \lambda_{1,p}(\Omega; V, V) = -p\beta_\Omega \int_{\partial K} (N - 1)Hv^2 ds - p\beta_\Omega \int_{\Omega_t} |\nabla \Lambda|^2 dx \\
= -p\beta_\Omega \int_{\partial K} Lv d\sigma - p\beta_\Omega (N - 1) \int_{\partial K} Hv^2 d\sigma
\]

Where \( \beta_\Omega \) is the Lagrange multiplier, here it is negative and \( p \) is allowed to range over \( 1 < p < \infty \), and \( \Lambda \) is the solution of the following boundary value problem:

\[
\begin{cases}
-\Delta \Lambda &= 0 \quad \text{in} \quad \Omega_t = D/K \\
\Lambda &= 0 \quad \text{on} \quad \partial D \\
\Lambda &= v \quad \text{on} \quad \partial K
\end{cases}
\] (16)

\( H \) is the mean curvature of \( \partial K \) and \( L \) is a pseudo differential operator known as the Steklov-Poincaré or capacity or Dirichlet to Neumann(see e.g [8]) operator, defined by \( Lv = \frac{\partial \Lambda}{\partial n} \) and \( n \) is the unit exterior normal of \( K \). In fact \( \Lambda \) is the harmonic extension of \( v \) in \( \Omega \).

**Proof**

The first derivative of the functional \( \lambda_{1,p}(\Omega) \) is given by the following equation

\[
\lambda'_{1,p}(\Omega, V) = -(p - 1) \int_{\partial K} |\nabla u|^p V(0) n d\sigma
\]

\[
\lambda'_{1,p}(\Omega, V) = -(p - 1) \int_{\Omega_t} div(|\nabla V(0)|) dx
\]

Using the hadamard formula we get

\[
d^2 \lambda_{1,p}(\Omega, V, V) = -(p - 1) \int_{\Omega_t} div(|\nabla V(0)|') dx + -(p - 1) \int_{\Omega_t} div(div(|\nabla V(0)|)V(0)) dx
\]
What gives after the simplification
\[
\frac{-1}{(p-1)} d^2\lambda_{1,p}(\Omega, V, V) = \int_{\Omega_t} d\nu (\nabla u, \nabla u'V(0)) dx + \int_{\Omega} d\nu (|\nabla u|^p V(0)) V(0) dx
\]
\[
\frac{-1}{(p-1)} d^2\lambda_{1,p}(\Omega, V, V) = \int_{\partial K} p|\nabla u|^p\nabla u'V(0) . nd\sigma + \int_{\partial K} \nabla (|\nabla u|^p) V(0) . nd\sigma
\]
Since \(|\nabla u| = \left(\frac{-\beta}{p-1}\right)^{\frac{1}{p}}\) on \(\partial K\) and \(\nabla u = -|\nabla u| \cdot n\) on \(\partial K\)
and \(u' = -\frac{\partial u}{\partial n} V . n = |\nabla u| \cdot v\) on \(\partial K\)
What gives after the simplification
\[
\frac{d^2\lambda_{1,p}(\Omega, V, V)}{(p-1)} = \int_{\partial K} [p|\nabla u|^{p-1} . \nabla u'V(0) . n - \nabla (|\nabla u|^p) V(0) . n] d\sigma \quad (17)
\]
Since \(u' = -\frac{\partial u}{\partial n} V . n = \left(\frac{-\beta}{p-1}\right)^{\frac{1}{p}} v\) on \(\partial K\) So \(n . \nabla u' = \left(\frac{-\beta}{p-1}\right)^{\frac{1}{p}} \frac{\partial u}{\partial n}\) on \(\partial K\)
Replacing in (17) We obtain the following relation :
\[
\frac{d^2\lambda_{1,p}(\Omega, V, V)}{(p-1)} = \int_{\partial K} [p\left(\frac{-\beta}{p-1}\right) \frac{\partial u}{\partial n} - \nabla (|\nabla u|^p) V(0) . n] d\sigma \quad (18)
\]
Replacing in (18) Which give:
\[
\frac{d^2\lambda_{1,p}(\Omega, V, V)}{(p-1)} = p\left(\frac{-\beta}{p-1}\right) \int_{\partial K} v L \nu d\sigma - \int_{\partial K} v^2 \nabla (|\nabla u|^p) . nd\sigma \quad (19)
\]
So we get the following relation:
\[
d^2\lambda_{1,p}(\Omega, V, V) = -p\beta \int_{\partial K} v L \nu d\sigma - (p-1) \int_{\partial K} v^2 \nabla (|\nabla u|^p) . nd\sigma \quad (20)
\]
Sine we assumed that \(\Omega\) is \(C^2\),so using the formula of the level motion set related to the mean curvature.
In fact \(\partial K = \{x \in \mathbb{R}^N; u(x) = 0\}\) and we have
\[-(N-1)H = \text{div}(\frac{\nabla u}{|\nabla u|}) = \frac{\Delta u}{|\nabla u|} - \frac{\nabla u. \nabla (|\nabla u|)}{|\nabla u|^2}.\]
The problem of obstacle for the first eigenvalue for the p-Laplacian operator

where $H$ is the mean curvature of $\partial K$. Furthermore, since $u = 0$ on $\partial K$, we have $\Delta_p u = 0$ on $\partial K$.

Finally we get

$$(N - 1)H = \frac{-|\nabla u|n}{|\nabla u|^2} \nabla(|\nabla u|) \quad \text{i.e.}$$

$$-(N - 1)H|\nabla u| = n.\nabla(|\nabla u|)$$

$$\nabla(|\nabla u|) = -(N - 1)H|\nabla u|.n$$

By multipliing by $|\nabla u|^{p-1}$ we are getting:

$$|\nabla u|^{p-1}\nabla(|\nabla u|) = -(N - 1)H|\nabla u|^p.n$$

Finally we get

$$\nabla(|\nabla u|^p) = -p(N - 1)H|\nabla u|^{p}.n$$

This gives the following relation:

$$\nabla(|\nabla u|^p).n = \frac{Hp(N - 1)\beta}{(p - 1)}$$

Replacing in (20) We obtain the following relation :

$$d^2\lambda_{1,p}(\Omega, V, V) = -p\beta \int_{\partial K} v Lv d\sigma - p\beta(N - 1) \int_{\partial K} Hv^2 d\sigma$$

(21)

Therefore the quadratic form of the functional $\lambda_{1,p}(\Omega, t)$ Is given by the following equation:

$$Q(v) = d^2\lambda_{(1,p)}(\Omega, V; V)$$

$$= -p\beta t \int_{\partial K} (N - 1)Hv^2 ds - p\beta t \int_{\Omega} |\nabla \Lambda|^2 dx$$

$$= -p\beta t \int_{\partial K} vLv d\sigma - p\beta t (N - 1) \int_{\partial K} Hv^2 d\sigma$$

5 Sufficient conditions for the minimum of the first eigenvalue of the p-Laplace operator

In [6], Michel Pierre and Marc Dambrine ((See as well [4],[5]) have shown that it is not enough to prove that the quadratic form is positive to say that a
critical form is a minimum. For $t \in [0,\epsilon]$, $\lambda_{(1,p)}(\Omega_t) = \lambda_{(1,p)}(\Omega) + \lambda'_{(1,p)}(\Omega_t;V)t + d^2\lambda_{(1,p)}(\Omega_t;V)Vt^2 + o(t^2)$. The amount $o(t^2)$ is expressed in terms of the norm of $C^2$. It appears in the expression $d^2J(\Omega,V,V)$ norm of $H^\frac{1}{2}(\partial\Omega)$. And these two norm are not equivalent. The amount $o(t^2)$ Is not lower than $||V||^2_{H^\frac{1}{2}(\partial\Omega)}$ see [6],[9].

next, such an argument does not guarantee that the critical point is A strict local minimum. For this we will use the main result [6] and The Taylor formula with With integral rest to see if $\Omega$ is a strict local minimum or not. To give sufficient conditions for a local minimum of basic worth, we first present the results we obtained in our paper [2].

Let $A$ be an operator defined in the following sense:

$$A : H^\frac{1}{2}(\partial\Omega) \rightarrow H^{-\frac{1}{2}}(\partial\Omega)$$

$$A = L + (N-1)(||H^-||_{\infty} + H)I,$$

where $I$ is the identity operator $L$ is the pseudo differential operator as defined in the proposition (4.1), and $H^- = max(0,-H)$.

**Remark 5.1**

As assumed $\partial\Omega$ is of class $C^2$, then the mean curvature $H$ is a continuous function on $\partial K$.

Let us set $\alpha(x) = (N-1)(||H^-||_{\infty} + H(x))$, $\forall x \in \partial\Omega$. We note that $\alpha$ is continous and $\forall x \in \partial\Omega$, $\alpha(x) \geq 0$ (moreover $\alpha(x) > 0$ on a sufficiently large set).

**Lemma 5.1**

1 - The operator $A$ is a bijection from $H^\frac{1}{2}(\partial\Omega)$ into $H^{-\frac{1}{2}}(\partial\Omega)$ and it is continous.

2 - The inverse operator $A^{-1}$ is compact and self adjoint from $H^{-\frac{1}{2}}(\partial\Omega)$ into $H^\frac{1}{2}(\partial\Omega)$.

**Proof**

For proof see [2]

**Remark 5.2** Since the inverse operator : $(\alpha I + L)^{-1}$ is compact, self adjoint, then there exists a Hilbert basis $(\phi_n)_{n \in \mathbb{N}} \subset H^\frac{1}{2}(\partial\Omega)$ and a decreasing sequence of eigenvalues $\mu_n$ which goes to 0.

**Proposition 5.1**

Let $\Omega_0$ the critical shape for $\lambda_{1,p}(\Omega_t)$ ( The first eigenvalue of the $p$-Laplace
The problem of obstacle for the first eigenvalue for the $p$-Laplacian operator is given by

$$
\lambda_{1,p}(\Omega_t) = \min_{u \in W^{1,p}_0(\Omega), u \neq 0} \frac{\int_{\Omega} |\nabla u|^p}{\int_{\Omega} |u|^p}
$$

With $\Omega_t$ solution of the following problem:

$$\begin{cases}
\Delta_p(u_t) = \lambda_{1,p}|u_t|^\alpha - 2u_t & \text{in } \Omega_t \\
u_t = 0 & \text{on } \partial \Omega_t
\end{cases}
$$

(22)

$\Omega_0$ is a local strict minimum of $\lambda_{1,p}(\Omega_t)$ if and only if

$$(N - 1)||H^-||_\infty < \frac{1}{\mu_0}$$

Proof

The proof is a direct consequence of the remarks(5.1) ,(5.2) and The results of [3].

Conclusion

Let $\Omega_t = \Omega \setminus K$ with $K$ An obstacle that moves Inside of $\Omega$ So $\Omega_0 = \Omega \setminus K$ Is a critical shape of the functional $\lambda_{1,p}(\Omega)$ If and if there is a lagrange multiplier $\beta < 0$ Depends on the domain $\Omega_0$ verifying the following relation:

$$|\nabla u| = \left(\frac{-\beta}{p-1}\right)^{\frac{1}{p}} \text{ on } \partial K$$

(23)

Let $\Omega_0$ be a critical form of the first eigenvalue ($\lambda_{1,p}(\Omega)$) of the P-Laplace operator with Dirichlet boundary conditions, from proposition (5.1), we conclude that:

if $(N - 1)||H^-||_\infty < \frac{1}{\mu_0}$, $\Omega$ is a local strict minimum for the first eigenvalue ($\lambda_{1,p}(\Omega)$) of the P-Laplace operator.

References


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