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Conditional Risk Measures Relying on International Accounting Standards

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Abstract

This paper is devoted to the definition of an Expected -Shortfall -like Risk Measure, relying mainly on IAS39, being applied in EU. The connection between certain reporting variables and corporate capital requirements is a subject that is not entirely studied. Since the reporting variables are statistically dependent, we propose a model of Principal Components Regression for the assets' forecast under a specific IAS. Consequently, a Coherent Risk Measure is needed for the calculation of the capital requirements which provide a securitization of the corporate assets.

Mathematics Subject Classifications: 91G70; 62P05; 91B30

Keywords: Principal Components Regression; Credit Risk; Expected Shortfall; Firm's Assets

1. International Accounting Standards -Introduction

International Accounting Standards are uniform addressing rules and recognition accounting issues and the formulation of economic conditions leading to the transparency, comparability and uniform valuation. They are issued by the International Accounting Standards Committee from 1973, till the end of 2001. The recent name of International Accounting Standards (IAS) are called International Financial Reporting Standards. The procedure for issuing an IAS is through the European Commission Regulators Securities for selecting topics for consideration and study and issue plans by theme. These plans are submitted to the Council for approval and since they are approved, they are sent to stakeholders (accounting organizations, governments, stock exchange authorities) for study and comments taken into account by the Council and thus made the necessary amendments to the original plans. Most studies mainly describe the situation only a particular country. In [12], [13], what was mentioned is that the accounting data from the accounting systems of developing countries are not related to the decision-making models of the least developed countries. Especially, IAS 39 -being suggested by European Commision- classifies financial assets of a firm into: (1) loans and receivables not held for trading; (2) held-to-maturity investments; (3) financial instruments held for trading, including derivatives; and (4) available-for-sale financial assets. Financial assets in (1), (2), (3), and (4) are recognized at, respectively, amortized cost; amortized cost subject to impairment; fair value with changes in fair value recognized in profit or loss; and fair value with changes in fair value recognized in other comprehensive income. Most financial liabilities are recognized at cost, except derivatives and liabilities held for trading, which are recognized at fair value with changes in fair value recognized in profit or loss. Armstrong et al. in [4] explain these implications of IAS39. The variables indicated by [4] to be the components of the Assets of a firm according to the IAS39, are in general dependent. A question which arises from the aspect of the credit risk is which is an appropriate monetary risk measure, being associated to IAS39 that provides a capital requirement value for any value of the components of the financial assets. By this article, we provide an answer to this question. Since the components of the assets are dependent, we cannot summarize them by using linear regression. We take the first *principal component* of the four components of the Assets, according to IAS39. Since the annual data are rare in this case, we may use quarterly data. Then we apply one -variable linear regression between the historical values of the first principal component $Z^{(1)}$ (the one which corresponds to the maximum eigenvalue of the $X^T X$-where X is the $n \times 4$ design matrix of the historical values of the 4 variables of the asset-components according to IAS39 for n quarters of years) and the Total Current Assets minus Cash Only Y (these are standard variables of a balance sheet of a firm), as the *response variable*. Hence the linear regression model is

$$Y_i = \beta_0 + \beta_1 Z_i^{(1)} + \epsilon_i, i = 1, 2, ..., n.$$

This linear regression gives us a model for the prediction of the *credit risk* of the firm. This model is well-known as *Principal Components Regression* and a seminal reference for it is [11]. The *determination coefficient* of the above model is

$$\frac{S_{yz}^2}{S_{zz} \cdot S_{yy}} = \frac{1}{\lambda^{(1)}} \cdot \frac{S_{yz}^2}{S_{yy}},$$

where $S_{yz} = \sum_{i=1}^{n} Y_i Z_i^{(1)}, S_{yy} = \sum_{i=1}^{n} Y_i^2$. The prediction under the above model for any value x_0 of the random vector of the four variables of the asset-components included in IAS39 is

$$Y_{x_0} = \hat{b}_0 + \hat{b}_1 u^{(1)} \cdot x_0.$$

Applying any of the well-known risk measure on the prediction variable Y_{x_0}, we take a capital requirement functional for the credit risk of a firm, relying on IAS39. For example, Expected Shortfall to be the coherent risk measure that replaces Value-at-Risk according to the Basel Committe [5] for regulatory capital purposes in the trading book. We assume that the uncertainty corresponds to a non-atomic probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Hence, we may define the IAS39- Expected Shortfall in the following form:

$$ES_a^{IAS39}(Y_{x_0}) := \max_{\pi \in [0, \frac{1}{a}\mathbf{1}]} \pi(-Y_{x_0}), a \in (0,1).$$

according to the dual representation of the Expected Shortfall in [10, Th.4.1], either the IAS39-Adjusted Expected Shortfall, which is defined as follows:

$$AES_{a,b}^{IAS39}(Y_{x_0} := \max_{\pi \in [\frac{1}{b}\mathbf{1}, \frac{1}{a}\mathbf{1}]} \pi(-Y_{x_0}), a \in (0,1), b > 1,$$

where π in both of cases is $\pi = \frac{d\mathbb{Q}_\pi}{d\mathbb{P}}$, and $\mathbb{Q}_\pi$ is a probability measure on the measurable space $(\Omega, \mathcal{F})$, according to the definition of the Adjusted Expected Shortfall in [9, Lem.6].

If we would like to recall the basics of Principal Components Analysis, we start with a $n \times p$ *design matrix*

$$X = [x_1, x_2, ...x_p],$$

where $x_i \in \mathbb{R}^n, i = 1, 2, ..., p$, n corresponds to the number of the observations and p corresponds to the number of the reporting variables, included in some IAS. As mentioned above, in IAS39 $p = 4$, while n is the number of quarters of tehe balance sheet of the firm. The *variance -covariance* matrix arising from this design matrix X is the $p \times p$ matrix $X^T \cdot X$, where A^T denotes the transpose of a matrix A. We notice that $X^T \cdot X$ is a symmetric matrix, hence its eigenvalues are positive. If we recall the notions defined in Multivalued Statistical Analysis, $\lambda^{(j)}$ is the j -ordered eigenvalue of $X^T \cdot X$ in the sense that

$$\lambda^{(1)} \geq ... \geq \lambda^{(p)},$$

while $u^{(j)} \in \mathbb{R}^p$ is the normalized eigenvector corresponding to $\lambda^{(j)}, j = 1, 2, ..., p$. Then the variable

$$z^{(j)} = \sum_{i=1}^{p} u_i^{(j)} x_i,$$

is the Principal Component corresponding to the eigenvalue $\lambda^{(j)}$ of $X^T \cdot X$. We also recall the following

Lemma 1.1.

$$S_{zz} = \sum_{j=1}^{n} (z_j^{(j)})^2 = \lambda^{(j)}.$$

Proof.

$$X \cdot u^{(j)} = \sum_{i=1}^{p} u_i^{(j)} x_i = z^{(j)},$$

hence

$$(z^{(j)})^T \cdot z^{(p)} = S_{zz} = (u^{(j)})^T \cdot X^T \cdot X \cdot u^{(j)}$$

$$= (u^{(j)})^T \cdot \lambda^{(j)} \cdot u^{(j)} = \lambda^{(j)}.$$

□

Other techniques of Multivariate Statistical Analysis, like Discriminant Analysis are also used mainly in the construction of the *scoring systems*, whose aim is the separation of clients of a financial institution (e.g. by a bank), according to the severity of the credit risk. For scoring systems' seminal references, see for example [16] and the references therein, both with [17] for the neural network approach on this topic. We use Principal Components Analysis, rather because our aim is to calculate the additional capital requirement of a firm, if its liquidity is limited, as it arises from the so-called *response variable* Y of the PCR model. This can be also understood as a model of severity of credit risk discrimination. For example, if ρ^{IAS39} is a coherent risk measure relying on IAS39, if $\rho(Y_{x_0}) < 0$, then no credit risk arises. If $\rho^{IAS39} \in [0, M]$, where M is a capital limit posed mainly by the specific bank, then the credit risk may be considered to be low. For questions regarding Multivariate Analysis, a reference is [14]. More specifically, [14, Ch.12] refers to Principal Components Analysis. Finally, the random variables $X : \Omega \to \mathbb{R}$ which belong to some L^p-space, sometimes are denoted by $X \in L^p_{\mathbb{R}}$, while the random vectors $X = (X_1, X_2, ..., X_n)$, where $X_i \in L^p_{\mathbb{R}}, i = 1, 2, ..., n$ are denoted by $X \in L^p_{\mathbb{R}^n}$. For questions about L^p-spaces, see in

2. Conditional Risk Measures relying on IAS39

According to the above Principal Component Regression Model, the *prediction* for a certain vector of the report variable $x_0 \in L^1_{\mathbb{R}^4}(\Omega, \mathbb{F}, \mathbb{P})$ is equal to

$$Y_{x_0} = \hat{b}_0 + \hat{b}_1 u^{(1)} \cdot x_0.$$

Since the distribution of the random variable Y_{x_0} relies on the distribution of x_0, $Y_{x_0} \in L^1(\Omega, \sigma(x_0), \mathbb{P})$, hence the Coherent Risk Measures being defined on this space may be called *Conditional Coherent*. For the Coherent Risk Measures, seminal references are [3] and [6], while for Conditional Coherent

Risk Measures, see [7], [15]. Also, about Expected Shortfall and its Coherence, see [1]. We may define the folowing Risk Measures:

Definition 2.1. *The Expected Shortfall relying on IAS39 is defined as follows:*

$$ES_a^{IAS39}(Y_{x_0}) := \max_{\pi \in [0, \frac{1}{a}\boldsymbol{1}]} \pi(-Y_{x_0}), a \in (0,1).$$

Definition 2.2. *The Adjusted Expected Shortfall relying on IAS39 is defined as follows:*

$$AES_a^{IAS39}(Y_{x_0}) := \max_{\pi \in [\frac{1}{b}\boldsymbol{1}, \frac{1}{a}\boldsymbol{1}]} \pi(-Y_{x_0}), a \in (0,1), b > 1.$$

Theorem 2.3. $ES_a^{IAS39}, a \in (0,1)$ *is well-defined and coherent on* $L^1_{\mathbb{R}}(\Omega, \mathcal{F}, \mathbb{P})$.

Proof. Let us verify the properties of coherence in the case of ES_a^{IAS39}.

(*i*) (Translation Invariance)

$$ES_a^{IAS39}(Y_{x_0} + t \cdot \mathbf{1}) = \max_{\pi \in [0, \frac{1}{a}\mathbf{1}]} \pi(-Y_{x_0} - t\mathbf{1}) =$$

$$\max_{\pi \in [0, \frac{1}{a}\mathbf{1}]} \pi(-Y_{x_0}) - t \max_{\pi \in [0, \frac{1}{a}\mathbf{1}]} \pi(-\mathbf{1}) = ES_a^{IAS39}(Y_{x_0}) - t,$$

due to the fact that

$$\pi(-\mathbf{1}) = \int_\Omega -\frac{d\mathbb{Q}_\pi}{d\mathbb{P}}(\omega) d\mathbb{P}(\omega) = -\int_\Omega d\mathbb{Q}_\pi(\omega) = -1.$$

(*ii*) (Subadditivity) If x_0, s_0 are two different random vectors of the reporting variables, then we obtain the prediction random variables Y_{x_0}, Y_{s_0}, respectively. Then,

$$ES_a^{IAS39}(Y_{x_0} + Y_{s_0}) = \max_{\pi \in [0, \frac{1}{a}\mathbf{1}]} \pi(-Y_{x_0} - Y_{s_0}) \leq$$

$$\max_{\pi \in [0, \frac{1}{a}\mathbf{1}]} \pi(-Y_{x_0}) + \max_{\pi \in [0, \frac{1}{a}\mathbf{1}]} \pi(-Y_{s_0}) = ES_a^{IAS39}(Y_{x_0}) + ES_a^{IAS39}(Y_{s_0}).$$

(*iii*) (Positive Homogeneity)

$$ES_a^{IAS39}(\lambda \cdot Y_{x_0}) = \max_{\pi \in [0, \frac{1}{a}\mathbf{1}]} \pi(-\lambda \cdot Y_{x_0}) = \lambda \cdot \max_{\pi \in [0, \frac{1}{a}\mathbf{1}]} \pi(-Y_{x_0}) =$$

$$= \lambda \cdot ES_a^{IAS39}(Y_{x_0}), \lambda \in \mathbb{R}_+.$$

(*iv*) (Monotonicity) If $Y_{s_0} \geq Y_{x_0}, \mathbb{P}-a.e.$, then $-Y_{x_0} \geq -Y_{s_0}, \mathbb{P}-a.e.$. Hence for any $\pi = \frac{d\mathbb{Q}_\pi}{d\mathbb{P}} \in [0, \frac{1}{a}\mathbf{1}], \mathbb{P}-a.e.$, such that $\mathbb{Q}_\pi$ is a probability measure on $(\Omega, \mathcal{F})$ we get that $\pi(-Y_{x_0}) \geq \pi(-Y_{s_0})$, in terms of the order in $\mathbb{R}$. By taking maxima all over the set of π, we take $ES_a^{IAS39}(Y_{x_0}) \geq ES_a^{IAS39}(Y_{s_0})$.

□

Theorem 2.4. $AES_{a,b}^{IAS39}, a \in (0,1), b > 1$ *is well-defined and coherent on* $L^1_{\mathbb{R}}(\Omega, \mathcal{F}, \mathbb{P})$.

Proof. Let us verify the properties of coherence in the case of $AES_{a,b}^{IAS39}$.

(i) (Translation Invariance)

$$AES_{a,b}^{IAS39}(Y_{x_0}+t\cdot\mathbf{1})=\max_{\pi\in[\frac{1}{b}\mathbf{1},\frac{1}{a}\mathbf{1}]}\pi(-Y_{x_0}-t\mathbf{1})=$$

$$\max_{\pi\in[\frac{1}{b}\mathbf{1},\frac{1}{a}\mathbf{1}]}\pi(-Y_{x_0})-t\max_{\pi\in[\frac{1}{b}\mathbf{1},\frac{1}{a}\mathbf{1}]}\pi(-\mathbf{1})=AES_{a,b}^{IAS39}(Y_{x_0})-t,$$

due to the fact that

$$\pi(-\mathbf{1})=\int_{\Omega}-\frac{d\mathbb{Q}_\pi}{d\mathbb{P}}(\omega)d\mathbb{P}(\omega)=-\int_{\Omega}d\mathbb{Q}_\pi(\omega)=-1.$$

(ii) (Subadditivity) If x_0, s_0 are two different random vectors of the reporting variables, then we obtain the prediction random variables Y_{x_0}, Y_{s_0}, respectively. Then,

$$AES_{a,b}^{IAS39}(Y_{x_0}+Y_{s_0})=\max_{\pi\in[\frac{1}{b}\mathbf{1},\frac{1}{a}\mathbf{1}]}\pi(-Y_{x_0}-Y_{s_0})\leq$$

$$\max_{\pi\in[\frac{1}{b}\mathbf{1},\frac{1}{a}\mathbf{1}]}\pi(-Y_{x_0})+\max_{\pi\in[\frac{1}{b}\mathbf{1},\frac{1}{a}\mathbf{1}]}\pi(-Y_{s_0})=AES_{a,b}^{IAS39}(Y_{x_0})+AES_{a,b}^{IAS39}(Y_{s_0}).$$

(iii) (Positive Homogeneity)

$$AES_{a,b}^{IAS39}(\lambda\cdot Y_{x_0})=\max_{\pi\in[\frac{1}{b}\mathbf{1},\frac{1}{a}\mathbf{1}]}\pi(-\lambda\cdot Y_{x_0})=\lambda\cdot\max_{\pi\in[\frac{1}{b}\mathbf{1},\frac{1}{a}\mathbf{1}]}\pi(-Y_{x_0})=$$

$$=\lambda\cdot AES_{a,b}^{IAS39}(Y_{x_0}),\lambda\in\mathbb{R}_+.$$

(iv) (Monotonicity) If $Y_{s_0}\geq Y_{x_0},\mathbb{P}-a.e.$, then $-Y_{x_0}\geq -Y_{s_0},\mathbb{P}-a.e.$. Hence for any $\pi=\frac{d\mathbb{Q}_\pi}{d\mathbb{P}}\in[\frac{1}{b}\mathbf{1},\frac{1}{a}\mathbf{1}],\mathbb{P}-a.e.$, such that $\mathbb{Q}_\pi$ is a probability measure on $(\Omega,\mathcal{F})$ we get that $\pi(-Y_{x_0})\geq\pi(-Y_{s_0})$, in terms of the order in $\mathbb{R}$. By taking maxima all over the set of π, we take $AES_{a,b}^{IAS39}(Y_{x_0})\geq AES_{a,b}^{IAS39}(Y_{s_0})$.

□

3. Practical Implications- Estimations

According to IAS39, the main reporting variables are $p=4$. On the other hand, the estimation of $ES_a^{IAS39}(Y)$, where $Y=(Y_1,Y_2,...,Y_n)$ is the sample of the variable which denotes the Total Assets minus Cash Only (namely the credit risk), may be made through the method implied by [8, Eq.(8)] and the estimation of $AES_{a,b}^{IAS39}(Y)$ by $\frac{1}{b}\overline{Y}$ (as it arises from the dual representation of it). Hence,

$$\hat{ES}_a^{IAS39}=-\frac{1}{n\cdot a}\sum_{i=1}^{[na]}Y_{i:n},$$

where $[na]$ is the integer part of $n\cdot a$ and $Y_{i:n}$ is the i-th ordered observation in the sample $(Y_1,Y_2,...,Y_n)$.

The balance -sheets of the firms contained in the various sites (Yahoo!-Finance, Bloomberg etc.), contain the following catalog of accounting variables:

(1) Cash And Equivalents

Short-Term Investments 2

Trading Asset Securities 4

TOTAL CASH AND SHORT TERM INVESTMENTS

(2) Accounts Receivable 1

Notes Receivable 1

Other Receivables 1

TOTAL RECEIVABLES 1

(3) Inventory 1
Finance Division Loans And Leases, Current 1
Finance Division Other Current Assets 4
Other Current Assets 3

(4) TOTAL CURRENT ASSETS

By TOTAL CURRENT ASSETS-Cash and Equivalents we obtain the direct liquidity frame of the firm. The other numbers are assigned, according to what what sort of component we think they could correspond in the accounting variables' of IAS39 characterization: (1) loans and receivables not held for trading; (2) held-to-maturity investments; (3) financial instruments held for trading, including derivatives; and (4) available-for-sale financial assets.

Also, we calculated the average yield of the 10-year-maturity bond of the Greek State, in order to caluclate the amortization factors by which every value in the balance sheet is transfered into the year 2003 so that all values to be camparable.

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