Dynamic Simulation of a Natural Circulation Drum Boiler Considering Stress in the Drum Wall

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Abstract
A mathematical model is developed to simulate the behavior of a natural circulation drum boiler. The present study investigates not only the fluid dynamics of the boiler and the heat conduction process in the drum shell but also the generated effective stresses in the drum metal wall. The model is established
based on physical laws. The results obtained by the simulation could address the understanding of interactions of state variables inside out. The performance parameters can be estimated at any operating conditions, which include the drum pressure, the volume of entire water, the steam quality at the riser top, the liquid level in the drum, and the distribution of temperature and stress in the drum shell. This study demonstrates that the model can describe the complex behavior of the natural circulation drum boiler with the given operating scenarios before actual plant implementation. The simulation result has been validated with the start-up data from a 2MWe circulating fluidized bed power plant.

**Keywords:** drum boiler; start-up; fluid dynamics; heat conduction; stress.

1. **Introduction**

Natural circulating drum boilers have widely been used in thermal power plants. A boiler is a complex unit in which the phase-change of working fluid (water-steam mixture) and heat transfer through walls occur, and the stress is generated inside the drum metal wall. Especially, the above processes take place more intensely during the startup process. In addition, a biomass boiler differs from other types of boilers such as gas- or oil-fired boiler since it takes a much longer time to fully burn the fuel [1]. This longer time to respond makes it more difficult to control. Therefore, well understanding of boiler behavior is very important for design and operation. Over the last decades, the dynamic simulation of the boiler has been received much attention from research groups.

F. Alobaid et al. [2] provided a review of the present state about the mathematical modeling of the thermal power plant. The authors mentioned the models and solution methods to simulate almost components in the plant and the processes of plants. They also considered the plant optimization, security, and safety. This review focused on the summary and highlight of the last studies. E.J. Adam and J.L. Marchetti [3] established a model including a dynamic evaporation sub-model and a phase separation sub-model in the drum. The model presented the drum liquid level and steam pressure in the drum. The work introduced a useful tool for the investigation to achieve good performance of the boiler system. K.J. Åström and R. D. Bell [4] developed a mathematical model with three state variables. The model is based on the principles of conservations, water-steam tables with the quadratic polynomials and the physical parameters of the boiler system. K.J. Åström and R. D. Bell [5] continued to develop a model with four state variables where the model used an empirical equation for calculating a flow of steam through the surface of the liquid in the drum. The volume of steam bubbles in the drum is the fourth state variable calculated by using that steam flow. Therefore, the model can simulate the high accuracy of the drum liquid level because the amount of steam in the water-steam mixture is computed exactly. H. Kim and S. Choi [6] improved the model developed by K.J. Åström and R. D.
Bell [5] to calculate the amount of steam inside the drum using two constitutional
equations for condensation flow and steam flow through the water surface [6].

B. Li et al [7] constructed a program to simulate the start-up behavior of the
circulation drum boiler system including economizers, superheaters, reheaters,
and evaporators. The model used in this program took into account the heat
transfer process, the mixture flow characteristics in the riser calculated using the
1D method, and using the lumped parameter method for drum and downcomer
model. Characteristics of flow and thermodynamics in the components could be
obtained from the program, therefore, it is useful to predict the boiler performance
and optimize the start-up process. P.U. Sunil et al. [8] proposed an integrated
model based on two models, one is developed by K.J. Åström and R. D. Bell [5],
another one is established by K.S. Bhambare, S.K. Mitra and U.N. Gaitonde [9].
This model presented three sub-models for simulation of drum boiler behavior
during start-up and load change period; the first one described the rigorous heat
transfer process for from the gas side through the riser wall to the liquid side, the
second one introduced the 1-D recirculation loop model, the last one is the lumped
drum model.

H.E. Emara-Shabaik et al. [10] studied the dynamics of circulation loop flow
in the drum boiler and the heat transfer process through the riser wall. That the
model focuses on the heat transfer phenomena between the inner riser metal wall
and the two-phase flow inside the riser tubes could calculate the real-time
temperature of the riser wall temperature [10]. Therefore, the model is a useful
application for checking the operating drum boiler to avoid overheating in riser
tubes. S. Bracco [11] established two simulation models based on the heat
conduction equation of Fourier for the steam drum wall. The models are solved by
the discretization of the equation in implicit and explicit methods. Those models
with the given steam temperature implemented in Matlab/Simulink environment
predicted the profile temperature of drum shell and stress distribution of metal
wall.

M. A. Habib et al. [12] considered the thermal stresses in the riser tube being
the thin-wall to determine the maximum boiler swing rate that guarantees the safe
operation of the drum boiler. Some researchers focused on the investigation of
thermal stress in the thick metal wall in the boiler system to design the control
system and to optimize the energy consumption and startup procedure of the
boiler system. For example, the model presented by Krüger et al. [13] took into
account the thermal stress of the drum metal wall but it calculated only one state
variable which is the radial thermal stress in the drum metal wall.

In the operation of the boiler, especially in the wide range of operating
conditions, several factors such as steam pressure, drum liquid level, and stresses
could negatively influence the safety of the operational process, start-up time and
life-span of the boiler. The adverse effects need to be comprehensively assessed.
However, few studies have focused on the simultaneous estimation of the above-
mentioned factors through the wide range of operating conditions as the start-up,
load changes and/or shut-down process. Therefore, it is necessary to introduce a
model that could concurrently estimate the effects of all major factors on the boiler.

In this paper, we present an integrated model to simulate and analyze the start-up process of natural circulating drum boilers from the data of the 2MW power pilot-scale system. Our proposed model is a more complete tool for studying different stages of boiler development such as a) design and material selection, b) behaviors analysis, c) operational control and optimization, d) operational training and efficiency increase, e) failures detection and risks containment [8].

2. The natural circulation drum boiler model.

A circulating fluidized bed (CFB) boiler power plant which consists of the CFB furnace, the steam turbine, the electrical generator, and the boiler system. The boiler system includes the economizer, superheater and boiler. The boiler is one of the most important modules in the thermal electric power plant [11].

The boiler is modeled in this study which is the natural circulating drum boiler as depicted in Fig 1. The drum boiler is separated into three main components: the riser tubes, the downcomer tubes, and the drum. The real system is much more complicated than the system shown in Fig. 1. The water is heated by the economizer or the heater before it goes into the drum where there are two phases: steam and water. The steam passes to the primary superheater and the water flows downward to downcomer tubes and then upward to riser tubes. The water inside the riser tubes is vaporized by heat transferred from the furnace. Therefore, it causes a mixture of water-steam inside the riser tubes. The density of the water-steam mixture inside the riser tubes is lower than the density of the water at riser tube inlet, thus, the water-steam mixture moves up through the risers and returns to the drum. As a result, there is a natural circulation loop of the working fluid taking place in the drum-downcomer-riser. Basically, the circulation loop of working fluid is created by the difference in density between the water in the downcomer tubes and the water-steam mixture in the riser tubes. The detailed description of the natural circulation loop in the drum boiler can be found elsewhere [3, 5, 6, 10, 14].

The drum is the horizontal cylindrical thick-wall component. In this study, the drum shell is considered that the thick-wall consists of two sections. The first section is a metal wall and the second one is an insulation wall.
The dynamic sub-model of working fluid flowing inside the drum boiler is established based on the balance equations of the mass and energy conservation for the whole drum boiler system and for the riser tubes sub-system. In addition, one mass balance equation is for steam bubbles under the drum level, and one momentum balance equation is for the working fluid in the circulation loop.

The heat conduction sub-model of the drum shell is derived from the fundamental laws of heat transfer and the energy balance equations of the layers in the drum shell [11]. The wall stress is calculated according to equations from M. A. Habib et al. [12].

The mathematical model of the natural circulating drum boiler is integrated including the dynamic sub-model of the working fluid flow inside the drum boiler, the heat conduction sub-model of the drum shell, and the stress calculation inside the drum wall. The integrated model can predict not only the dynamics of two-phase flow but also the temperature in each layer of the drum shell and the stress in each layer of the metal wall.

2.1 General assumptions

The sub-models are simplified based on the following assumptions.

- A lumped parameter method is used in the models.
- The temperature of feedwater is assumed no change.
- All quantities are assumed uniformly in a cross-section of the tube
There is no pressure drop and temperature gradients along the tubes.
- The ratio between the drum length and its diameter is assumed sufficiently high, therefore the only radial variations of the heat flux and temperature are considered.
- The metal wall is the same temperature as the working fluid.

Each sub-model is described in detail in the following section.

2.2 Sub-model for the fluid dynamics in the drum boiler

The model for the fluid dynamics in the drum boiler is derived from physical laws and referred to the empirical equation of the steam flow rate out of the drum liquid level surface [5]. The detailed governing equations of the fluid dynamic sub-model are given below.

2.2.1 Governing equations for the whole system

The mass conservation equation can be written as follows:

$$\frac{d}{dt} [\rho_s V_s + \rho_w V_w] = W_f - W_s. \quad (1)$$

The energy conservation equation:

$$\frac{d}{dt} [\rho_s u_s V_s + \rho_w u_w V_w + m_t C_p t_m] = Q + W_f h_f - W_s h_s, \quad (2)$$

where the internal energy, $u$, is calculated by Eq. (3)

$$u = h - \frac{p}{\rho}, \quad (3)$$

by replacing Eq. (3) into Eq. (2), the energy conservation equation for the whole system is rewritten as

$$\frac{d}{dt} [\rho_s h_s V_s + \rho_w h_w V_w - P V_t + m_t C_p t_s] = Q + W_f h_f - W_s h_s, \quad (4)$$

where the entire volume of all components in the system is

$$V_t = V_s + V_w. \quad (5)$$

2.2.2 Governing equations for the riser tubes

The mass conservation equation can be expressed as follows:
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\[
\frac{d}{dt}(\rho_s \bar{x}_v V_r + \rho_w (1 - \bar{x}_v) V_r) = W_{dc} - W_r. \tag{6}
\]

The energy conservation equation of the riser tubes section is defined as

\[
\frac{d}{dt}(\rho_s h_s \bar{x}_v V_r + \rho_w h_w (1 - \bar{x}_v) V_r - p V_r + m_r C_p t_s) = Q + W h_w - (x_r h_c + h_w) W_r. \tag{7}
\]

Another form of energy balance is expressed in Eq. (8) resulting from the combination of Eqs. (6 and 7) in which \(W_r\) is eliminated for the convenience in solving the model.

\[
h_c (1 - x_r) \frac{d}{dt} (\rho_s \bar{x}_v V_r) + \rho_w (1 - \bar{x}_v) V_r \frac{dh_w}{dt} - x_r h_c \frac{d}{dt} (\rho_w (1 - \bar{x}_v) V_r) + \\
\rho_s \bar{x}_v V_r \frac{dh_s}{dt} = V_r \frac{dp}{dt} + m_r C_p \frac{dx_r}{dt} = Q - x_r h_{lt} W_{dc}. \tag{8}
\]

where \(h_{lt}\) is the latent heat of condensation, \(h_{lt} = h_s - h_w\).

From equation (6), the flow rate of working fluid in the riser tubes can be computed from Eq. (9) where the rate of change of steam pressure in the drum and steam mass fraction at the outlet of riser tubes is taken into account.

\[
W_r = W_{dc} - V_r \left( \bar{x}_v \frac{\partial \rho_s}{\partial p} + (1 - \bar{x}_v) \frac{\partial \rho_w}{\partial p} + (\rho_s - \rho_w) \frac{\partial \bar{x}_v}{\partial p} \right) \frac{dp}{dt} + V_r (\rho_w - \rho_s) \frac{\partial \bar{x}_v}{\partial x_r} \frac{dx_r}{dt}. \tag{9}
\]

2.2.2.1 The fraction of steam in risers

The steam mass fraction in the steam-water mixture flow at a specific location \((z)\) along the risers is given by

\[
x_z = \frac{QA}{Wh_{lt} V z}. \tag{10}
\]

After rearrangement and using dimensionless length, the local mass fraction of steam can be represented as a function of the location of riser tubes and the mass fraction of steam at the top of riser tubes

\[
x_z(\varepsilon) = x_r \varepsilon, 0 \leq \varepsilon \leq 1.
\]

The relation between volume fraction and mass fractions of steam is calculated by

\[
x_v = f(x_z) = \frac{\rho_w x_z}{\rho_s + (\rho_w - \rho_s) x_z}. \tag{11}
\]
Therefore, the average volume fraction $\bar{x}_v$ is obtained from the integration in Eq. (13)

$$\bar{x}_v = \int_0^1 x_v(\varepsilon) = \frac{1}{x_r} \int_0^{x_r} f(\varepsilon) d\varepsilon = \frac{\rho_w}{\rho_w - \rho_s} \left[ 1 - \frac{\rho_s}{(\rho_w - \rho_s) x_r} \right] \ln \left( 1 + \frac{\rho_w - \rho_s}{\rho_s} x_r \right).$$  (12)

### 2.2.3 Mass balance equation of steam bubbles inside the vapor-water mixture in the drum

The equation of mass equilibrium for the steam bubbles in the vapor-water mixture inside the drum is

$$\frac{d}{dt} (\rho_s V_{sd}) = x_r W_r - W_{sd} - W_{cd},$$  (13)

where $W_{cd}$ is the mass flow rate of condensed water, and $W_{sd}$ is the mass flow rate of the steam out of the drum liquid surface that can be calculated by Eqs. (15) and (16), respectively.

#### 2.2.3.1 Energy equilibrium for phase change of steam within the drum

The energy equilibrium equation of the phase change of steam within the drum is based on the laws of thermodynamics.

$$W_{cd} h_{lt} + (h_f - h_w) W_f = \rho_s V_{sd} \frac{dh_s}{dt} + \rho_w V_{wa} \frac{dh_w}{dt} + m_d C_p \frac{dt_s}{dt} - V_a \frac{dP}{dt}.$$  (14)

From Eq. (11) the rate of the condensed water flow is calculated by

$$W_{cd} = \frac{1}{h_{lt}} (h_f - h_w) W_f + \frac{1}{h_{lt}} (\rho_s V_{sd} \frac{dh_s}{dt} + \rho_w V_{wa} \frac{dh_w}{dt} + m_d C_p \frac{dt_s}{dt} - V_a \frac{dP}{dt}).$$  (15)

The rate of the steam flow out of the drum liquid level surface, $W_{sd}$, is determined from an empirical model as expressed in Eq. (16)

$$W_{sd} = \frac{\rho_s}{T_d} (V_{sd} - V_{sd}^0) + x_r W_{dc} + x_r \beta (W_{dc} - W_r).$$  (16)
2.2.4 Circulation flow of the drum-downcomer-riser loop

The momentum equilibrium equation for the drum-downcomer – riser loop is

\[ (L_r + L_{dc}) \frac{dW_{dc}}{dt} = (\rho_w - \rho_s)\bar{x}_w V_r g - \frac{f}{2 \rho_w A_{dc}} W_{dc}^2. \]  

(17)

At steady-state operation of the system, the flow rate of downcomer flow is calculated by Eq. (18) as follows.

\[ W_{dc} = \sqrt{\frac{2 \rho_w A_{dc} (\rho_w - \rho_s) g \bar{x}_w V_r}{f}}. \]  

(18)

2.2.5 The fluid dynamics model and parameters in summary.

Based on the fundamental equations analyzed above the overall fluid dynamics model of the natural circulating drum boiler is established by the four differential equations. The system of differential equations with four state variables including the drum pressure, P; the total volume of entire water in the system, \( V_w \); the steam quality at the riser top, \( x_r \); and the total volume of steam bubbles in the steam-water mixture below the drum liquid level, \( V_{sd} \); can be written as follows:

\[
\begin{bmatrix}
  a_{11} & a_{12} & 0 & 0 \\
  a_{21} & a_{22} & 0 & 0 \\
  a_{31} & 0 & a_{33} & 0 \\
  a_{41} & 0 & a_{43} & a_{44}
\end{bmatrix}
\begin{bmatrix}
  \frac{dP}{dt} \\
  \frac{dV_w}{dt} \\
  \frac{dx_r}{dt} \\
  \frac{dV_{sd}}{dt}
\end{bmatrix}
= \begin{bmatrix}
  W_f - W_s \\
  Q + W_f h_f - W_s h_s \\
  Q - x_r h_{lt} W_{dc} \\
  \frac{\rho_s}{T_d} (V_{sd}^0 - V_{sd}) + \frac{1}{h_{lt}} (h_f - h_{w}) W_f
\end{bmatrix}
\]

(19)

where

\[ a_{11} = \left( V_w \frac{\partial \rho_w}{\partial P} + V_s \frac{\partial \rho_s}{\partial P} \right), \]
\[ a_{12} = (\rho_w - \rho_s), \]
\[ a_{21} = (\rho_w V_w \frac{\partial h_w}{\partial P} + V_w h_w \frac{\partial \rho_w}{\partial P} + \rho_s V_s \frac{\partial h_s}{\partial P} + h_s V_s \frac{\partial \rho_s}{\partial P} - V_t + m_t C_p \frac{\partial t_s}{\partial P}), \]
\[ a_{22} = (\rho_w h_w - \rho_s h_s), \]
\[ a_{31} = \left( \rho_w \frac{\partial h_w}{\partial P} - x_r h_{lt} \frac{\partial \rho_w}{\partial P} \right) (1 - \bar{x}_v) V_r + \left( (1 - x_r) h_{lt} \frac{\partial \rho_s}{\partial P} + \rho_s \frac{\partial h_s}{\partial P} \right) \bar{x}_v V_r \]
\[ + (\rho_s + (\rho_w - \rho_s) x_r) h_{lt} V_r \frac{\partial \bar{x}_v}{\partial P} - V_t + m_r C_p \frac{\partial t_s}{\partial P} \right], \]
\[ a_{33} = (1 - x_r) \rho_s + x_r \rho_w \right) h_{lt} \frac{\partial \bar{x}_v}{\partial x_r}, \]
\[ a_{41} = V_{sd} \frac{\partial \rho_s}{\partial P} + \frac{1}{h_{lt}} \left( \rho_s V_s \frac{\partial h_s}{\partial P} + \rho_w V_w \frac{\partial h_w}{\partial P} + m_a C_p \frac{\partial t_s}{\partial P} - (V_{sd} + V_{wd}) \right) \]
\[ + x_r (1 + \beta) V_r \bar{x}_v \frac{\partial \bar{x}_v}{\partial P} + (1 - \bar{x}_v) \frac{\partial \rho_w}{\partial P} + (\rho_s - \rho_w) \frac{\partial \bar{x}_v}{\partial P}, \]
\[ a_{43} = x_r (1 + \beta) V_r (\rho_s - \rho_w) \frac{\partial \bar{x}_v}{\partial x_r}, \]
\[ a_{44} = \rho_s. \]

At the steady-state operation, the right-hand side of the equation system (19) becomes zero leading to four algebraic equations

\[ W_f = W_s, \quad (20) \]
\[ Q = W_s h_s - W_f h_f, \quad (21) \]
\[ Q = W_{dc} x_r h_{lt}, \quad (22) \]
\[ V_{sd} = V_{sd}^0 - \frac{T_d (h_w - h_f)}{\rho_s h_{lt}} q_f. \quad (23) \]

The individual derivative of the average of the volume fraction of steam in riser tubes according to pressure, P is expressed by

\[ \frac{\partial \bar{x}_v}{\partial P} = \frac{1}{(\rho_w - \rho_s)^2} \left( \rho_w \frac{\partial \rho_s}{\partial P} - \rho_s \frac{\partial \rho_w}{\partial P} \right) \left( 1 + \rho_w \frac{1}{\rho_s 1+\delta} - \rho_s - \rho_w \frac{\partial \rho}{\partial \rho_s} \ln(1 + \delta) \right), \quad (24) \]

where \( \delta \) is defined as

\[ \delta = \frac{x_r (\rho_w - \rho_s)}{\rho_s}. \quad (25) \]
The individual derivative of the average of the volume fraction of steam in the steam-water mixture inside the riser tubes according to the mass fraction of steam at the top of riser tubes, $x_r$, is calculated by

$$\frac{\partial \bar{x}_v}{\partial x_r} = \frac{\rho_w}{\rho_s \delta} \left( \frac{1}{\delta} \ln(1 + \delta) - \frac{1}{1 + \delta} \right).$$ (26)

The entire water volume inside the steam drum is determined from

$$V_{wd} = V_w - V_{dc} - (1 - \bar{x}_v)V_r.$$ (27)

and the stay time of steam in the steam drum is calculated from

$$T_d = \frac{\rho_s V_{sd}^0}{W_{sd}}.$$ (28)

The drum liquid level can be calculated by the equation as:

$$l = \frac{V_{wd} + V_{sd}}{A_d} = \frac{V_{wd}}{A_d} + \frac{V_{sd}}{A_d} = l_{wd} + l_{sd},$$ (29)

In which the term $l_{wd}$ describes the level variations effecting the total volume of water in the steam drum, term $l_{sd}$ represents the steam level variations effecting the total volume of steam bubbles below the water level in the steam drum.

2.3. **Heat conduction sub-model of the drum shell**

Since the steam drum is designed in the form of the pressure cylindrical vessel, a heat conduction model is developed based on the cylindrical coordinate ($r$, $\theta$, and $z$) for convenience.

2.3.1 **Energy balance of solid layers of uniform material**

The general energy balance for an element of control solid volume can be written as:

$$\sum_i dQ_{ini} = \sum_j dQ_{outj} + c\rho dV \frac{\partial T}{\partial t}$$ (30)

where:

$$\sum_i dQ_{ini} : \text{Sum of the inlet thermal flows.}$$
\[ \sum_{i} dQ_{\text{out}_i} : \text{Sum of the outlet thermal flow} \]

\[ q^o = cpdV \frac{\partial T}{\partial t} : \text{Rate of heat accumulation within the controlled volume} \]

And the equation of heat conduction along general \( x \)-direction can be expressed as:

\[ dQ_x = -k A_x \frac{\partial T}{\partial x} \] (31)

The summation of the inlet thermal flows is calculated from Eq. (32)

\[ \sum_{i} dQ_{\text{in}_i} = dQ_r + dQ_\theta + dQ_z. \] (32)

where:

\( dQ_r \): Inlet heat flow in the \( r \) direction

\( dQ_\theta \): Inlet heat flow in the \( \theta \) direction

\( dQ_z \): Inlet heat flow in the \( z \) direction

The heat conduction equations written for each direction of the cylindrical coordinated are described in Eqs. (33), (34) and (35).

\[ dQ_r = -kdA_r \frac{\partial T}{\partial r} = -k \frac{\partial T}{\partial r} rd\theta dz. \] (33)

\[ dQ_\theta = -\frac{k}{r} dA_\theta \frac{\partial T}{\partial \theta} = -\frac{k}{r} \frac{\partial T}{\partial \theta} dr dz. \] (34)

\[ dQ_z = -kdA_z \frac{\partial T}{\partial z} = -k \frac{\partial T}{\partial z} r d\theta dr. \] (35)

By substituting Eqs. (33), (34), (35) into Eq. (32) the total inlet thermal flow can be expressed as Eq. (36).

\[ \sum_{i} dQ_{\text{in}_i} = -k \frac{\partial T}{\partial r} rd\theta dz - \frac{k}{r} \frac{\partial T}{\partial \theta} dr dz - k \frac{\partial T}{\partial z} r d\theta dr. \] (36)

Similarly, the summation of outlet thermal flows is:
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\[ \sum_j dQ_{out,j} = dQ_{r+dr} + dQ_{\theta+d\theta} + dQ_{z+dz}. \]  \hspace{1cm} (37)

where:

- \(dQ_{r+dr}\): Outlet heat flow in the r direction
- \(dQ_{\theta+d\theta}\): Outlet heat flow in the \(\theta\) direction
- \(dQ_{z+dz}\): Outlet heat flow in the z direction

The directional heat conduction flows at the outlet surface of the elemental controlled volume (\(dV = r dr d\theta dz\)) can be described in Eqs. (38), (39), and (40).

\[ dQ_{r+dr} = -k \frac{\partial T}{\partial r} r d\theta dz - \frac{k}{r} \frac{\partial T}{\partial r} dV - k \frac{\partial^2 T}{\partial r^2} dV. \]  \hspace{1cm} (38)

\[ dQ_{\theta+d\theta} = -\frac{k}{r} \frac{\partial T}{\partial \theta} dr dz - \frac{k}{r^2} \frac{\partial^2 T}{\partial \theta^2} dV. \]  \hspace{1cm} (39)

\[ dQ_{z+dz} = -k \frac{\partial T}{\partial z} r d\theta dr - k \frac{\partial^2 T}{\partial z^2} dV. \]  \hspace{1cm} (40)

Then Eq. (41) is the expression of the directional summation obtained from the substitution of Eqs. (38), (39), and (40) into Eq. (37).

\[ \sum_j dQ_{out,j} = -k \frac{\partial T}{\partial r} r d\theta dz - \frac{k}{r} \frac{\partial T}{\partial r} dV - k \frac{\partial^2 T}{\partial r^2} dV - k \frac{\partial T}{\partial \theta} dr dz - \frac{k}{r^2} \frac{\partial^2 T}{\partial \theta^2} dV - \frac{k}{r^2} \frac{\partial^2 T}{\partial \theta^2} dV. \]  \hspace{1cm} (41)

Finally, the general energy balance of Eq. (30) can be written as below by employing Eqs. (36) and (41).

\[ c\rho dV \frac{\partial T}{\partial t} = k \frac{\partial T}{\partial r} dV + k \frac{\partial^2 T}{\partial r^2} dV + k \frac{\partial T}{\partial \theta} dV + k \frac{\partial^2 T}{\partial \theta^2} dV + k \frac{\partial^2 T}{\partial z^2} dV. \]  \hspace{1cm} (42)

\[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{a} \frac{\partial T}{\partial t}. \]  \hspace{1cm} (43)

Where the thermal diffusivity is calculated from:

\[ a = \frac{k}{c_p \rho}. \]  \hspace{1cm} (44)
According to the proposed assumptions, the heat transfer equation by conduction through a cylindrical single wall is simplified as

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{\partial T}{\partial t}.
\]  

(45)

2.3.2 Energy balance of solid layers of two different materials (metal and insulation material)

It is noted that Eq. (45) can only be used to solve for a single uniformed material. Hence, an additional energy balance equation should be derived to predict the variation of the temperature along both the drum metal wall and its insulation cover in transient conditions because the boundary temperature at such the interface is not well known in practice. The description of the heat conduction through the interface is depicted in Fig. 6.

![Fig. 2. Heat transfer model between metal and insulation.](image)

The energy balance of the controlled volume consisting of the final layer of metal and the first layer of the insulation is

\[
\sum_i dQ_{in_i} = \sum_j dQ_{out_j} + q_{met}^o + q_{ins}^o.
\]  

(46)

where:

\[\sum_i dQ_{in_i}: \text{Sum of the inlet thermal flows.}\]

\[\sum_j dQ_{out_j}: \text{Sum of the outlet thermal flows.}\]

\[q_{met}^o = c_{met} \rho_{met} dV \frac{\partial T}{\partial t} : \text{Rate of heat accumulation within the metal layer}\]

\[q_{ins}^o = c_{ins} \rho_{ins} dV \frac{\partial T}{\partial t} : \text{Rate of heat accumulation within the insulation layer}\]
The inlet and outlet thermal flows with respect to the radial direction are
\[
\sum_i dQ_{in_i} = -k_{met} \frac{\partial T}{\partial r} r d\theta dz. \quad (47)
\]
\[
\sum_j dQ_{out_j} = -k_{ins} \frac{\partial T}{\partial r} r d\theta dz - \frac{k_{ins}}{r} \frac{\partial T}{\partial r} dV - k_{ins} \frac{\partial^2 T}{\partial r^2} dV. \quad (48)
\]
Then the energy balance of heat conduction through a solid with two different materials can be expressed as Eq. (49) or Eq. (50) by the combination of Eqs. (46), (47), and (48).
\[
-k_{met} \frac{\partial T}{\partial r} r d\theta = -k_{ins} \frac{\partial T}{\partial r} r d\theta dz - \frac{k_{ins}}{r} \frac{\partial T}{\partial r} dV - k_{ins} \frac{\partial^2 T}{\partial r^2} dV + c_{met} \rho_{met} dV \frac{\partial T}{\partial t} + c_{ins} \rho_{ins} dV \frac{\partial T}{\partial t}. \quad (49)
\]
\[
\left(\frac{k_{met}}{a_{met}} + \frac{k_{ins}}{a_{ins}}\right) \frac{\partial T}{\partial t} = -k_{met} \frac{\partial T}{\partial r} \frac{1}{dr} dr + k_{ins} \frac{\partial T}{\partial r} \frac{1}{dr} dr + \frac{k_{ins}}{r} \frac{\partial T}{\partial r} + k_{ins} \frac{\partial^2 T}{\partial r^2}. \quad (50)
\]

2.3.3 Discretized methods

In order to analyze the temperature distribution in the drum shell that is consisted of metal wall and insulation wall, the energy balance equations, Eqs. (45) and (50) can be solved by the finite difference method (FDM) in the Matlab environment. The metal wall and insulation wall are discretized by layers as depicted in Fig. 7.

---

**Fig. 3.** The drum shell discretization into coaxial layers.
2.3.3.1 Steady-state conditions

At steady-state conditions, Eq. (45) becomes
\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = 0.
\]  \hspace{1cm} (51)

the integration Eq. (51) is straightforwardly derived as
\[
T = C_1 \ln(r) + C_2. \tag{52}
\]

2.3.3.2 Transient conditions

Eq. (45) can be discretized by adapting the first-order and second-order numerical derivatives and then the final form of Eq. (56) is obtained.

The first order of the individual derivative of wall temperature according to the radial direction
\[
\left. \frac{\partial T}{\partial r} \right|_i = \frac{T(r_i + \Delta r) - T(r_i)}{\Delta r} = \frac{T_{i+1}^t - T_i^t}{\Delta r}. \tag{53}
\]

The second order of the individual derivative of the wall temperature according to the radial direction
\[
\left. \frac{\partial ^2 T}{\partial r^2} \right|_i = \frac{T_{i+1}^t - 2T_i^t + T_{i-1}^t}{(\Delta r)^2}. \tag{54}
\]

The first order of the individual derivative of wall temperature according to time
\[
\left. \frac{\partial T}{\partial t} \right|_i = \frac{T_{i+1}^{t+\Delta t} - T_i^t}{\Delta t}. \tag{55}
\]

The result of the derivation equation of temperature
\[
T_{i+1}^{t+\Delta t} = T_i^t + \frac{a\Delta t}{(\Delta r)^2} \left[ T_{i+1}^t \left( 1 + \frac{\Delta r}{\tilde{r}_i} \right) - T_i^t \left( 2 + \frac{\Delta r}{\tilde{r}_i} \right) + T_{i-1}^t \right]. \tag{56}
\]

To calculate the temperature at the interface between the two different materials, Eq. (50) should also be discretized by FDM as follows
\[
T_{n,\text{met}}^{t+\Delta t} = T_{n,\text{met}}^t + \Delta t \left[ \frac{-k_{\text{met}} \frac{r_{n,\text{met}}^t - r_{n-1,\text{met}}^t}{\Delta r_{\text{met}}} + k_{\text{ins}} \frac{r_{n,\text{ins}}^t - r_{n,\text{met}}^t}{\Delta r_{\text{ins}}} + k_{\text{met}} \frac{r_{n,\text{met}}^t - r_{n-1,\text{met}}^t}{\Delta r_{\text{met}}} + k_{\text{ins}} \frac{r_{n,\text{ins}}^t - r_{n,\text{met}}^t}{\Delta r_{\text{ins}}} \frac{r_{n,\text{met}}^t - r_{n-1,\text{met}}^t}{\Delta r_{\text{met}}(\Delta r_{\text{ins}} + \Delta r_{\text{met}})} + k_{\text{met}} \frac{r_{n,\text{met}}^t - r_{n-1,\text{met}}^t}{\Delta r_{\text{met}}(\Delta r_{\text{ins}} + \Delta r_{\text{met}})} + k_{\text{ins}} \frac{r_{n,\text{ins}}^t - r_{n,\text{met}}^t}{\Delta r_{\text{ins}}(\Delta r_{\text{ins}} + \Delta r_{\text{met}})} \frac{r_{n,\text{met}}^t - r_{n-1,\text{met}}^t}{\Delta r_{\text{met}}(\Delta r_{\text{ins}} + \Delta r_{\text{met}})}} \right]. \tag{57}
\]
The system of algebraic equations with numerical derivatives can be set as Eq. (58) by applying Eq. (56) to layers of metal and layers of insulation, and Eq. (57) to one interface layer.

\[
\begin{align*}
T_{1\text{, met}}^{t+\Delta t} &= T_{1\text{, met}}^t + \frac{a_{\text{met}}\Delta t}{(\Delta r_{\text{met}})^2} \left[ T_{2\text{, met}}^t \left( 1 + \frac{\Delta r_{\text{met}}}{r_{1\text{, met}}} \right) 
- T_{1\text{, met}}^t \left( 2 + \frac{\Delta r_{\text{met}}}{r_{1\text{, met}}} \right) + T_{0\text{, met}}^t \right] \\
T_{i\text{, met}}^{t+\Delta t} &= T_{i\text{, met}}^t + \frac{a_{\text{met}}\Delta t}{(\Delta r_{\text{met}})^2} \left[ T_{i+1\text{, met}}^t \left( 1 + \frac{\Delta r_{\text{met}}}{r_{i\text{, met}}} \right) 
- T_{i\text{, met}}^t \left( 2 + \frac{\Delta r_{\text{met}}}{r_{i\text{, met}}} \right) + T_{i-1\text{, met}}^t \right] \\
T_{n-1\text{, met}}^{t+\Delta t} &= T_{n-1\text{, met}}^t + \frac{a_{\text{met}}\Delta t}{(\Delta r_{\text{met}})^2} \left[ T_{n\text{, met}}^t \left( 1 + \frac{\Delta r_{\text{met}}}{r_{n-1\text{, met}}} \right) 
- T_{n-1\text{, met}}^t \left( 2 + \frac{\Delta r_{\text{met}}}{r_{n-1\text{, met}}} \right) + T_{n-2\text{, met}}^t \right] \\
T_{n\text{, met}}^{t+\Delta t} &= T_{n\text{, met}}^t + \frac{a_{\text{met}}\Delta t}{(\Delta r_{\text{met}})^2} \left[ T_{n-1\text{, met}}^t \left( 1 + \frac{\Delta r_{\text{met}}}{r_{n-1\text{, met}}} \right) 
- T_{n-1\text{, met}}^t \left( 2 + \frac{\Delta r_{\text{met}}}{r_{n-1\text{, met}}} \right) + T_{n-2\text{, met}}^t \right] \\
T_{1\text{, ins}}^{t+\Delta t} &= T_{1\text{, ins}}^t + \frac{a_{\text{ins}}\Delta t}{(\Delta r_{\text{ins}})^2} \left[ T_{2\text{, ins}}^t \left( 1 + \frac{\Delta r_{\text{ins}}}{r_{1\text{, ins}}} \right) 
- T_{1\text{, ins}}^t \left( 2 + \frac{\Delta r_{\text{ins}}}{r_{1\text{, ins}}} \right) + T_{n\text{, met}}^t \right] \\
T_{i\text{, ins}}^{t+\Delta t} &= T_{i\text{, ins}}^t + \frac{a_{\text{ins}}\Delta t}{(\Delta r_{\text{ins}})^2} \left[ T_{i+1\text{, ins}}^t \left( 1 + \frac{\Delta r_{\text{ins}}}{r_{i\text{, ins}}} \right) 
- T_{i\text{, ins}}^t \left( 2 + \frac{\Delta r_{\text{ins}}}{r_{i\text{, ins}}} \right) + T_{i-1\text{, ins}}^t \right] \\
T_{n-1\text{, ins}}^{t+\Delta t} &= T_{n-1\text{, ins}}^t + \frac{a_{\text{ins}}\Delta t}{(\Delta r_{\text{ins}})^2} \left[ T_{n\text{, ins}}^t \left( 1 + \frac{\Delta r_{\text{ins}}}{r_{n-1\text{, ins}}} \right) 
- T_{n-1\text{, ins}}^t \left( 2 + \frac{\Delta r_{\text{ins}}}{r_{n-1\text{, ins}}} \right) + T_{n-2\text{, ins}}^t \right]
\end{align*}
\]

(58)

Where, the parameters \(k_{\text{met}}, a_{\text{met}}, k_{\text{ins}}, a_{\text{ins}}, \Delta r_{\text{met}}, \Delta r_{\text{ins}}\) are as given in Table 1.
### Table 1
Major parameters of the steam drum shell

<table>
<thead>
<tr>
<th>No</th>
<th>Name of parameters</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Thermal conductivity of metal [15]</td>
<td>$k_{\text{met}}$</td>
<td>$54 - 3.33 \times 10^{-2}T$</td>
<td>W/m.K</td>
</tr>
<tr>
<td>02</td>
<td>Metal specific heat capacity [15]</td>
<td>$c_{\text{met}}$</td>
<td>$(425 + 7.73 \times 10^{-1}T - 1.69 \times 10^{-3}T^2 + 2.22 \times 10^{-6}T^3) \times 10^3$</td>
<td>J/Kg.K</td>
</tr>
<tr>
<td>03</td>
<td>Density of the metal [15]</td>
<td>$\rho_{\text{met}}$</td>
<td>7850</td>
<td>kg/m³</td>
</tr>
<tr>
<td>04</td>
<td>Thermal diffusivity of metal</td>
<td>$a_{\text{met}}$</td>
<td>$k_{\text{met}}/(c_{\text{met}}\rho_{\text{met}})$</td>
<td>N/m²</td>
</tr>
<tr>
<td>05</td>
<td>Thermal conductivity of the insulation [16]</td>
<td>$k_{\text{ins}}$</td>
<td>0.055</td>
<td>W/m.K</td>
</tr>
<tr>
<td>06</td>
<td>Thermal diffusivity of insulation [16]</td>
<td>$a_{\text{ins}}$</td>
<td>$1.2 \times 10^{-6}$</td>
<td>m²/s</td>
</tr>
<tr>
<td>07</td>
<td>Metal thickness</td>
<td>$B_{\text{met}}$</td>
<td>$60 \times 10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>08</td>
<td>Number of layers in the metal wall</td>
<td>$n_{\text{met}}$</td>
<td>18</td>
<td>-</td>
</tr>
<tr>
<td>09</td>
<td>The thickness of the layer in the metal wall</td>
<td>$\Delta r_{\text{met}}$</td>
<td>$B_{\text{met}}/n_{\text{met}}$</td>
<td>m</td>
</tr>
<tr>
<td>10</td>
<td>Insulation thickness</td>
<td>$B_{\text{ins}}$</td>
<td>$150 \times 10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>11</td>
<td>Number of layers in insulation wall</td>
<td>$n_{\text{ins}}$</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>The thickness of the layer in insulation wall</td>
<td>$\Delta r_{\text{ins}}$</td>
<td>$B_{\text{ins}}/n_{\text{ins}}$</td>
<td>m</td>
</tr>
</tbody>
</table>

### 2.4 Stress calculation of the drum metal wall

The generative stress inside the drum metal wall is thermal stress and mechanical stress.

#### 2.4.1 Thermal stress calculation

The stresses are generated due to the temperature variance between the internal layer and the external layer in the metal wall. The thermal stress generation inside the drum metal wall can be calculated from the following equations by applying the temperature distribution data from Eq. (58). The directional thermal stresses of the cylindrical wall are determined by Eqs. (59) to (61).
Dynamic simulation of a natural circulation drum boiler

Tangential stress:

\[
\sigma_{T-\theta} = \frac{E\alpha}{(1-v)r^2} \left[ \frac{r^2 + r_{int}^2}{r^2 - r_{int}^2} \int_{r_{int}}^{r^*} T(r)rdr \right] (59)
\]

Radial stress:

\[
\sigma_{T-r} = \frac{E\alpha}{(1-v)r^2} \left[ \frac{r^2 - r_{int}^2}{r^2 - r_{int}^2} \int_{r_{int}}^{r^*} T(r)rdr \right] (60)
\]

Longitudinal stress:

\[
\sigma_{T-z} = \frac{E\alpha}{(1-v)} \left[ \frac{2}{r^2 - r_{int}^2} \int_{r_{int}}^{r^*} T(r)rdr \right] (61)
\]

The thermal effective stress based on the von-Misses theory can be calculated from Eq. (62)

\[
\sigma_{T-eff} = \left[ \sigma_{T-\theta}^2 + \sigma_{T-r}^2 + \sigma_{T-z}^2 - (\sigma_{T-\theta}\sigma_{T-r} + \sigma_{T-\theta}\sigma_{T-z} + \sigma_{T-r}\sigma_{T-z}) \right]^{1/2} (62)
\]

where the physical values of the metal wall are as given in Table 2.

<table>
<thead>
<tr>
<th>No</th>
<th>Name of parameters</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Modulus of elasticity of the metal wall material [17]</td>
<td>E</td>
<td>210*10^9</td>
<td>Pa</td>
</tr>
<tr>
<td>02</td>
<td>The thermal expansion coefficient of the metal wall material [17]</td>
<td>a</td>
<td>11.7*10^-6</td>
<td>1/K</td>
</tr>
<tr>
<td>03</td>
<td>Poisson’s ratio [17]</td>
<td>v</td>
<td>0.3</td>
<td>-</td>
</tr>
</tbody>
</table>

2.4.2 Mechanical stress

The mechanical stress is caused by the pressure inside the drum. The direction mechanical stresses of the cylindrical wall are determined by Eqs. (63) to (65).

Tangential stress:

\[
\sigma_{P-\theta} = \frac{r_{int}^2 * P}{(r_{out}^2 - r_{int}^2)} \left[ 1 + \frac{r_o^2}{r^2} \right] (63)
\]
Radial stress:

\[ \sigma_{p-r} = \frac{r_{int}^2 \cdot P}{(r_{out}^2 - r_{int}^2)} \left[ 1 - \frac{r_{int}^2}{r^2} \right] \]  

(64)

Longitudinal stress:

\[ \sigma_{p-z} = 0 \]  

(65)

The effective mechanical stress based on the von-Misses theory can be calculated from Eq. (66)

\[ \sigma_{p_{-\text{eff}}} = \left[ \sigma_{p_{-\theta}}^2 + \sigma_{p_{-r}}^2 + \sigma_{p_{-z}}^2 - (\sigma_{p_{-\theta}} \sigma_{p_{-r}} + \sigma_{p_{-\theta}} \sigma_{p_{-z}} + \sigma_{p_{-r}} \sigma_{p_{-z}}) \right]^{1/2} \]  

(66)

2.4.3 Thermo-mechanical stress

The thermo-mechanical stresses are the sums of the thermal stress and the mechanical stress

Tangential stress:

\[ \sigma_{\theta} = \sigma_{p-\theta} + \sigma_{p-\theta} \]  

(67)

Radial stress:

\[ \sigma_{r} = \sigma_{p-r} + \sigma_{p-r} \]  

(68)

Longitudinal stress:

\[ \sigma_{z} = \sigma_{p-z} + \sigma_{p-z} \]  

(69)

Effective thermo-mechanical stress can be calculated by von-Misses theory as equation follows

\[ \sigma_{\text{eff}} = \left[ \sigma_{\theta}^2 + \sigma_{r}^2 + \sigma_{z}^2 - (\sigma_{\theta} \sigma_{r} + \sigma_{\theta} \sigma_{z} + \sigma_{r} \sigma_{z}) \right]^{1/2} \]  

(70)

2.5 Solution procedure

The solution procedure of the drum boiler model is described in Fig. 8. Based on the given input data of boiler operation including heat flow added to the riser, the rate of feedwater flow and the rate of steam demand flow, the fluid dynamic sub-model with a system of four difference equations (19) is solved whose simulated results are the total volume of water, pressure of working fluid, steam mass
Dynamic simulation of a natural circulation drum boiler

fraction at the outlet of riser tubes and total volume of steam bubbles under the water level in drum. From the pressure of the working fluid, the temperature of the working fluid is interpolated by the saturated water and steam tables. Then using the predicted steam temperature, the temperature distribution in the drum wall is calculated by the heat conduction sub-model with a system of equations (58). Finally, the temperature distribution is used to calculate the distribution of stress by using stress calculation of the drum metal wall including equations (63-70).

![Diagram of the solution procedure of the drum boiler model.](image)

3. Simulation results and discussions

The dynamic response of the drum boiler is simulated under several operating conditions and parameters given in Table 3.

**Table 3**
The normal operation data of 2MW CFBC drum boiler system at the Korea Institute of Energy Research

<table>
<thead>
<tr>
<th>No</th>
<th>Name of parameters</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Amount of heat flux added to the system</td>
<td>$Q$</td>
<td>$4.3*10^6$</td>
<td>W</td>
</tr>
<tr>
<td>02</td>
<td>Flow rate of feedwater</td>
<td>$W_f$</td>
<td>2</td>
<td>kg/s</td>
</tr>
<tr>
<td>03</td>
<td>Temperature of feedwater</td>
<td>$T_{fw}$</td>
<td>426</td>
<td>K</td>
</tr>
<tr>
<td>04</td>
<td>Steam flow rate</td>
<td>$W_s$</td>
<td>2</td>
<td>kg/s</td>
</tr>
<tr>
<td>05</td>
<td>Drum pressure</td>
<td>$P$</td>
<td>$4.5*10^6$</td>
<td>Pa</td>
</tr>
<tr>
<td>06</td>
<td>Total water volume in the system</td>
<td>$V_w$</td>
<td>2.2</td>
<td>m$^3$</td>
</tr>
<tr>
<td>07</td>
<td>Steam quality at the riser top</td>
<td>$x_r$</td>
<td>0.0248</td>
<td>[-]</td>
</tr>
</tbody>
</table>
Table 3 (continued)
The normal operation data of 2MW CFBC drum boiler system at the Korea Institute of Energy Research

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>08</td>
<td>Steam bubbles volume under the liquid level</td>
<td>$V_{sd}$</td>
<td>0.189 m$^3$</td>
</tr>
<tr>
<td>09</td>
<td>Volume of the drum</td>
<td>$V_d$</td>
<td>2.27 m$^3$</td>
</tr>
<tr>
<td>10</td>
<td>Volume of the riser tubes</td>
<td>$V_r$</td>
<td>1.5 m$^3$</td>
</tr>
<tr>
<td>11</td>
<td>Volume of the down-comer tubes</td>
<td>$V_{dc}$</td>
<td>0.73 m$^3$</td>
</tr>
<tr>
<td>12</td>
<td>Drum liquid surface area at the normal operating level</td>
<td>$A_d$</td>
<td>2.7 m$^2$</td>
</tr>
<tr>
<td>13</td>
<td>Downcomer area</td>
<td>$A_{dc}$</td>
<td>0.052 m$^2$</td>
</tr>
<tr>
<td>14</td>
<td>Total mass of riser tubes metal</td>
<td>$m_r$</td>
<td>11,210 kg</td>
</tr>
<tr>
<td>15</td>
<td>Drum metal mass</td>
<td>$m_d$</td>
<td>7320 kg</td>
</tr>
<tr>
<td>16</td>
<td>Total metal mass of the system</td>
<td>$m_t$</td>
<td>21,570 kg</td>
</tr>
<tr>
<td>17</td>
<td>The friction coefficient of fluid flow in the downcomer-riser loop</td>
<td>$f$</td>
<td>25 -</td>
</tr>
<tr>
<td>18</td>
<td>The stayed time of steam in the drum</td>
<td>$T_d$</td>
<td>3 second</td>
</tr>
<tr>
<td>19</td>
<td>The parameter in the empirical equation to calculate the flow rate of the steam flow out of the drum liquid level surface</td>
<td>$\beta$</td>
<td>0.3 -</td>
</tr>
<tr>
<td>20</td>
<td>Total volume of steam bubbles in the drum in the case no steam condensation phenomena in the steam drum</td>
<td>$V_{sd}^0$</td>
<td>0.3 m$^3$</td>
</tr>
<tr>
<td>21</td>
<td>Drum inner diameter</td>
<td>$D$</td>
<td>1.2 m</td>
</tr>
<tr>
<td>22</td>
<td>Ambient temperature</td>
<td>$T_{amb}$</td>
<td>313 K</td>
</tr>
</tbody>
</table>

3.1 Simulation results of a step-change

To unravel the behavior of the drum boiler dynamics and to understand the interaction between state variables, a step change is given in each input like the heat flow, the feedwater flow, and the steam demand flow. A step change of 10% increase at the normal operating conditions and only one signal is changed but the others are not changed. The corresponding results are shown below.

3.1.1 A step change of 10% in heat addition to the riser.

Fig. 5-a shows that the flow rate of heat is increased stepwise with 10% at 50th second. The responses of the state variables ($P$, $V_w$, $x_r$, $V_{sd}$), the drum liquid level, the mass flow rate of water-steam mixture in the riser, the mass flow rate of water inside downcomer, and the mass flow rate of condensation in the drum are shown in the other subplots of Fig. 5. The drum pressure $P$ shown in Fig. 5-b increases almost linearly. This phenomenon is caused by increasing steam in the drum. The volume of entire water $V_w$ shown in the subplot-c increases because of the increas-
ing condensation flow which is caused by the increase of drum pressure. At the
first stage of the 10% heat-up process, the steam quality at the riser top \( x_r \) shown
in Fig. 5-d shoots up because more steam is produced in the riser. After that, its
increase becomes more gradual because of the increase in pressure. The volume of
steam bubbles below the liquid level in the drum \( V_{sd} \) shown in Fig. 5-e first
increases because of the fast increase of the steam flow going from the riser tubes
to the drum, then it decreases because of the increase of the condensation flow. It
is indicated in Eq (29) that the level of liquid in the drum is calculated by the sum
of the total volume of water in the drum and the total volume of the steam bubbles
below the interface. As shown in Fig. 5-f, the liquid level in the drum increases at
the very first moment at 50\(^{th}\) second because of the increased volume of the steam
bubble and the water in the drum. This increased volume stems from the suddenly
increased flow rate of the steam-water mixture from the riser tubes. After that, the
liquid level decreases because the steam-water mixture gradually decreases. From
80\(^{th}\) second, the drum liquid level increases again because of the increase in the
condensation flow. Fig. 5-g shows the mass flow rate of water-steam mixture out
of the riser tubes and mass flow rate of water inside the downcomer tubes. The
mass flow rate out of the riser increases suddenly because of the step increase in
heat flow. The sudden heat addition, that causes more vaporization of water in the
riser tubes, makes the difference between water-steam mixture density in the riser
tubes and water density in downcomer tubes larger (the driving force for the
circulation loop of working fluid through the riser-downcomer). Just after about
30 seconds, the riser flow rate decreases and approaches gradually to the
downcomer flow rate because the circulation loop equilibrium is reestablished.
According to Fig. 5-h, the condensation flow rate inside the drum increases
because the more riser flow with higher steam quality is coming up to the drum
and the drum pressure increases.

3.1.2 A step change of 10% in steam demand.

Fig. 6-a shows that the steam demand is increased stepwise with 10% and the
other subplots show the simulation results. In Fig. 6-b, when the flow rate of the
steam out of the drum increases stepwise, the pressure of the steam in the drum
decreases because of the decrease of steam inside the drum. The volume of the
entire water in the system shown in Fig. 6-c also decreases because the water in
the system is increasingly vaporized. The evaporation increases because of
decreases in the drum pressure. The steam quality at the top of the riser shown in
Fig. 6-d first increases quickly because the pressure of the steam-water mixture in
the riser decreases suddenly, and then it decreases because the water from the
downcomer tubes comes to the riser tubes increasingly. Fig. 6-e shows the volume
of steam bubbles below the interface \( V_{sd} \) which increases because the pressure of
steam in the drum decreases. As shown in Fig. 6-f, the liquid level of the inside
drum increases at the very first moment around 50\(^{th}\) second because the volume of
steam bubbles in the drum increases very fast and the flow rate of the steam-water
mixture from the riser increases. After that, the drum liquid level decreases
because the volume of water in the drum decreases due to the decrease of condensation flow. Fig. 6-g shows the mass flow rate of the water-steam mixture in the riser tubes and mass flow rate of the water in the downcomer tubes. The mass flow rate of the water-steam mixture in the riser tubes increases quickly at the first moment because the pressure in the drum suddenly decreases. After that, the rate of mass flow rate in the riser tubes equals the mass flow rate of water in the down-comer tubes. The two mass flow rates increase steadily because of the linear decrease in the drum pressure. The flow rate of condensation shown Fig. 6-g drops stepwise because of the linear decrease of steam pressure in the drum.

3.1.3 A step change of 10% in feedwater flow rate.

As shown in Fig. 7-a, the flow rate of feedwater increases stepwise by 10% and the other subplots show the responses to step increase in the feedwater. The drum pressure shown in Fig. 7-b decreases gradually because of the increasing feedwater into the drum. This is caused by the reduction of working fluid temperature. The total water volume shown in Fig. 7-c clearly increases because of the feedwater addition. Fig. 7-d indicates that steam quality at the riser outlet increases at around 50th second due to the sudden decrease of the drum pressure and decrease after 15 seconds because of the decrease of condensation flow. The volume of steam bubbles under the interface shown in Fig. 7-e first decreases rapidly because of the decrease in temperature of working fluid in the drum. Fig. 7-f shows the drum liquid level slightly decreasing at the very first moment around the 50th second because the total volume of steam bubbles in the drum decreases very fast. After that, the liquid level gradually increases because the more feedwater comes in the drum.

3.2 Simulation of the drum boiler start-up.

The behavior of the natural circulation drum boiler for its start-up process (from steady-state to on-load condition) is presented in this section. Processes taking place during the start-up procedure are: 1) The augmenting of the combustion process in the furnace, 2) the increase of the heat transfer process from the furnace to the riser, 3) the evaporation of the water inside the system, 4) the formation of the circulation loop of water-steam mixture in the drum-downcomer-riser, 5) the change of drum liquid level, 6) the gradual increase of the pressure, 7) the fluctuation of the stress in the metal wall. All parts in the boiler system transit from a static state to a normal operating state. Therefore, based on the boiler system parameters and the start-up data from the 2MW CFB thermal power pilot-scale system, the influences of those parameters and data on the drum pressure, the drum liquid level, the temperature and the stress distribution in the drum shell are presented and analyzed below.
Fig. 5. The responses to a stepwise increase of 10% in heat flow

(a). Heat flow to the riser

(b). Drum pressure

(c). Volume of entire water

(d). Steam quality at riser top

(e). Steam bubbles volume

(f). Drum liquid level (l)

(g). Riser (W_r) and downcomer (W_dc) flow

(h). Condensation flow (W_cd)
Fig. 6. Responses to a stepwise increase of 10% in steam demand flow
Fig. 7. Responses to a stepwise increase of 10% in the feedwater flow.

### 3.2.1 Fluid dynamics in the drum boiler

Fig. 8 shows the data of the three input signals during the start-up period. As shown in Fig. 8-b, from beginning to around 7,000th second, the water is not supplied into the boiler system because the drum has some amount of water already. After that, the water is fed into the system to compensate for the water amount evaporated and going out of the drum. When the combustion process takes place, the heat, which is produced in the furnace, is transferred to the riser that can be seen in Fig. 8-a. The furnace heat is transferred through the riser wall to the rising working fluid. Therefore, steam is produced gradually as shown in the subplot-c of Fig. 8. The main simulation results are shown in Fig. 9. It has a
good agreement of drum pressure between the calculated and experimental data as shown in Fig. 9-a. Fig. 9-b, c, and d describe the dynamics of the total volume of water in the boiler, the steam quality at the riser top, and the volume of steam bubbles under the liquid level in the drum, respectively. According to the calculation of the drum liquid level shown in Fig. 9-e, the drum liquid level was around the normal value of 0 at the start, but the level gradually decreases, and approaches -0.2m at 7,000th second because part water is evaporated while there is no feedwater addition. From 7,000th to 14,000th second, the drum liquid level is changed corresponding to the feedwater input. Then the drum liquid level reaches a nearly normal level because the start-up process becomes stable operation.

3.2.2 The dynamics of temperature distribution in the drum shell

The heat conduction sub-model developed in the model section is applied to simulate the temperature distribution in the drum metal wall and insulation cover.

3.2.2.1 The steam temperature variation during the start-up period

Based on the fluid dynamic sub-model and the steam tables, the steam temperature during the start-up is calculated. Fig. 10-a shows the steam temperature during the start-up which is essential to examine the temperature distribution in the drum shell. It is needed to know that there are always some variations in steam temperature for 6,000th to 17,000th second as can be seen in zoomed Fig. 10-b.

3.2.2.2 The temperature distribution in the drum shell during the start-up period.

The temperature at the outer insulation wall is assumed the same as ambient temperature. This temperature, together with the steam temperature, is the boundary condition for the heat conduction model. The temperature distribution in the metal wall and the insulation cover are shown together in Fig. 11. The temperature in the metal layers for 6,000th to 17,000th second is zoomed in and described in Fig. 12. During the increase in steam temperature, the simulation results indicate that temperatures in the metal layers are lower than the steam temperature. And the temperature in the 18-layer is the lowest. When the steam temperature decreases, at first, the temperatures in the metal layers are still lower than the steam temperature due to the thermal inertia of the drum metal wall. Then, after a delay time of 700 seconds from 7,000th second, the layers temperatures are higher than the steam temperature. This phenomenon caused by thermal inertia reversely occurs in case of a sudden increase in the steam temperature.
Fig. 8. The input data for the start-up period.

(a). Heat flow transferred to the riser in the drum boiler (Q)

(b). Feedwater flow into the drum (q_f)

(c). Steam flow out of the drum (q_s)
Fig. 9. Simulation results of fluid dynamics in the drum boiler
Dynamic simulation of a natural circulation drum boiler

Fig. 10. Steam temperature history during the start-up period.

(a). For interval time from the start-up to 17,000th second

(b). For interval time from 6,000th second to 17,000th second

Fig. 11. Dynamic simulation of temperature in both the metal layers and the insulation layers
3.2.3 The dynamics of stress distribution in the drum metal wall

In this section, the stresses including the thermal stress, mechanical stress and combined thermal-mechanical stress, which distributed in the drum metal wall, are demonstrated by using the equations of the stress calculation.

3.2.3.1 The thermal stress distribution

The temperature gradient inside the metal wall could lead to very high thermal stress. Fig. 13 shows the simulated thermal stresses distribution in the drum metal wall. The result indicates that the von Mises thermal effective stress reaches the maximum values in the first layer which is contacted with the steam. It is caused by the difference in temperature between the internal and external of this layer. This difference temperature is the biggest value among the layers. The thermal effective stress reaches its maximum value at around 7,000th second because at that time the temperature gradient found to be the maximum value. The thermal stresses are almost null when the steam temperature approaches steady-state conditions.

3.2.3.2 The mechanical stress distribution

The mechanical stress is caused by the drum pressure. The von Mises effective mechanical stress is varied corresponding to the steam pressure as shown in Fig. 14. It is important to remark that the highest value of the mechanical stress is found in the first layer that is directly contacted with the steam at high pressure.
3.2.3.3 The combined thermal-mechanical stress distribution

The drum boiler operates at the transient conditions, especially during the start-up period. It also works in severe operations (high pressure, high temperature, and continuous operation). These operating conditions lead to the lifetime limit of the drum material. This lifetime is affected by the generated thermal-mechanical stress in the drum metal wall. The stress causes the creep and fatigue of the material. The repeat of stress peaks causes the fatigue and the high stress occurred under steady loading conditions results in the creep. The drum material is SA-515 Grade 70 carbon steel; therefore, it has the ultimate strength of 483 MPa [17]. The tolerance limit of the stress is calculated as less than 50% of the ultimate strength. Therefore, the allowable stress in the drum metal wall is around 240 MPa. The durability of boiler material and its lifetime is maintained if the generated stress has values smaller than the allowable limit value. Fig. 15 shows the von Mises effective thermal-mechanical stress distribution in the drum metal wall and Figure
16 shows the directions and effective thermal-mechanical stress plotted in 3-D graphs. It is clear to see that the stress value of the first layer among all the layers is the highest. The maximum stress can be noticed at around 7,000\textsuperscript{th} second because the thermal-mechanical effective stresses are the combination of both thermal stress and mechanical stress. At that moment, both the thermal and mechanical stresses also reach their highest values. However, the highest value is just around 87 MPa which is lower than the allowable limit value of 240 MPa.

Fig. 15. Effective thermal-mechanical stress distribution in the drum metal wall.

Fig. 16. Direction thermal-mechanical stress distribution in the drum metal wall. (a) Effective stress; (b) Tangential stress; (c) Radial stress; (d) Longitudinal stress.
4. Conclusions

The present paper develops a model of natural circulating drum boilers, which can describe the detailed fluid dynamics of flow through the boiler modules and the temperature distributions in the drum shell. In addition, the combined thermal-mechanical stress distribution in the drum metal wall is also taken into account. The present model is able to predict not only the drum pressure and the drum liquid level but also the stress distribution in the drum metal wall during its operation, thus this model can be a base model for control purpose. The behavior of the boiler is demonstrated under not only the step changes in the input parameters but also the start-up process. The simulation results of the drum pressure show a good agreement with the data from a 2MWe CFB boiler power plant. During the start-up process, the simulation results show that the effective thermo-mechanical stress of the first layer in the drum metal wall is the highest where the temperature gradient is the biggest. The stress reaches a maximum value of 87MPa at around 7,000th second. As this model can calculate the three main factors, the pressure, the liquid level, and the effective thermal-mechanical stress, it is possible to check the safety of a given operating scenario. The simulation results of the present model are found to be useful for the lifetime evaluation and the design of the drum boiler.

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Nomenclature

- \( a \): Thermal diffusivity of the material [m\(^2\)/s]
- \( A \): Heat exchange area [m\(^2\)]
- \( A_d \): Liquid surface area of the drum [m\(^2\)]
- \( A_{dc} \): Cross-sectional area of the downcomer tubes [m\(^2\)]
- \( B \): Thickness of the drum metal wall [m]
- \( C_p \): Metal’s specific heat [J/kg K]
- \( D \): Diameter of drum [m]
- \( E \): Modulus of elasticity [N/m\(^2\)]
- \( f \): Friction coefficient of fluid flow in the downcomer-riser loop [-]
- \( h_f \): Specific enthalpy of feedwater [J/kg]
- \( h_s \): Specific enthalpy of steam leaving the boiler [J/kg]
- \( h_w \): Specific enthalpy of saturated water [J/kg]
- \( h_{lv} \): Latent heat vaporization [J/kg]
- \( k \): Thermal conductive of material [W/m K]
- \( L \): Length [m]
- \( L_{dc} \): Length of the downcomer [m]
- \( L_r \): Length of the riser [m]
Drum liquid level [m]
Number of layers [-]
Total mass of the steam drum metal [kg]
Total mass of the downcomer tubes metal [kg]
Total mass of the riser tubes metal [kg]
Total mass of the system metal [kg]
Steam pressure in the drum [Pa]
Heat flow rate [W]
Mass flow rate [kg/s]
Condensation mass flow in the drum [kg/s]
Downcomer mass flow rate [kg/s]
Feedwater mass flow rate [kg/s]
Mass flow rate of the steam-water mixture out of the risers [kg/s]
Mass flow rate of the steam out of the boiler [kg/s]
Mass flow rate of steam through the liquid surface in the drum [kg/s]
Radius [m]
Residence time of steam in the drum [s]
Temperature [K]
Temperature of saturated steam [K]
Time [s]
Internal energy [J/kg]
Internal energy of the steam [J/kg]
Internal energy of the water [J/kg]
Volume [m³]
Volume of the drum [m³]
Volume of the downcomers [m³]
Volume of the risers [m³]
Poisson’s number [-]
Volume of steam bubbles under the water liquid in the drum [m³]
Volume of steam bubbles under assuming no condensation of steam in the drum [m³]
Volume of the entire steam in the system [m³]
Volume of the entire water volume in the system [m³]
Volume of the drum boiler system [m³]
Volume of water in the steam drum [m³]
Coefficient of thermal expansion [1/K]
Steam quality (mass fraction of steam) [-]
Steam quality at a specific location along the riser [-]
Steam quality at the riser top [-]
Volume fraction of steam at the riser top [-]
Average volume fraction of steam inside the riser [-]
Empirical parameter of the equation for calculation of steam flow through the liquid surface in the drum [-]
Normalized length coordinate along with the risers [-]
Density of the material [kg/m³]
Stress [Pa]
Subscripts

c  Condensation
d  Drum
dc  Downcomer
ext  External
f  Feedwater
in  Inlet
ins  Insulation
int  Internal
met  Metal
out  Outer
r  Radial direction
s  Steam
t  Total system
v  Volume
w  Water
z  Longitudinal direction
θ  Tangential direction

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