New Structural Engineering Construction Methods

by Remove q Points in PG(2,q)

Aidan Essa Mustafa Sulaimaan

Department of Mathematics, College of Computer Sciences and Mathematics
University of Mosul, Mosul, Iraq

Nada Yassen Kasm

Department of Mathematics, College of Education for Pure Sciences
University of Mosul, Mosul, Iraq

Corresponding author

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Abstract

In this work we were able to find a new approach of Engineering Building methods of the double Blocking sets by Remove q points over Galois field of order 11. Thus, we obtained a new \{36,2\}-blocking set, and according to the theorem (1.2.1), we obtained the linear \([97,3,87]_{11}\) Griesmer code, theorem (2.1.1) giving some examples on arcs of the Galois field GF(q); q=11.

Keywords: projective plane, (k,r)-arc, double blocking set, Linear[n,k,d] code, \{\lambda ,t\}- blocking set, Galois field GF(q)

1. Introduction

Let GF(q) denote the Galois field of q elements and V (3, q) be the vector space of row vectors of length three with entries in GF(q). Let PG(2, q) be the corresponding projective plane [3].

1.1 Explore Importance of the Problem

Definition (1.1.1): A projective plane consists of a set of lines, a set of points, and a relation between points and lines called incidence, having the following properties:
Given any two distinct points, there is exactly one line incident with both of them. Given any two distinct lines, there is exactly one point incident with both of them. There are four points such that no line is incident with more than two of them. The second condition means that there are no parallel lines. The last condition excludes the so-called degenerate cases. The term "incidence" is used to emphasize the symmetric nature of the relationship between points and lines. Thus the expression "point P is incident with line ℓ" is used instead of either "P is on ℓ" or "ℓ passes through P".

**Definition (1.1.2):** A \((k, r)\)-arc in PG\((n, q)\) is a set \(K\) of \(k\) points with \(k \geq n + 1\) such that no \(n + 1\) points lie in a hyperplane.

**Definition (1.1.3):** An arc \(K\) is complete if it is not properly contained in a larger arc.

**Definition (1.1.4):** A \(\{\lambda, t\}\)-blocking set \(S\) in \(\pi = PG(2, q)\) is a set of \(\lambda\) points such that every line of \(PG(2, q)\) intersects \(S\) in at least \(n\) points, and there is a line intersecting \(S\) in exactly \(n\) points. Note that a \(k\)-arc is the complement of a \((q^2 + q + 1 - k, q + 1 - r)\)-blocking set in a projective plane and conversely. [5].

**Definition (1.1.5):** Let \(\mu\) be a set of points in any plane An \(i\)-secant is a line meeting \(M\) in exactly \(I\) points. Define \(\tau_i\) as the number of \(i\)-secants to a set \(M\). [3].

**Theorem (1.1.6):** Let \(B\) be a double blocking set in the projection plane PG\((2, q)\):
1. If \(q < 9\), \(B\) has less than \(3q\) of points.
2. If \(q = 11, 13, 17\) or \(19\), then \(|B| \geq (5q + 7)/2\).
3. If \(19 < q = p^{2d+1}\) then \(|B| \geq 2q + p^d [(p^{d+1} + 1)/(p^d + 1)] + 2\).
4. If \(4 < q\) is a square then \(|B| \geq 2q + 2\sqrt{q} + 2\) (Ball, 2018).

**Theorem (1.1.7):** To be the \((k, r)\)-arc in the projection plane PG\((2, q)\) the relationship
\[(q+1-r)T_r \geq q^2 + q + 1 - k\].

Besides the combinatorial relationships shared by \((k, r)\)-arcs and algebraic curves, the existence and classification of \((k, r)\)-arcs is motivated by their close association with the theory of linear codes.

If \(V\) is an \(m\)-dimensional vector space defined over a finite field \(F_q\), the weight of an non-zero vector \(v \in V\) is the number of its non-zero coordinates. Over a field \(F_q\), a linear \([m, s, d]_q\)-code \(C\) of length \(m\), dimension \(s\) and minimum distance \(d\), is an \(s\)-dimensional subspace of \(V\) with all non-zero vectors \(v \in C\) of weight \(w = w(v) \geq d\). Here, the integers \(m\), \(s\) and \(d\) are the parameters of the code \(C\) and its elements are words or codewords. A matrix \(G\) is a generator matrix for the linear code \(C\) if its rows are a basis for \(C\). Also, the dual code, \(C^1\), of an \(F_q\)-linear code \(C\), is the linear code consisting of all \(y \in V\) orthogonal to every \(x \in C\). Here, \(x = (x_1, x_2, ..., x_m)\) and \(y = (y_1, y_2, ..., y_m)\) are said to be orthogonal in \(V\) if \(x_1y_1 + x_2y_2 + ... + x_my_m = 0\). Now, it is known, see [38], that a linear \([m, s, d]_q\)-code \(C\) satisfies
the following bound due to Singleton:
\[ d \leq m - s + 1 \]

1.2 Describe Relevant Scholarship
Theorem (1.2.1) there exists a projective \([n, 3, d]_q\) code if and only if there exists an \((n, n-d)\)-arc in \(PG(2, q)\). In this paper we consider the case \(q=11\) and the elements of \(GF(11)\) are denoted by \(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\). [8].

1.3 Geometric Building Approach
In the next theorem we shall present new example. They constructed the \((36, 2)\)-blocking set in \(PG(2,11)\), which is geometrically constructed by the researcher method
1) Get new the linear \([97, 3, 87]_{11}\) Griesmer code 2) Get a new \((97, 10)\)-arc.

2. Method (Engineering Building methods)
We construction of new \((97, 10)\)-arc and new projective \([97, 3, 87]_{11}\) code and getting:
Theorem (2.1.1): There exists a \((36, 2)\)-blocking set in \(PG(2,11)\) and a \((97, 10)\)-arc. Consider the accompanying 42 points in \(PG(2,11)\) as shown in the table 1 and table 2 and table 3 and table 4.

<table>
<thead>
<tr>
<th>Table 1. (L_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
</tr>
<tr>
<td>(Mi)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. (L_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
</tr>
<tr>
<td>(Ni)</td>
</tr>
</tbody>
</table>

| | 9 | 10 | 11 | 12 |
| \(Ni\) | \((1,8,5)\) | \((1,4,2)\) | \((0,1,9)\) | \((1,9,3)\) |
Table 3. L_5

<table>
<thead>
<tr>
<th>i</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pi</td>
<td>(1,2,7)</td>
<td>(1,6,3)</td>
<td>(1,5,4)</td>
<td>(1,7,2)</td>
<td>(1,0,9)</td>
<td>(1,3,6)</td>
<td>(1,8,1)</td>
<td>(0,1,10)</td>
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</table>

Table 4. L_6

<table>
<thead>
<tr>
<th>i</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pi</td>
<td>(1,4,5)</td>
<td>(1,9,0)</td>
<td>(1,10,10)</td>
<td>(1,1,8)</td>
</tr>
</tbody>
</table>

The lines \(Lu :aiU+biV+ciW=0, (i=1,2,3,4; u=46,47,50,51)\) are selected so that each line \(Lu\) contains the point \((ai, bi, ci), (i=1,2,3,4)\). The points M_i (i=1,2,...,12) belong to the line L46: U+6V+2W=0. The points N_i (i=1,2,...,12) belong to the line L47: U+13V+W=0. The points P_i (i=1,2,...,12) lie on the line L50: U+6V+6W=0, and the points Q_i (i=1,2,...,12) are the points of the line L51: U+5V+8W=0. The four lines intersect pairwise at the points M_2=N_1, M_6=Q_5, M_11=P_10, N_2=P_12, N_11=Q_10 and P_2=Q_1, i.e. they are lines in general position.

In \(\text{PG}(2,11)\) the 12 lines which pass through the point \((1,0,0)\) have equations:

- P_1: V=0,
- P_7: V+5W=0,
- P_2: W=0,
- P_8: V+6W=0,
- P_3: V+W=0,
- P_9: V+7W=0,
- P_4: V+2W=0,
- P_10: V+8W=0,
- P_5: V+3W=0,
- P_11: V+9W=0,
- P_6: V+4W=0,
- P_12: V+10W=0.

The careful analysis of the lines L46,L47,L50 and L51 shows that each quadruple (in the case of \(i=1,2\) —each triple, and in the case of \(i=11,12\) and 6,5 and 2,12— each pair) of points M_i,N_i,P_i,Q_i (i=1,2,...,12) belongs to one of the 12 lines p_i.(For example, the four points M_4,N_5,P_11,Q_3 lie on the line p_12: V+10W=0.)

Now let us set the following task: Remove 11 points from the set L46U,L47U,L50U,L51, two or three points from each line, so that:

1. There is no line in \(\text{PG}(2,11)\) which is different from LU and which contains four of the removed points.
2. The lines that contain three of the removed points intersect at most five new points A_1,A_2,A_3,A_4.
3. To satisfy the conditions a 2-blocking set will adding one point A_5.
4. The lines that contain just two of the removed points do not pass through the
intersection points M2,M6,M11,N2,N11 and P11. The conditions (a)–(c) will guarantee that adding the points A1,A2,A3,A4 to the set of remaining points of the lines, we shall obtain a 2-blocking set with no more than 36 points.

Let us take out the following 11 points:

- from the line L46 – M5,M8,
- from the line L47 – N3,N5,N6,
- from the line L50 – P4,P7,P11,
- and from the line L51 – Q2,Q4,Q11.

The set of removed points $W=\{(1,7,6), (0,1,8), (1,3,4), (1,7,7), (1,10,1), (1,7, 2), (1,8,1), (1,10,10), (1,3,9), (1,10, 6), (1,2,0)\}$ is a(11,4)–arc in $PG(2,11)$ and has the following secant distribution:

\[
T_0=42, \quad T_1=64, \quad T_2=13, \quad T_3=14, \quad T_4=0
\]

Since $T_4=0$, condition (a) is satisfied. The thirteen 2-secants of $W$ are

1. $U+5V+5W=0$
2. $U+7V=0$
3. $U+5V+3W=0$
4. $U+8V+10W=0$
5. $U+2V+9W=0$
6. $U+9W=0$
7. $U+9V+5W=0$
8. $U+7V+6W=0$
9. $U+4V+8W=0$
10. $U+10W=0$
11. $U+4V+5W=0$
12. $V+10W=0$
13. $X+6V+2W=0$

It is easy to check now that none of them contains an intersection point of lines $Lu$; thus condition (c) is fulfilled.

Let us look at condition (b). The 3-secants of $W$, i.e. the lines different from $Lu$, such that each contains three of the removed points, are

1. $g_1: U+V=0, \ \ P11,Q4,N6 \in g_1$
2. $g_2: U+5V+4W=0, \ \ Q11,P4,N6 \in g_2$
3. $g_3: U+5V+7W=0, \ \ P11,Q11,N3 \in g_3$
4. $g_4: U+8V+7W=0, \ \ Q2,M5,N6 \in g_4$
5. $g_5: U+9V+3W=0, \ \ M8,P11,Q2 \in g_5$
6. $g_6: V+2W=0, \ \ P4,Q4,N3 \in g_6$
7. $g_7: U+3V=0, \ \ M2,N5,P4 \in g_7$
8. $g_8: U+5V+9W=0, \ \ M8,Q11,N5 \in g_8$
9. $g_9: U+V+2W=0, \ \ Q2,N5,P7 \in g_9$
10. $g_{10}: U+9V+4W=0, \ \ P7,N3,M5 \in g_{10}$
11. $g_{11}: U+10V+7W=0, \ \ Q4,P7,M8 \in g_{11}$

Each line $g_i$ intersects some line $Lu$ at a point not in the set $W$. Indeed:

\[
\begin{align*}
g_1 \cap L46 &= 122= (1,10,8), \quad g_2 \cap L46=46=(1,3,7), \quad g_3 \cap L46=44=(1,5,1), \quad g_4 \cap L50=50=(1,2,7), \\
g_5 \cap L47=123=(1,8,5), \quad g_6 \cap L46=107=(1,2,10), \quad g_7 \cap L51=85=(1,7,1), \\
g_8 \cap L50=126=(1,4,5), \quad g_9 \cap L46=26=(1,0,5), \quad g_{10} \cap L51=61=(1,5,5), \quad g_{11} \cap L47=27=(1,4,2).
\end{align*}
\]

Furthermore, the lines $g_i$ intersect one another in quadruples at the points (1,10,2), (1,0,3), (1,9,1), (1,7,8), (1,4,3). More precisely $g_1 \cap g_{10}=(1,10,2), g_3 \cap g_{11}=(1,0,3), g_5 \cap g_8 = (1,9,1), g_5 \cap g_7 = (1,7,8), g_2 \cap g_9=(1,4,3)$. Therefore, (1,10,2),(1,0,3),(1,9,1),(1,7,8),(1,4,3) are the points $A1,A2,A3,A4,A5$.

Adding these five points to the rest 31 points, we obtain the set $V=\{(1,6,9), (1,1,2), (1,9,0), \ldots\}$.
which is a(36,2)-blocking set in PG(2,11) and has the following secant distribution:

\[ T_2=46, \quad T_3=46, \quad T_4=23, \quad T_5=11, \quad T_6=3, \quad T_9=3, \quad T_{10}=1. \]

The complement of the set W is a (97,10)-arc. It follows now by Theorem 1.1 that there exists a[97,3,87]_{11} which is required as shown in Table 5, Table 6, Table 7.

Table 5. Projection Level Points in PG(2,11)

<table>
<thead>
<tr>
<th>i</th>
<th>(P_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 0 0</td>
</tr>
<tr>
<td>2</td>
<td>0 1 0</td>
</tr>
<tr>
<td>3</td>
<td>0 0 1</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>133</td>
<td>0 1 1</td>
</tr>
</tbody>
</table>

Table 6. Projection Level Lines in PG(2,11)

<table>
<thead>
<tr>
<th>L_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2,11,24,30,35,62,70,77,114,118,132,</td>
</tr>
<tr>
<td>L_2</td>
</tr>
<tr>
<td>2,3,12,25,31,36,63,71,78,115,119,133</td>
</tr>
<tr>
<td>L_3</td>
</tr>
<tr>
<td>3,4,13,26,32,37,64,72,79,91,116,120,1</td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>L_{133}</td>
</tr>
<tr>
<td>133,1,10,23,29,34,61,69,76,113,117,131</td>
</tr>
</tbody>
</table>

Table 7. The bound of linear codes [1]

<table>
<thead>
<tr>
<th>q</th>
<th>11</th>
<th>13</th>
<th>16</th>
<th>17</th>
<th>19</th>
</tr>
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<tbody>
<tr>
<td>2</td>
<td>12</td>
<td>14</td>
<td>18</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>23</td>
<td>28</td>
<td>28-33</td>
<td>31-39</td>
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<tr>
<td>4</td>
<td>32</td>
<td>38-40</td>
<td>52</td>
<td>48-52</td>
<td>52-58</td>
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<tr>
<td>5</td>
<td>43-45</td>
<td>49-53</td>
<td>65</td>
<td>61-69</td>
<td>68-77</td>
</tr>
<tr>
<td>6</td>
<td>56</td>
<td>64-66</td>
<td>78-82</td>
<td>79-86</td>
<td>86-96</td>
</tr>
<tr>
<td>7</td>
<td>67</td>
<td>79</td>
<td>93-97</td>
<td>95-103</td>
<td>105-115</td>
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<tr>
<td>8</td>
<td>78</td>
<td>92</td>
<td>120</td>
<td>114-120</td>
<td>126-134</td>
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<tr>
<td>9</td>
<td>89-90</td>
<td>105</td>
<td>129-131</td>
<td>137</td>
<td>147-153</td>
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<tr>
<td>10</td>
<td>100-102</td>
<td>118-119</td>
<td>142-148</td>
<td>154</td>
<td>172</td>
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<tr>
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<td>132-133</td>
<td>159-164</td>
<td>166-171</td>
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<tr>
<td>12</td>
<td>143-147</td>
<td>180-181</td>
<td>183-189</td>
<td>204-210</td>
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<td>13</td>
<td>195-199</td>
<td>205-207</td>
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<td>14</td>
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<td>221-225</td>
<td>243-250</td>
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<td>286-290</td>
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<td>18</td>
<td></td>
<td>324-330</td>
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</table>
3. Conclusions

1) At the projective plane PG (2,11). There existent new (97 , 10) - arc and
new(36 , 2)- Blocking sets.
2) Improvement of linear codes in the projection plane PG (2,11) Theorem (2.1.1)
[97,3,87].

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