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Note on Soft Modules

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Abstract

In this short note, we explore soft ideal of a soft ring R implies soft submodule of R. However, the converse doesn't exist. We also introduce the term soft multiplicative module.

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1. Introduction and Preliminaries

Soft set theory was introduced in 1999 by Molodtsov[1] for dealing to remove uncertainties in the given data. Afterward, soft groups, soft rings, soft modules, soft semirings, soft near rings etc. are introduced in the literature [see [3], [2], [5], [13], [17] & [8]]. Different operations of soft sets have been introduced [[4], [10], [11]], and many authors also introduced the wide-range of applications towards decision making theory [[6], [12], [14]].

For basic of a module theory we refer [7]. For soft module theory we refer [9] & [13].

Definition 1. Let η is the mapping from A to $\wp(U)$, then the pair (η, A) is called a soft set over the universe U[1].

Definition 2. Let $\eta_A, \eta_B \in S(U)$. Then, η_A is a soft subset of η_B, is denoted by $\eta_A \mathrel{\tilde{\subset}} \eta_B$, if $\eta_A(x) \subset \eta_B(x)$ for all $x \in E$ [15].

Definition 3. Let η_A and η_B be soft sets over U. Then, intersection of η_A and η_B, denoted by $\eta_A \widetilde{\cap} \eta_B$, is defined by $\eta_A \widetilde{\cap} \eta_B = \eta_{A\widetilde{\cap}B}$, where $\eta_{A\widetilde{\cap}B}(x) = \eta_A(x) \cap \eta_B(x)$ for all $x \in E$ [15].

Definition 4. Following [15], if η_A and η_B be the two soft sets over universal set U, then $\wedge$-product of η_A and η_B i.e., $\eta_A \wedge \eta_B$ is defined as,$\eta_{A\wedge B} : E \times E \to \wp(U)$, and $\eta_{A\wedge B}(x,\, y) = \eta_A(x) \cap \eta_B(y)$.

Definition 5. If η_A and η_B be the two soft sets over the universal set U, then $\vee$-product of η_A and η_B is defined by $\eta_{A\vee B} : E \times E \to \wp(U)$ where $\eta_{A\vee B}(x,\, y) = \eta_A(x) \cup \eta_B(y)$[15].

Definition 6. Let Ψ be a function from A to B and $\eta_A, \eta_B \in S(U)$. Then the soft subsets $\Psi(\eta_A) \in S(U)$ and $\Psi^{-1}(\eta_B) \in S(U)$ defined by

$$\Psi(\eta_A)(y) = \begin{cases} \cup\{\eta_A(x) : x \in A,\ \Psi(x) = y\}, & \text{if } y \in \Psi(A) \\ \emptyset, & \text{if } y \notin \Psi(A) \end{cases}$$

for all $y \in B$ and $\Psi^{-1}(\eta_B)(x) = \eta_B(\Psi(x))$ for all $x \in A$. Then $\Psi(\eta_A)$ is called a soft image of η_A under $\Psi^{-1}(\eta_B)$ is called the soft pre-image (or soft inverse image) of η_B under Ψ [16].

Definition 7. Let η_A be a soft set over U and α be a subset of U. Then upper α-inclusion of soft set η_A is defined by $\eta_A^{\supseteq\alpha} = \{x \in A : \eta_A(x) \supseteq \alpha\}$ [16].

Definition 8. Let $(\eta,\, A)$ be the soft right nearsemiring over right nearsemiring R_1 and $(\zeta,\, B)$ be the soft left nearsemiring over the left nearsemiring R_2. Suppose $f : R_1 \to R_2$ and $g : A \to B$ be the two mappings. Then the pair $(f,\, g)$ is called a soft nearsemiring anti-homomorphism if it satisfies the following conditions.

(i) f is an anti-epimorphism of nearsemirings.

(ii) g is a surjective mapping.

(iii) $f(\eta(x)) = \zeta(g(x))$ for all $x \in A$.

If f is an anti-isomorphism then and g is bijective then we call $(f,\, g)$ a soft nearsemiring anti-isomorphism [18].

Definition 9. Let $(F,\, A)$ be a soft set over M. Then, $(F,\, A)$ is said to be a soft module over M if and only if $F(x)$ is a submodule of M for all $x \in A$ [18].

2. Main Result

In this section, we provide the main result that is, every soft ideal of a soft ring is a soft submodule, however every soft submodule is not a soft ideal. An ideal is just a subset of R but a submodule consists of the ring R, an abelian group M and an operation defined between R and M whereas I is just a subset of R that satisfies some criterion. Similary, an ideal being a subset of a ring R must be closed w.r.t multiplication as well.

Theorem 1. *Every soft ideal of a soft ring is a soft submodule, however every soft submodule is not a soft ideal.*

We provide two examples in the support of the above theorem.

Example 1. Let M and N be $\mathbb{Z}$-module and $F(x) = \{y : x\rho y \Leftrightarrow x + y \in \mathbb{Z}\}$. Then, for all $y_1, y_2 \in F(x)$, $x \in N$ and $r \in R = \mathbb{Z}$, we have $(x+y_1)+(x+y_2) = x+(y_1+y_2) \in \mathbb{Z}$, so $y_1+y_2 \in F(x)$, $r(x+y) = rx+ry \in \mathbb{Z}$, implies $yr \in F(x)$ for all $r \in R$. Thus, for each $x \in N$, we can easily verify that $F(x)$ is a submodule of M and hence, (F, A) is a soft module over M. Since multiplication in $F(x)$ is not defined hence it is clear that (F, A) is not an ideal of $R = \mathbb{Z}$.

Example 2. Let $R = A = \mathbb{Z}_4 = \{0, 1, 2, 3\}$ and $I = \{0, 1, 2\}$. The set-valued function $F : A \to P(R)$ given by $F(x) = \{y \in R : x, y \in \{0, 2\}\}$. Then $F(0) = R$, $F(1) = \{0\}$, $F(2) = \mathbb{Z}_4$ and $F(3) = \{0, 2\}$. Since all these sets are subrings of R. Hence, (F, A) is a soft ring over R. On the other hand, consider the function $\gamma : I \to P(R)$ given by $\gamma(x) = \{y \in R : x.y = 0\}$. As we see, $\gamma(0) = R \lhd R$, $\gamma(1) = \{0\} \lhd F(1) = \{0\}$ and $\gamma(2) = \{0, 2\} \lhd F(2) = \mathbb{Z}_4$. Hence, (γ, I) is a soft ideal of (F, A). Similarly it is easy to verify that each of them are soft submodules over (F, A).

Definition 10. Let R be a ring and M be a module over R. (F, A) be a soft module over the module M and (G, B) be the soft ideal. Then the soft multiplicative module $(F, A) \times (G, B) = (H, A \times B)$ is defined as $H(x, y) = F(x) \cap G(y)$ for all $(x, y) \in A \times B$.

The following results can be easily produced about soft multiplication modules.

Proposition 1. *Let (F, A) and (G, B) be two soft multiplicative modules over M. Then,*

(i) $(F, A) \tilde{\cap} (G, B)$ is a multiplicative soft module.

(ii) $(F, A) \tilde{\cup} (G, B)$ is a multiplicative soft module.

Proof. Straightforward. □

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