Simulation of the Incidence of Zika with Prevention and Control

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Abstract

A simulation model is formulated in three dimensions, following the formalism of S. Ronald Ross for the transmission of malaria. The model includes the populations of pregnant women infected with the Zika virus, pregnant women whose fetuses have developed microcephaly and the vector, female mosquitoes carrying the Zika virus. The simulations were executed in Maple for six different prevention strategies, resulting that the best strategy is when 80% of pregnant women are prevented from mosquito bites and 20% of mosquitoes are eliminated by the use of traps.

Keywords: Zika; Microcephaly; Aedes aegypti; biological control; Aedes mosquito traps; mathematical model

1 Introduction

Since the identification of Zika virus infection in Brazil in May 2015, the virus has spread throughout the Americas. Until May 19, 2016, according to the WHO 60 countries and territories reported the persistence of transmission by mosquitoes [1, 2]. Although this infection often leads to mild disease, its appearance in the Americas has coincided with a sharp increase in the number of patients developing Guillain-Barre syndrome and the birth of children with
congenital microcephaly [3, 4, 5]. Reports of high rates of primary microcepha-
ly and Guillain-Barré syndrome associated with Zika virus infection in
French Polynesia and Brazil have led to consider this infection as a threat in
public health. Until this moment there are no vaccines, therapies or preventive
drugs available for this infection.

2 The model

In the construction of the model the Ross-Macdonald formalism is followed, ex-
tending the dynamic system that describes the infectious process to dimension
three, by including a third ordinary differential equation for the population
of pregnant women infected with Zika virus that present fetuses with micro-
cephaly. In modeling, it is considered that the host population is the population
of pregnant women susceptible to being infected with the Zika virus when they
are bitten by the female mosquito 

The terms of transmission are formulated based on the force of the infection in
the susceptible hosts and in non-carrier female mosquitoes. A fraction of micro-
cephaly development is included in pregnant women infected with Zika virus.
Two controls are also considered: a prevention fraction of pregnant women
who are bitten by the mosquito and a fraction of mosquitoes that die when
trapped with used traps inside the houses and their surroundings. It is impor-
tant to note that, due to the prevention of mosquito bites, it is assumed that
women do not contribute to the transmission of Zika to non-carrier mosquitoes
of virus. The modeling is done considering an endemic region to Zika infection.

The model presents the following variables and parameters: \( x \): fraction of
pregnant women infected with the Zika virus; \( x_m \): fraction of women in preg-
nancy whose fetuses have developed microcephaly, \( 1 - x - x_m \): fraction of
susceptible pregnant women, \( y \): fraction of female mosquitoes carrying the
Zika virus, \( 1 - y \): fraction of female mosquitoes not carrying Zika virus at
a time \( t \), respectively. \( \lambda(y) \): force of the infection in pregnant women, \( \lambda(x) \):
force of the infection in the female mosquito, \( \omega_m \): rate of development of mi-
crocephaly in fetuses of pregnant women infected with Zika virus, \( \theta \): recovery
rate of women infected with Zika virus, \( \mu \): death rate of women with fetuses
who develop microcephaly, \( \epsilon \): death rate of female mosquitoes due to environ-
mental conditions, \( \beta \): probability of transmission of Zika virus to susceptible
women in pregnancy, \( \sigma \): probability of transmission of Zika virus to non-carrier
mosquitoes by the bite of these to pregnant women infected with Zika, \( h_1 \): cov-
erage of women in pregnancy who are prevented from mosquito bites and \( h_2 \):
fraction of mosquitoes that die when trapped with used traps inside homes
and their surroundings.
The nonlinear ordinary differential equations that interpret the dynamics are:

\[ \frac{dx}{dt} = \lambda_h(y)(1 - h_1)(1 - x - x_m) - \rho x \]  

(1)

\[ \frac{dx_m}{dt} = \omega_m x - \mu x_m \]  

(2)

\[ \frac{dy}{dt} = \lambda_v(x)(1 - y) - (\epsilon + h_2)y \]  

(3)

where the forces of infection are: \( \lambda_h(y) = \beta y, \lambda_v(x) = \sigma x, \rho = \theta + \omega_m, \)

\( 0 < \beta, \sigma < 1, \epsilon, \mu, \theta, \omega_m > 0 \) and \( 0 \leq h_1, h_2 < 1. \)

This system becomes the next one,

\[ \frac{dx}{dt} = \beta(1 - h_1)(1 - x - x_m)y - (\theta + \omega_m)x \]  

(4)

\[ \frac{dx_m}{dt} = \omega_m x - \mu x_m \]  

(5)

\[ \frac{dy}{dt} = \sigma(1 - y)x - (\epsilon + h_2)y \]  

(6)

The positive trajectories of this system are in the epidemiological sense region,

\( \Pi = \{(x, x_m, y) \in \mathbb{R}_+^3 : 0 < x + x_m \leq 1, \ 0 \leq y \leq 1\}. \)

Using the mathematical definition, we calculate the \( R_0 \), as the spectral radius of a matrix, called the next generation, that is, the dominant eigenvalue of said matrix [6, 7],

\[ R_0 = \rho(G) = \max\{\lambda_i\}, \quad i = 1, 2, 3 \]

where \( G \) is the next-generation matrix, \( \rho \) is the dominant eigenvalue and \( \lambda_i \) are the eigenvalues. Thus,

\[ \tilde{R}_0 = \rho(G) = \max\{\lambda_{1,2}\} = \sqrt{\frac{\beta \sigma (1 - h_1)}{(\theta + \omega_m)(\epsilon + h_2)}}. \]

Table 2 contains, for different strategies \((h_1, h_2)\), the value of the epidemic threshold. Without any control or with a low control \((0.3, 0.2)\), the threshold value is greater than one, which indicates prevalence of Zika virus.

The best strategy is when 80% of pregnant women are prevented from mosquito bites and 20% of mosquitoes are eliminated by the use of traps.
Table 1: The $R_0$, prevention and control strategies.

<table>
<thead>
<tr>
<th>Estrategia</th>
<th>Umbral $R_0(h_1, h_2)$</th>
<th>0.92678</th>
<th>0.7496</th>
<th>0.7703</th>
<th>0.6935</th>
<th>2.427</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>23.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.96, 0)</td>
<td>(0.6, 0.4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.4, 0.6)</td>
<td>(0.8, 0.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.3, 0.2)</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

3 Numerical simulations

For an average temperature of 23°C, the probabilities of transmission $\sigma$, $\beta$ and the death rate of the mosquito were calculated, using the functions contained in [8, 9, 10]; with a life expectancy in Colombia of 75 years, an average transmissibility period of 7 days $\left(E[x_2]\right)$ and considering the theory of the Poisson process in epidemiology $\left(E[x_2] = \frac{1}{\theta+\mu}\right)$, the parameters $\mu$ and $\theta$ were calculated; the values of the parameters $\sigma$, $\alpha$ were assigned according to the parameters previously calculated, as shown in Table 2.

Table 2: Estimated parameters of the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$\epsilon$</th>
<th>$\omega_m$</th>
<th>$\mu$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.7913</td>
<td>0.7730</td>
<td>0.0352</td>
<td>0.7</td>
<td>0.010</td>
<td>0.0500</td>
</tr>
</tbody>
</table>

The simulations of the dynamic system (4)-(6), are plotted in the Maple software with the values of the calculated parameters that appear in Table 2, for the different strategies:

1. Without both prevention and control ($h_1 = 0$ and $h_2 = 0$).
2. 96% of prevention of women in pregnancy and no control over mosquito ($h_1 = 0$).
3. 60% of prevention of women in pregnancy and 40% control of mosquitoes.
4. 40% of prevention of women in pregnancy and 60% of control of mosquitoes.
5. 80% of prevention of women in pregnancy and 20% of control of mosquitoes.
6. 30% of prevention of women in pregnancy and 20% of control of mosquitoes.

In the first graph of Figure 1 without both prevention and control, there is an epidemic with an $R_0 = 23.17$ and the cases of microcephaly stabilize at a level above 90% and higher than the population of infected women with an epidemic peak of 15% at 10 days, later decreases and stabilizes around 20
Figure 1: Strategies: \((h_1, h_2) = (0, 0)\) with \(R_0 = 23.17\) and \((h_1, h_2) = (0.96, 0)\) with \(R_0 = 0.92678\); \(x\) (continuous line), \(x_m\): (segmented line) and \(y\): (dotted line).

4 Conclusions

It is concluded from this study that the best strategy is when 80% of pregnant women are prevented from mosquito bites and 20% of mosquitoes are eliminated by the use of traps; similarly for a 60% and 40%. 

days; while the population of mosquitoes carrying the Zika virus stabilizes approximately 25% after 80 days. That is, the infection reflects an endemic state.

Applying the second strategy with only prevention, the second graph of Figure 1 shows qualitative changes in population trajectories breaking their stability; that is, there is an epidemiological effect on the incidence of the infection.

The graphs in Figure 2 show a very similar behavior for strategies 3 and 4. With threshold values of 0.7496 and 0.7703 the infection tends to die out over time. In this case, any of the two strategies is more advisable than strategy three.

The first graph of Figure 3 shows a behavior similar to the behavior of the first graph, Figure 2 and in the second graph of Figure 3, oscillatory behaviors are observed, very interesting in the infectious process that is investigated.
Figure 2: Strategies: \((h_1, h_2) = (0.6, 0.4)\) with \(R_0 = 0.7496\) and \((h_1, h_2) = (0.4, 0.6)\) with \(R_0 = 0.7703\); \(x\) (continuous line), \(x_m\): (segmented line) and \(y\): (dotted line).

Figure 3: Strategies: \((h_1, h_2) = (0.8, 0.2)\) with \(R_0 = 0.6935\) and \((h_1, h_2) = (0.3, 0.2)\) with \(R_0 = 2.427\); \(x\) (continuous line), \(x_m\): (segmented line) and \(y\): (dotted line).

References


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