Aedes aegypti Population Model with Integrated Control

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Abstract

A mathematical model to the Aedes aegypti population dynamics, is proposed and analyzed. In the present model the integrated control (IC) along the all early stages egg, (larva-pupa) and adult stage of the mosquito, is included. The model is development through a Nonlinear Differential Equation System which describes the population dynamics of the mosquito when IC is applied. The equilibrium points and stability analysis through Routh-Hurwitz criterion are obtained and done, respectively. In addition, simulations under different conditions will allow to determine the IC efficacy on this population, is performed.

Keywords: Population Dynamics, Aedes aegypti, Integrated Control, Routh-Hurwitz Criterion, Stability

1 Introduction

Aedes aegypti mosquito has been studied for many years given the capacities of adaptation that has shown, which has led to a wide distribution of this mosquito worldwide. On the other hand, this is the main transmitter of viruses that cause diseases such as Dengue, Yellow Fever, Chikungunya and Zika, all them of easy spread and incidence [1]. The use of mathematical models has
contributed to the strategies implementation to control the spread of viruses, helping to make models that involve the population dynamics of the main (A. aegypti) transmitter of these viruses [1]. Currently, most of the efforts to control the mosquito population increase, consists of performing fumigation for the removal of the adult mosquito, as well as the eradication of the possible breedings, to not allow that those immature stages became to the adult mosquito [1].

In this paper some mathematical models are studied, then the system of ordinary differential equations in shown, variables and parameters are defined, equilibrium points are found and the stability analysis of each one are obtained. Then is performed simulations using varied controls to determine the influence that the IC has in the A. aegypti population under these considerations of the model.

Different mathematical models are suggested and analyzed based on ordinary differential equations, which study the population behavior and A. aegypti control, which provide relevant information. In one of these works is designed and analyzed a mathematical model for the population dynamics of the A. aegypti, which includes the life cycle of the vector and two control strategies including: Chemical control to the adult mosquito and biological control to the immature stages [6]. Also, there is a work where is modeling the growth dynamics of the mosquito subjected to adulticide control and with resistance to the chemical [7]. Another of the works is refered to a mathematical model that shows the transmission dynamics of dengue, with the objective of studying the population behavior of A. aegypti and affected human population [5]. Finally, the work in which is modeled the transmission dynamics of classical dengue in an endemic region considering the use of preventive measures and of mechanical control in reducing the transmission of disease [8].

2 The Model

In order to determine the influence of a IC into the A. aegypti population dynamics in endemic regions, we propose and analyze a simulation model based on nonlinear ordinary differential equations, that includes the mosquito population in all its stages such as egg, larva-pupa and adult.
The compartmentalized diagram of the dynamics is:

\[ \begin{align*}
    \dot{x} & = \beta x - \mu x \\
    \dot{y} & = \phi f(1-g)(1-\frac{y+z}{k})x - (\sigma + \theta)y \\
    \dot{z} & = \theta y - (\pi + \delta)z - \omega z
\end{align*} \]

and the associated differential equation system is:

\[ \begin{align*}
    \frac{dx}{dt} & = \omega z - (\beta + \mu)x \\
    \frac{dy}{dt} & = \phi f(1-g) \left(1 - \frac{y+z}{k}\right) x - (\sigma + \theta)y \\
    \frac{dz}{dt} & = \theta y - (\pi + \delta)z - \omega z
\end{align*} \]

With initial conditions \( x(0) = x_0, y(0) = y_0, z(0) = z_0 \), and biological sense region: \( \Omega = \{(x, y, z) \in \mathbb{R}^3_+ : x > 0, 0 < y + z \leq k\} \), the model has biological sense when the populations are not negative and the immature mosquito populations do not exceed the carrying capacity of the breedings.

The model considers a divided population in three stages where: \( x \) is the average number of adult mosquitoes in a \( t \) time, \( y \) is the average number of eggs in a \( t \) time and \( z \) is the average number of larvae-pupae in a \( t \) time. In addition, the parameter meanings are as follow, \( \phi \) : oviposition rate, \( \mu \) : natural mortality rate of adult mosquitoes, \( g \) : fraction of eliminated eggs by ovitraps, \( f \) : rate of eggs which become adults, \( \beta \) : adult mortality rate by adulticide, \( \sigma \) : Invalidity rate of eggs, \( \theta \) : rate of eggs become larvae-pupae, \( \pi \) : natural mortality rate of larva-pupa stage, \( \delta \) : mortality rate of larva-pupa stage by chemicals and \( \omega \) : rate of larva-pupa become adult stage.
2.1 Stability analysis

Whereas the system (1-3) when there is no variation in the time, the homogeneous system is as follow:

\[ 0 = \omega z - (\beta + \mu)x \]  \hspace{1cm} (4)
\[ 0 = \phi f(1 - g) \left(1 - \frac{y + z}{k}\right)x - (\sigma + \theta)y \]  \hspace{1cm} (5)
\[ 0 = \theta y - (\pi + \delta)z - \omega z \]  \hspace{1cm} (6)

Direct calculation shows that their solutions are:

\[ E_0 = (0, 0, 0) \quad y \quad E_1 = (\hat{x}, \hat{y}, \hat{z}) \]

With:

\[ \hat{x} = \frac{k\omega \theta(\eta - 1)}{\eta(\pi + \delta + \omega + \theta)(\beta + \mu)}, \quad \hat{y} = \frac{k(\eta - 1)(\pi + \delta + \omega)}{\eta(\pi + \delta + \omega + \theta)}, \quad \text{and} \quad \hat{z} = \frac{k\theta(\eta - 1)}{\eta(\pi + \delta + \omega + \theta)} \]

In addition

\[ \eta = \frac{\phi f(1 - g)\omega \theta}{(\beta + \mu)(\pi + \delta + \omega)(\sigma + \theta)}, \quad \eta > 1 \]

Being \( \eta \) the threshold of the mosquito population growth. This number determines if the population continues growing, or whether on the contrary disappears from the environment.

2.1.1 Equilibrium point without coexistence \( E_0 \)

The Jacobian Matrix associated to the system (1-3), is obtained

\[
J(x, y, z) = \begin{pmatrix}
-(\beta + \mu) & 0 & \omega \\
\phi f(1 - g) \left(1 - \frac{y + z}{k}\right) - \left[\frac{\phi f(1 - g)x}{k}\right] + (\sigma + \theta) & 0 & -\frac{\phi f(1 - g)x}{k} \\
0 & \theta & -(\pi + \delta + \omega)
\end{pmatrix}
\]

Now, the equilibrium \( E_0 \) is evaluated in the matrix and is obtained the characteristic polynomial:

\[ P(\lambda) = \begin{vmatrix}
-(\beta + \mu + \lambda) & 0 & \omega \\
\phi f(1 - g) & -(\sigma + \theta + \lambda) & 0 \\
0 & \theta & -(\pi + \delta + \omega + \lambda)
\end{vmatrix} \]

This is,

\[ P(\lambda) = \lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0 \]

With:

\[ a_1 = (\theta + \sigma + \omega + \delta + \pi) + (\beta + \mu) \]
\[ a_2 = (\omega + \delta + \pi)(\theta + \sigma) + (\beta + \mu)(\theta + \sigma + \omega + \delta + \pi) \]
\[ a_3 = (\omega + \delta + \pi)(\theta + \sigma)(\beta + \mu)(1 - \eta) \]

Therefore, using the Routh-Hurwitz criterion [3] is that:
• If $\eta < 1$ then $a_1 > 0$, $a_3 > 0$ and $a_1 a_2 > a_3$, which implies that $E_0$ is local and asymptotically steady.

• If $\eta > 1$ then $a_3 < 0$, which implies that $E_0$ is unsteady.

### 2.1.2 Equilibrium point with coexistence $E_1$

Evaluating the equilibrium $E_1$ and finding the characteristic polynomial:

$$P(\lambda) = \begin{vmatrix}
-(\beta + \mu + \lambda) & 0 & \omega \\
\phi f(1 - g) \left(1 - \frac{y + z}{k}\right) & -\phi f(1 - g) x & -\phi f(1 - g) y \\
0 & \theta & -(\pi + \delta + \omega + \lambda)
\end{vmatrix}$$

Obtaining,

$$P(\lambda) = \lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0$$

Then,

$$a_1 = (\theta + \sigma) + (\omega + \delta + \pi) + (\beta + \mu) + \left(\phi f(1 - g) x\right)$$

$$a_2 = (\omega + \delta + \pi)(\theta + \sigma + \beta + \mu) + (\beta + \mu)(\theta + \sigma) + (\omega + \delta + \pi + \beta + \mu + \theta) \left(\phi f(1 - g) x\right)$$

$$a_3 = (\omega + \delta + \pi)(\theta + \sigma)(\beta + \mu) \left(1 - \eta + \frac{y + z}{k}\right) + (\beta + \mu)(\omega + \delta + \pi + \theta) \left(\phi f(1 - g) y\right)$$

Therefore, using the Routh-Hurwitz criterion [3] is that:

• If $\eta > 1$ then $a_1 > 0$, $a_3 > 0$ and $a_1 a_2 > a_3$, which implies that $E_1$ is local and asymptotically steady.

• If $\eta < 1$ then $a_1 < 0$ and $a_3 < 0$, which implies that $E_1$ is unsteady.

### 2.1.3 Simulations

The simulations corresponding to the system (1-3), are performed, using several initial conditions and some values for the parameters.

**Simulations using IC and different initial conditions**

Figure 1 shows that regardless of the initial conditions of the eggs, larvae-pupae and adult mosquito populations, they disappear within a period of 30 to 35 days due to implementation of the IC. It demonstrates the effectiveness of the IC used in egg, larva-pupa and adult mosquito stages in endemic regions.
Simulations using IC in the egg, larva-pupa and adult mosquito stages

Now, we used fixed initial conditions and three different values to the parameters of the controls, which are varied in such a way that the influence of IC in the population dynamics of *A. aegypti*, is observed.

From figure 2, we conclude that under this model, the IC is the most effective in the three populations. However, analyzing population dynamics using control only in the juvenile stages, it appears that these controls are as effective as their implementation in the three stages. In adult mosquito population, it is shown that the use of a single control method makes the population decrease in longer period to which occurs when two or more control methods are used. It is concluded that apply 40% control in the larva-pupa stage and 43% in the egg stage (under these conditions), populations tend to disappear in a period from 30 to 35 days.

Simulations using control in the egg and larva-pupa stages

Figure 2 also shows that the most effective controls are applied to egg and larva-pupa stages. Therefore, it is again used fixed initial conditions and applies control in these stages (without applying control in adult mosquito) but by varying the values of the parameters.
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Figure 2: Population dynamics of A. aegypti in the egg, larva-pupa and adult mosquito stages, using different values to the controls $g$, $\delta$ and $\beta$. The parameter values: $\phi = 6$, $\mu = 0.143$, $f = 0.8$, $\sigma = 0.4$, $\theta = 0.2$, $\pi = 0.4$, $\omega = 0.2$ and initial conditions: $x_0 = 3000$, $y_0 = 5000$ and $z_0 = 4000$

Figure 3: Population dynamics of A. aegypti in the egg, larva-pupa and adult mosquito stages, using different values to the controls $g$ and $\delta$. The parameter values: $\phi = 6$, $\mu = 0.143$, $f = 0.8$, $\sigma = 0.4$, $\theta = 0.2$, $\pi = 0.4$, $\omega = 0.2$ and initial conditions: $x_0 = 3000$, $y_0 = 5000$ and $z_0 = 4000$

- Here, we concluded, that the most effective control is in which is used a high percentage of control in the two populations. The use of a high
control in the larvae-pupae stage and a low control in the egg stage, both generate that the adult mosquito population disappeared more quickly than if the control were applied contrary.

- According simulations, there are two possibilities to eliminate the adult mosquito population quickly: 1. Using a 50% control in the egg and larva-pupa stages, or 2. using a 20% control in egg stage and 70% control in the larva-pupa stage.

3 Main Results

Comparing simulations made with the different controls, is observed that:

- The IC method is the best strategy to takes into account to fight the populations of *A. aegypti* in endemic regions.

- The best control by itself is the larvicide, but acting only is not enough, that is why the joint use of several controls is required.

- It is not necessary to use the three controls, since the use of control in the immature is almost as effective as the control in the three stages considered. This could have a favorable economic influence.

- To apply this strategy of IC is recommended to use a higher control in larva-pupa stage and one lower than to the egg stage, since with this strategy would be an option to have a medium control in both stages and equivalent efficiency.

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References


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