Semidetached $B$-Algebras

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Abstract

The notion of semidetached $B$-algebra is introduced, and their properties are investigated. Several conditions for a semidetached structure to be a semidetached $B$-algebra are provided. Characterizations of semidetached $B$-algebras are considered.

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1 Introduction

The idea of quasi-coincidence of a fuzzy point with a fuzzy set, which is mentioned in [10], played a vital role to generate some different types of fuzzy subgroups, called $(\alpha, \beta)$-fuzzy subgroups, introduced by Bhakat and Das [1].

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In particular, \((\in, \in \lor q)\)-fuzzy subgroup is an important and useful generalization of Rosenfeld’s fuzzy subgroup. In \(B\)-algebras and \(BCK/BCI\)-algebras, the concept of \((\alpha, \beta)\)-fuzzy notions, which are studied in the papers [2], [3], [4], [5], [6], [7], [11], and [12] is also important and useful generalization of the well-known concepts, called fuzzy subalgebras. Jun et al. [9] introduced the notion of semidetached structure and applied it to \(BCK/BCI\)-algebras.

In this paper, we introduce the notion of semidetached \(B\)-algebras, and investigate their properties. We provide several conditions for a semidetached structure to be a semidetached \(B\)-algebra. We consider characterization of a semidetached \(B\)-algebra.

2 Preliminaries

A \(B\)-algebra is a set \(X\) with a constant 0 and a binary operation \('\ast'\) satisfying the axioms:

(a1) \(x \ast x = 0\),

(a2) \(x \ast 0 = x\),

(a3) \((x \ast y) \ast z = x \ast (z \ast (0 \ast y))\)

for all \(x, y, z \in X\).

A nonempty subset \(S\) of a \(B\)-algebra \(X\) is called a subalgebra of \(X\) if \(x \ast y \in S\) for all \(x, y \in S\).

A fuzzy set \(\lambda\) in a \(B\)-algebra \(X\) is called a fuzzy \(B\)-algebra of \(X\) (see [8]) if it satisfies:

\[
(\forall x, y \in X) (\lambda(x \ast y) \geq \min\{\lambda(x), \lambda(y)\}).
\]

(2.1)

For any fuzzy set \(\lambda\) in a set \(X\) and any \(t \in [0, 1]\), the set

\[
\lambda_t = \{x \in X \mid \lambda(x) \geq t\}
\]

is called a level subset of \(\lambda\).

Note that a fuzzy set \(\lambda\) in \(X\) is a fuzzy \(B\)-algebra of \(X\) if and only if \(\lambda_t\) is a subalgebra of \(X\) for all \(t \in (0, 1]\).

A fuzzy set \(\lambda\) in a set \(X\) of the form

\[
\lambda(y) := \begin{cases} t \in (0, 1] & \text{if } y = x, \\ 0 & \text{if } y \neq x, \end{cases}
\]

(2.2)

is said to be a fuzzy point with support \(x\) and value \(t\) and is denoted by \(x_t\).

For a fuzzy set \(\lambda\) in a set \(X\), a fuzzy point \(x_t\) is said to
• contained in λ, denoted by \( x_t \in \lambda \) (see [10]), if \( \lambda(x) \geq t \).
• be quasi-coincident with λ, denoted by \( x_t q \lambda \) (see [10]), if \( \lambda(x) + t > 1 \).
• \( x_t \in \triangledown q \lambda \) if \( x_t \in \lambda \) or \( x_t q \lambda \).

3 Semidetached \( B \)-algebras

In what follows, let \( X \) denote a \( B \)-algebra unless otherwise specified.

Definition 3.1 ([7]). A fuzzy set \( \lambda \) in \( X \) is called an \( (\in, \in \triangledown q) \)-fuzzy \( B \)-algebra of \( X \) if it satisfies:

\[
x_t \in \lambda, \ y_r \in \lambda \Rightarrow (x \ast y)_{\min(t,r)} \in \triangledown q \lambda
\]

for all \( x, y \in X \) and \( t, r \in (0, 1] \).

Lemma 3.2 ([7]). A fuzzy set \( \lambda \) in \( X \) is an \( (\in, \in \triangledown q) \)-fuzzy \( B \)-algebra of \( X \) if and only if it satisfies:

\[
\lambda(x \ast y) \geq \min\{\lambda(x), \lambda(y), 0.5\}
\]

for all \( x, y \in X \).

Given a set \( X \) and a subinterval \( \Omega \) of \([0, 1]\), a semidetached structure over \( \Omega \) is defined to be a pair \((X, f)\) where \( f : \Omega \to \mathcal{P}(X) \) is a mapping (see [9]).

Definition 3.3. A semidetached structure \((X, f)\) is called a semidetached \( B \)-algebra over \( \Omega \) with respect to \( t \in \Omega \) (briefly, \( t \)-semidetached \( B \)-algebra over \( \Omega \)) if \( f(t) \) is a \( B \)-subalgebra of \( X \) where \( \mathcal{P}(X) \) is the power set of \( X \).

We say that \((X, f)\) is a semidetached \( B \)-algebra over \( \Omega \) if it is a \( t \)-semidetached \( B \)-algebra over \( \Omega \) for all \( t \in \Omega \).

Given a fuzzy set \( \lambda \) in \( X \), consider the following mappings

\[
\mathcal{A}_U^\lambda : \Omega \to \mathcal{P}(X), \ t \mapsto \lambda_t,
\]

\[
\mathcal{A}_Q^\lambda : \Omega \to \mathcal{P}(X), \ t \mapsto Q(\lambda;t),
\]

\[
\mathcal{A}_E^\lambda : \Omega \to \mathcal{P}(X), \ t \mapsto E(\lambda;t),
\]

where \( Q(\lambda;t) := \{x \in X \mid x_t q \lambda\} \) and \( E(\lambda;t) := \{x \in X \mid x_t \in \triangledown q \lambda\} \) which are called the \( q \)-set and \( \in \triangledown q \)-set with respect to \( t \) (briefly, \( t \)-\( q \)-set and \( t \)-\( \in \triangledown q \)-set), respectively, of \( \lambda \).

Note that, for any \( t, r \in (0, 1] \), if \( t \geq r \) then every \( r \)-\( q \)-set is contained in the \( t \)-\( q \)-set, that is, \( Q(\lambda;r) \subseteq Q(\lambda;t) \). Obviously, \( E(\lambda;t) = \lambda_t \cup Q(\lambda;t) \).
**Theorem 3.4.** A semidetached structure \( (X, \mathcal{A}_Q^\lambda) \) is a semidetached \( B \)-algebra over \( \Omega = (0, 1] \) if and only if \( \lambda \) is a fuzzy \( B \)-algebra of \( X \).

**Proof.** Straightforward. \( \square \)

**Theorem 3.5.** If \( \lambda \) is a fuzzy \( B \)-algebra of \( X \), then a semidetached structure \( (X, \mathcal{A}_Q^\lambda) \) is a semidetached \( B \)-algebra over \( \Omega = (0, 1] \).

**Proof.** Let \( x, y \in \mathcal{A}_Q^\lambda(t) \) for \( t \in \Omega = (0, 1] \). Then \( x_t q \lambda \) and \( y_t q \lambda \), that is, \( \lambda(x) + t > 1 \) and \( \lambda(y) + t > 1 \). It follows from (2.1) that

\[
\lambda(x \ast y) + t \geq \min\{\lambda(x), \lambda(y)\} + t = \min\{\lambda(x) + t, \lambda(y) + t\} > 1.
\]

Hence \( (x \ast y)_t q \lambda \), and so \( x \ast y \in \mathcal{A}_Q^\lambda(t) \). Therefore \( \mathcal{A}_Q^\lambda(t) \) is a subalgebra of \( X \) for all \( t \in \Omega \). Consequently \( (X, \mathcal{A}_Q^\lambda) \) is a semidetached \( B \)-algebra over \( \Omega = (0, 1] \). \( \square \)

Using Theorems 3.4 and 3.5, we have the following corollary.

**Corollary 3.6.** If the semidetached structure \( (X, \mathcal{A}_Q^\lambda) \) is a semidetached \( B \)-algebra over \( \Omega = (0, 1] \), then the semidetached structure \( (X, \mathcal{A}_Q^\lambda) \) is a semidetached \( B \)-algebra over \( \Omega = (0, 1] \).

**Definition 3.7.** A fuzzy set \( \lambda \) in \( X \) is called a right \((q, \in \vee q)\)-fuzzy \( B \)-algebra of \( X \) if it satisfies:

\[
x_t q \lambda, y_r q \lambda \Rightarrow (x \ast y)_{\min\{t,r\}} \in \vee q \lambda \tag{3.6}
\]

for all \( x, y \in X \) and \( t, r \in (0.5, 1] \).

If \( \lambda \) satisfies the condition (3.6) for all \( x, y \in X \) and \( t, r \in (0, 0.5] \), then we say that \( \lambda \) is a left \((q, \in \vee q)\)-fuzzy \( B \)-algebra of \( X \). If \( \lambda \) satisfies the condition (3.6) for all \( x, y \in X \) and \( t, r \in (0, 1] \), then we say that \( \lambda \) is a \((q, \in \vee q)\)-fuzzy \( B \)-algebra of \( X \).

**Theorem 3.8.** Every right \((q, \in \vee q)\)-fuzzy \( B \)-algebra is an \((\in, \in \vee q)\)-fuzzy \( B \)-algebra.

**Proof.** Let \( \lambda \) be a right \((q, \in \vee q)\)-fuzzy \( B \)-algebra of \( X \). Let \( x, y \in X \) and \( t, r \in (0, 1] \) be such that \( x_t \in \lambda \) and \( y_r \in \lambda \). Then \( \lambda(x) \geq t \) and \( \lambda(y) \geq r \). If \( (x \ast y)_{\min\{t,r\}} \in \vee q \lambda \), then \( \lambda(x \ast y) < \min\{t, r\} \) and \( \lambda(x \ast y) + \min\{t, r\} \leq 1 \). It follows that \( \lambda(x \ast y) < 0.5 \) and so that \( \lambda(x \ast y) < \min\{t, r, 0.5\} \). Hence

\[
1 - \lambda(x \ast y) > 1 - \min\{t, r, 0.5\} = \max\{1 - t, 1 - r, 1 - 0.5\} \geq \max\{1 - \lambda(x), 1 - \lambda(y), 0.5\},
\]

and thus there exists \( \delta \in (0, 1] \) such that

\[
1 - \lambda(x \ast y) \geq \delta > \max\{1 - \lambda(x), 1 - \lambda(y), 0.5\}. \tag{3.7}
\]
The right inequality in (3.7) implies that \( \delta > 0.5, \lambda(x) + \delta > 1 \) and \( \lambda(y) + \delta > 1 \), that is, \( x_\delta q \lambda \) and \( y_\delta q \lambda \). Since \( \lambda \) is a right \((q, \in \lor q)\)-fuzzy \(B\)-algebra of \(X\), it follows that \( (x * y)_\delta \in \lor q \lambda \). On the other hand, the left inequality in (3.7) implies that \( \lambda(x * y) + \delta \leq 1 \), that is, \( (x * y)_\delta q \lambda \) and \( \lambda(x * y) \leq 1 - \delta < 1 - 0.5 = 0.5 < \delta \), i.e., \( (x * y)_\delta \in \lor q \lambda \). Hence \( (x * y)_\delta \in \lor q \lambda \), which is a contradiction. Therefore \( (x * y)_{\min\{t,r\}} \in \lor q \lambda \), and thus \( \lambda \) is an \((\in, \lor q)\)-fuzzy \(B\)-algebra of \(X\).

**Theorem 3.9.** If every fuzzy point has the value \( t \in (0, 0.5] \), then every \((\in, \lor q)\)-fuzzy \(B\)-algebra is a left \((q, \in \lor q)\)-fuzzy \(B\)-algebra.

*Proof.* Let \( \lambda \) be an \((\in, \lor q)\)-fuzzy \(B\)-algebra of \(X\). Let \( x, y \in X \) and \( t, r \in (0, 0.5] \) be such that \( x_\in q \lambda \) and \( y_\in q \lambda \). Then \( \lambda(x) + t > 1 \) and \( \lambda(y) + r > 1 \). Since \( t, r \in (0, 0.5] \), it follows that \( \lambda(x) > 1 - t \geq 0.5 \geq t \) and \( \lambda(y) > 1 - r \geq 0.5 \geq r \), that is, \( x_\in q \lambda \) and \( y_\in q \lambda \). It follows from (3.1) that \( (x * y)_{\min\{t,r\}} \in \lor q \lambda \). Therefore \( \lambda \) is a left \((q, \in \lor q)\)-fuzzy \(B\)-algebra of \(X\).

**Corollary 3.10.** If every fuzzy point has the value \( t \in (0, 0.5] \), then every right \((q, \in \lor q)\)-fuzzy \(B\)-algebra is a left \((q, \in \lor q)\)-fuzzy \(B\)-algebra.

**Proposition 3.11.** If \( (X, \mathcal{A}_Q^\lambda) \) is a semidetached \(B\)-algebra over \( \Omega = (0.5, 1] \), then \( \lambda \) satisfies:

\[
x_t \in \lambda, \ y_r \in \lambda \Rightarrow (x * y)_{\max\{t,r\}} q \lambda
\]

for all \( x, y \in X \) and \( t, r \in \Omega \).

*Proof.* Let \( x, y \in X \) and \( t, r \in \Omega = (0.5, 1] \) be such that \( x_\in q \lambda \) and \( y_\in q \lambda \). Then \( \lambda(x) \geq t > 0.5 \) and \( \lambda(y) \geq r > 0.5 \), which imply that \( \lambda(x) + t > 1 \) and \( \lambda(y) + r > 1 \), that is, \( x_\in q \lambda \) and \( y_\in q \lambda \). It follows that \( x, y \in \mathcal{A}_Q^\lambda(\max\{t,r\}) \) and \( \max\{t,r\} \in (0.5, 1] \). Since \( \mathcal{A}_Q^\lambda(\max\{t,r\}) \) is a subalgebra of \( X \) by assumption, we have \( x * y \in \mathcal{A}_Q^\lambda(\max\{t,r\}) \) and so \( (x * y)_{\max\{t,r\}} q \lambda \).

**Proposition 3.12.** If \( (X, \mathcal{A}_Q^\lambda) \) is a semidetached \(B\)-algebra over \( \Omega = (0, 0.5] \), then \( \lambda \) satisfies:

\[
x_t q \lambda, \ y_r q \lambda \Rightarrow (x * y)_{\max\{t,r\}} q \lambda, \ (x * y)_{\max\{t,r\}} \in \lambda
\]

for all \( x, y \in X \) and \( t, r \in \Omega \).

*Proof.* Let \( x, y \in X \) and \( t, r \in \Omega = (0, 0.5] \) be such that \( x_\in q \lambda \) and \( y_\in q \lambda \). Then \( x \in \mathcal{A}_Q^\lambda(t) \) and \( y \in \mathcal{A}_Q^\lambda(r) \). It follows that \( x, y \in \mathcal{A}_Q^\lambda(\max\{t,r\}) \) and \( \max\{t,r\} \in \Omega = (0, 0.5] \). Thus \( x * y \in \mathcal{A}_Q^\lambda(\max\{t,r\}) \) since \( \mathcal{A}_Q^\lambda(\max\{t,r\}) \) is a subalgebra of \( X \) by the assumption. Hence \( (x * y)_{\max\{t,r\}} q \lambda \). Also, \( \lambda(x * y) > 1 - \max\{t,r\} \geq 0.5 \geq \max\{t,r\} \). Thus \( (x * y)_{\max\{t,r\}} \in \lambda \), and (3.9) is valid.
Corollary 3.13. If $(X, A_Q^λ)$ is a semidetached $B$-algebra over $Ω = (0, 0.5]$, then $λ$ satisfies:

$$x_t q λ, \ y_r q λ \Rightarrow (x * y)_{\max\{t, r\}} \in \land q λ$$

(3.10)

for all $x, y \in X$ and $t, r \in Ω$.

Theorem 3.14. If $λ$ is a right $(q, ∈\lor q)$-fuzzy $B$-algebra of $X$, then $(X, A_Q^λ)$ is a semidetached $B$-algebra over $Ω = (0.5, 1]$

Proof. Let $x, y \in A_Q^λ(t)$ for $t \in (0.5, 1]$. Then $x_t q λ$ and $y_r q λ$. Since $λ$ is a right $(q, ∈\lor q)$-fuzzy $B$-algebra of $X$, we have $(x * y)_t ∈ \lor q λ$, that is, $(x * y)_t ∈ λ$ or $(x * y)_t q λ$. If $(x * y)_t ∈ λ$, then $λ(x * y) ≥ t > 0.5 > 1 − t$ and so $λ(x * y) + t > 1$, i.e., $(x * y)_t q λ$. Hence $x * y \in A_Q^λ(t)$. If $(x * y)_t q λ$, then $x * y \in A_Q^λ(t)$. Therefore $A_Q^λ(t)$ is a subalgebra of $X$ for all $t \in (0.5, 1]$, and consequently $(X, A_Q^λ)$ is a semidetached $B$-algebra over $Ω = (0.5, 1]$. □

Corollary 3.15. Every right $(q, ∈\lor q)$-fuzzy $B$-algebra $λ$ of $X$ satisfies the condition (3.8).

Proof. It is by Proposition 3.11 and Theorem 3.14. □

Theorem 3.16. For a subalgebra $S$ of $X$, let $λ$ be a fuzzy set in $X$ such that

(1) $λ(x) ≥ 0.5$ for all $x ∈ S$,

(2) $λ(x) = 0$ for all $x ∈ X \setminus S$.

Then $λ$ is a left $(q, ∈\lor q)$-fuzzy $B$-algebra of $X$.

Proof. Let $x, y \in X$ and $t, r ∈ (0, 0.5]$ be such that $x_t q λ$ and $y_r q λ$. Then $λ(x) + t > 1$ and $λ(y) + r > 1$, which imply that $λ(x) > 1 − t ≥ 0.5$ and $λ(y) > 1 − r ≥ 0.5$. Hence $x ∈ S$ and $y ∈ S$. Since $S$ is a subalgebra of $X$, we get $x * y ∈ S$ and so $λ(x * y) ≥ 0.5 ≥ \max\{t, r\}$. Thus $(x * y)_{\max\{t, r\}} ∈ λ$, and so $(x * y)_{\max\{t, r\}} \in \lor q λ$. Therefore $λ$ is a left $(q, ∈\lor q)$-fuzzy $B$-algebra of $X$. □

Theorem 3.17. If a fuzzy set $λ$ in $X$ satisfies the condition

$$x_t ∈ λ, \ y_r ∈ λ \Rightarrow (x * y)_{\min\{t, r\}} q λ$$

(3.11)

for all $x, y ∈ X$ and $t, r ∈ Ω = (0, 0.5]$, then $(X, A_Q^λ)$ is a semidetached $B$-algebra over $Ω = (0, 0.5]$.

Proof. Let $x, y ∈ A_Q^λ(t)$ for $t ∈ Ω = (0, 0.5]$. Then $x_t q λ$ and $y_r q λ$, that is, $λ(x) + t > 1$ and $λ(y) + r > 1$. Since $t ≤ 0.5$, it follows that $λ(x) > 1 − t ≥ t$ and $λ(y) > 1 − t ≥ t$. Thus $x_t ∈ λ$ and $y_r ∈ λ$, which imply from (3.11) that $(x * y)_t q λ$. Hence $x * y ∈ A_Q^λ(t)$ and so $A_Q^λ(t)$ is a subalgebra of $X$ for all $t ∈ Ω = (0, 0.5]$. Therefore $(X, A_Q^λ)$ is a semidetached $B$-algebra over $Ω = (0, 0.5]$. □
Corollary 3.18. If a fuzzy set $\lambda$ in $X$ satisfies the condition (3.11), then it satisfies the condition (3.9).

Proof. It is by Proposition 3.12 and Theorem 3.17.

Proposition 3.19. If $(X, A^\lambda_5)$ is a semidetached $B$-algebra over $\Omega = (0, 1]$, then $\lambda$ satisfies:

$$x_t q \lambda, y_r q \lambda \Rightarrow (x * y)_{\max\{t, r\}} \in \lor q \lambda \quad (3.12)$$

for all $x, y \in X$ and $t, r \in \Omega$.

Proof. Let $x, y \in X$ and $t, r \in \Omega = (0, 1]$ be such that $x_t q \lambda$ and $y_r q \lambda$. Then $x \in A^\lambda_5(t) \subseteq A^\lambda_5(t)$ and $y \in A^\lambda_5(r) \subseteq A^\lambda_5(r)$. It follows that $x, y \in A^\lambda_5(\max\{t, r\})$ and so from the hypothesis that $x * y \in A^\lambda_5(\max\{t, r\})$. Hence $(x * y)_{\max\{t, r\}} \in \lor q \lambda$, and consequently (3.12) is valid.

Theorem 3.20. If $\lambda$ is an $(\in, \lor q)$-fuzzy $B$-algebra of $X$, then $(X, A^\lambda_5)$ is a semidetached $B$-algebra over $\Omega = (0.5, 1]$.

Proof. Let $x, y \in A^\lambda_5(t)$ for $t \in \Omega = (0.5, 1]$. Then $x_t q \lambda$ and $y_t q \lambda$, that is, $\lambda(x) + t > 1$ and $\lambda(y) + t > 1$. It follows that $\lambda(x * y) + t \geq \min\{\lambda(x), \lambda(y), 0.5\} + t = \min\{\lambda(x) + t, \lambda(y) + t, 0.5 + t\} > 1$ by Lemma 3.2. Hence $(x * y)_t q \lambda$, and so $x * y \in A^\lambda_5(t)$. Therefore $A^\lambda_5(t)$ is a subalgebra of $X$ for all $t \in (0.5, 1]$, that is, $(X, A^\lambda_5)$ is a semidetached $B$-algebra over $\Omega = (0.5, 1]$.

Corollary 3.21. Every $(\in, \lor q)$-fuzzy $B$-algebra $\lambda$ of $X$ satisfies the condition (3.8).

Proof. It is by Proposition 3.11 and Theorem 3.20.

Theorem 3.22. If $(X, A^\lambda_5)$ is a semidetached $B$-algebra over $\Omega = (0, 1]$, then $\lambda$ is an $(\in, \lor q)$-fuzzy $B$-algebra of $X$.

Proof. For a semidetached $B$-algebra $(X, A^\lambda_5)$ over $\Omega = (0, 1]$, assume that there exists $a, b \in X$ such that $\lambda(a * b) < \min\{\lambda(a), \lambda(b), 0.5\} = t_0$. Then $t_0 \in (0, 0.5]$ and $a, b \in U(\lambda; t_0) \subseteq A^\lambda_5(t_0)$, which implies that $a * b \in A^\lambda_5(t_0)$. Hence $\lambda(a * b) \geq t_0$ or $\lambda(a * b) + t_0 > 1$. This is a contradiction. Thus $\lambda(x * y) \geq \min\{\lambda(x), \lambda(y), 0.5\}$ for all $x, y \in X$. It follows from Lemma 3.2 that $\lambda$ is an $(\in, \lor q)$-fuzzy $B$-algebra of $X$.

Corollary 3.23. If $(X, A^\lambda_5)$ is a semidetached $B$-algebra over $\Omega = (0, 1]$, then $(X, A^\lambda_5)$ is a semidetached $B$-algebra over $\Omega = (0.5, 1]$.

Theorem 3.24. If $\lambda$ is an $(\in, \lor q)$-fuzzy $B$-algebra of $X$, then $(X, A^\lambda_5)$ is a semidetached $B$-algebra over $\Omega = (0, 0.5]$. 
Proof. Let \( x, y \in \mathcal{A}_E^t(t) \) for \( t \in \Omega = (0, 0.5] \). Then \( x_t \in \vee q \lambda \) and \( y_t \in \vee q \lambda \). Hence we have the following four cases:

(1) \( x_t \in \lambda \) and \( y_t \in \lambda \),

(2) \( x_t \in \lambda \) and \( y_t q \lambda \),

(3) \( x_t q \lambda \) and \( y_t \in \lambda \),

(4) \( x_t q \lambda \) and \( y_t q \lambda \).

The first case implies that \( (x \ast y)_t \in \vee q \lambda \) and so \( x \ast y \in \mathcal{A}_E^t(t) \). For the second case, \( y_t q \lambda \) induces \( \lambda(y) > 1 - t \geq t \), i.e., \( y_t \in \lambda \). Hence \( (x \ast y)_t \in \vee q \lambda \) and so \( x \ast y \in \mathcal{A}_E^t(t) \). Similarly, the third case implies \( x \ast y \in \mathcal{A}_E^t(t) \). The last case induces \( \lambda(x) > 1 - t \geq t \) and \( \lambda(y) > 1 - t \geq t \), that is, \( x_t \in \lambda \) and \( y_t \in \lambda \). It follows that \( (x \ast y)_t \in \vee q \lambda \) and so \( x \ast y \in \mathcal{A}_E^t(t) \). Therefore \( \mathcal{A}_E^t(t) \) is a subalgebra of \( X \) for all \( t \in (0, 0.5] \). Hence \( (X, \mathcal{A}_E^t) \) is a semidetached \( B \)-algebra over \( \Omega = (0, 0.5] \).

Corollary 3.25. If \( \lambda \) is a right \((q, \in \vee q)\)-fuzzy \( B \)-algebra of \( X \), then \( (X, \mathcal{A}_E^t) \) is a semidetached \( B \)-algebra over \( \Omega = (0, 0.5] \).

Proof. It is by Theorems 3.8 and 3.24.

Theorem 3.26. If \( (X, \mathcal{A}_E^t) \) is a semidetached subalgebra over \( \Omega := (0.5, 1] \), then so is \( (X, \mathcal{A}_Q^t) \).

Proof. Let \( x, y \in \mathcal{A}_E^t(t) \) for \( t \in \Omega = (0.5, 1] \). Then \( x_t q \lambda \) and \( y_t q \lambda \), which imply that \( x_t \in \vee q \lambda \) and \( y_t \in \vee q \lambda \). Hence \( x, y \in \mathcal{A}_E^t(t) \), and so \( x \ast y \in \mathcal{A}_E^t(t) \) since \( \mathcal{A}_E^t(t) \) is a subalgebra of \( X \) for all \( t \in \Omega = (0.5, 1] \). It follows that \( (x \ast y)_t \in \vee q \lambda \), that is \( (x \ast y)_t \in \lambda \) or \( (x \ast y)_t q \lambda \). In either case, we have \( (x \ast y)_t q \lambda \) since \( \Omega = (0.5, 1] \). Hence \( x \ast y \in \mathcal{A}_E^t(t) \) for all \( t \in \Omega = (0.5, 1] \), and therefore \( (X, \mathcal{A}_E^t) \) is a semidetached subalgebra over \( \Omega := (0.5, 1] \).

Theorem 3.27. If \( \lambda \) is a right \((q, \in \vee q)\)-fuzzy \( B \)-algebra of \( X \), then \( (X, \mathcal{A}_E^t) \) is a semidetached \( B \)-algebra over \( \Omega = (0.5, 1] \).

Proof. Let \( x, y \in \mathcal{A}_E^t(t) \) for \( t \in \Omega = (0.5, 1] \). Then \( x_t \in \vee q \lambda \) and \( y_t \in \vee q \lambda \). Hence we have the following four cases:

(1) \( x_t \in \lambda \) and \( y_t \in \lambda \),

(2) \( x_t \in \lambda \) and \( y_t q \lambda \),

(3) \( x_t q \lambda \) and \( y_t \in \lambda \),

(4) \( x_t q \lambda \) and \( y_t q \lambda \).
For the first case, we have $\lambda(x) + t \geq 2t > 1$ and $\lambda(y) + t \geq 2t > 1$, that is, $x, q \lambda$ and $y, q \lambda$. Hence $(x \ast y)_t \in \bigvee q \lambda$, and so $x \ast y \in A^\lambda(t)$. In the case (2), $x, t \in \lambda$ implies $\lambda(x) + t \geq 2t > 1$, i.e., $x, q \lambda$. Hence $(x \ast y)_t \in \bigvee q \lambda$, and so $x \ast y \in A^\lambda(t)$. Similarly, the third case implies $x \ast y \in A^\lambda(t)$. For the last case, we have $(x \ast y)_t \in \bigvee q \lambda$, and so $x \ast y \in A^\lambda(t)$. Consequently, $A^\lambda(t)$ is a subalgebra of $X$ for all $t \in \Omega = (0, 1]$. Therefore $(X, A^\lambda)$ is a semidetached $B$-algebra over $\Omega = (0.5, 1]$. □

By Theorems 3.26 and 3.27, we have the following corollary.

**Corollary 3.28.** If $\lambda$ is a right $(q, \in \vee q)$-fuzzy $B$-algebra of $X$, then $(X, A^\lambda_Q)$ is a semidetached $B$-algebra over $\Omega = (0.5, 1]$. 

**References**


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