An Optimal Strategy for Liquidity Management in Banking

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Abstract

During the global financial crisis, banks were under severe pressure to maintain adequate liquidity. This resulted in demands for cash from various sources, including counterparties, short-term creditors, and, especially, existing borrowers. Also, banks were forced to maintain more liquid assets in the event of unexpected future losses that may occur from securities write-downs. This can cause a detrimental effect on its investment portfolio and lending activities. In response to the liquidity crises the Basel Committee on Banking Supervision (BCBS) designed a set of precautionary measures (known as Basel III) imposed on banks and one of its purposes is to protect the economy from deteriorating. Recently, bank regulators wanted banks to depend on sources such as core deposits and long-term funding from small businesses and less on short-term wholesale funding. This was an attempt to protect banks from being vulnerable to another financial crises. In order to address the aforementioned problem, we investigate the money supply process between a central bank and a commercial bank and how it influences the liquidity of a bank. In particular, we formulate a stochastic control problem involving cash which is held as deposits at a central bank. The solution of this problem is readily obtained from existing literature. Using simulations we investigate the manner in which this interplay affects the liquidity coverage ratio of a bank. Finally, we make a few concluding remarks and discuss possibilities for further research.

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1 Introduction

In situations when a financial firm starts raising voluntary savings and using those deposits to finance an asset portfolio, the liquidity and asset-liability management of that institution becomes more complex. For instance, a commercial bank not only has to manage the fluctuating demand and varying interest rates and terms on loans, but it also has to deal with erratic deposit demands, withdrawals, changing interest rates and terms of savings. The management of liquidity should ensure that the commercial bank maintains sufficient cash and liquid assets to satisfy client demand for loans and savings withdrawals, and to pay the bank’s expenses. Liquidity refers to the ability of an institution to meet immediate financial obligations. Bank assets should be readily and easily convertible into cash to finance these demands. Daily analysis and detailed estimation of the size and timing of cash inflows and outflows over the coming days and weeks are performed to minimize the risk that savers will be unable to access their deposits in the moments they demand them. In order for a commercial bank to make liquidity projections and realistic growth, certain information is needed such as the history of deposit and loan inflows and outflows and overall daily cash demands to determine the amount of cash that needs to be kept on-site and in demand deposit type accounts. A liquidity shortage can cause great damage to a bank. In particular, a liquidity crisis can result in client confidence to deteriorate over time. The 2007–09 financial crisis caused the banking system in the United States to collapse for the first time, raising fundamental questions about liquidity risk. The global financial system experienced urgent demands for cash from various sources, including counterparties, short-term creditors, and especially, existing borrowers. Central bank emergency lending programs probably mitigated the decline. During the financial crisis, banks that were more exposed to liquidity risk increased their holdings of liquid assets. Liquidity exposure affected behavior along several dimensions. On the asset side, banks holding securities with low liquidity, such as mortgage-backed securities, expanded their cash buffers during the crisis and decreased new lending. Such banks were worried about their ability to finance securitized assets. They protected themselves by hoarding liquidity, to the detriment of borrowers. On the liability side, banks that relied more on wholesale sources of funding, cut new lending significantly more than banks that relied predominantly on traditional deposits and equity capital for funding.

In order to provide some sort of relief for internationally active banks, in
December, 2010 the Basel Committee on Banking Supervision (BCBS) issued the Basel III: International framework for liquidity risk measurement, standards, and monitoring. Although Basel II regulation established procedures for assessing credit, market, and operational risk, it did not provide effective protocols for managing liquidity and systemic risks.

Current liquidity risk management procedures can be classified as micro- or macro-prudential. In the case of the former, simple liquidity ratios such as credit-to-deposit ratios (net stable funding ratio), liquidity coverage ratios and the assessment of the gap between short-term liabilities and assets are appropriate to cover the objectives of bank balance sheet analysis. The ratio approach for liquidity risk management is a quantitative internationally accepted standard for alerting banks about any possible adverse economic downturns. In this paper, we mainly focus on the liquidity coverage ratio, which is designed to ensure that commercial banks have the necessary assets on hand to ride out short-term liquidity disruptions. Banks are required to hold an amount of highly-liquid assets, such as cash or Treasury bonds, equal to or greater than their net cash over a 30 day period (having, at least, 100% coverage). The liquidity coverage ratio (LCR) started to be regulated and measured in 2011, but the full 100% minimum would not be enforced until 2015. The LCR is defined as

\[
\text{LCR} = \frac{\text{Total Stock of High-Quality Liquid Assets (HQLAs)}}{\text{Total Nett Cash Outflows (TNCOF) over the next 30 calendar days}}
\]

and it is required by BCBS that \( \text{LCR} \geq 1 \). The HQLAs in (1.1) refers to the stock of unencumbered (not pledged) high quality liquid assets banks must hold to cover the total net cash outflows over a 30-day period ([1]; [2]; [3]; [4] and [5]). The implementation is intended to favor those assets that are counted as liquid, and at the same time reduce incentives to hold assets that are considered less liquid. The Committee has liberalized the definition of what counts as a liquid asset in their liquidity framework. The high-quality liquid assets can be divided into two categories, namely; Level 1 assets and Level 2 assets. Level 1 assets (L1As) includes cash, central bank reserves, and Government bonds with 0% risk weight under Basel II. Level 2 assets (L2As) mainly comprises government bonds with a 20% risk weight under Basel II and at least AA-rated corporate bonds (issued by a non-bank), and covered bonds which have a proven track record as a reliable source of liquidity (repo or sale) in the capital market. L2As is further categorized into Level 2A assets (L2AAs) and Level 2B assets (L2BAs). L2AAs are subjected to a 15% haircut while L2BAs are subjected to a 50% haircut. The latter assets include corporate debt securities, unencumbered equities, and residential mortgage-backed securities. L2As are limited to a maximum of 40% of the overall liquid asset pool when
computing the LCR. In other words, the quantity of L2As included in the calculation of HQLA can be at most \( \frac{2}{3} \) of the quantity of L1As. In addition, a 15\% haircut is applied to the current market value of each Level 2A asset held in the stock of HQLA. (see [4]).

The reason why there are so many haircuts on L2As is to ensure that the majority of a banking organizations’ HQLAs consist of Level 1 assets. The amount of Level 1 assets thus acts as a constraint on the recognition of Level 2 assets as HQLAs. Based on the description for HQLAs above, a bank’s stock of HQLAs can then be written as

\[
\text{HQLAs} = L1A + \min \left( 0.85 \times L2A, \frac{2}{3} \times L1A \right).
\]  

(1.1)

In the denominator of the LCR are the TNCOF. TNCOF over a 30-day time period are determined by the total expected cash outflows minus total expected cash inflows and reflect the net amount of funding that may not be realized within the 30 days under a stress scenario. Total expected cash outflows are calculated by multiplying the outstanding balances of various types of liabilities and off-balance sheet commitments by rates at which they are expected to run off or be drawn down (i.e. the amount of funding maturing in the 30-day period that will not be rolled over). Total expected cash inflows are calculated by multiplying the outstanding balances of various categories of contractual receivables by the rates at which they are expected to flow in under the scenario up to an aggregate cap of 75\% of total expected cash outflows. The aforementioned haircut prevents bank’s from relying solely on these inflows for its liquidity. Thus, it ensures that a bank holds a minimum stock of HQLAs equal to 25\% of cash outflows. Symbolically, this means

\[
\text{Total Expected Cash Inflows} \leq 0.75 \times \text{Total Expected Cash Outflows}.
\]

The NCOF can be calculated as

\[
\text{NCOF} = \text{Outflows} - \min \left( \text{Inflows}, 0.75 \times \text{Outflows} \right).
\]  

(1.2)

The formulas for HQLAs and NCOF in (1.1) and (1.2), respectively were obtained from [22].

Our contribution has connections with [9]; [10]; [12]; [14]; [18] and [20].

In the paper [9], the authors explore the relationships between Shareholder Cash Flow Rights (SCFRs), capital stability and liquidity via the Net Stable Funding Ratio (NSFR) and LCR, respectively. In particular, they investigate the effects of shareholder cash flow rights on the aforementioned funding ratio and a non-Basel III liquidity coverage ratio for certain developing countries during the period 2005 Q1 to 2009 Q4. The working paper [10] examines large
capital injections by U.S. financial institutions from 2000 to 2009. These infusions include private as well as government cash injections under the Troubled Asset Relief Program (TARP). The sample period covers both business cycle expansions and contractions, and the recent financial crisis. Elyasiani et. al. [10] show that more financially constrained institutions were more likely to have raised capital through private-market offerings during the period prior to TARP, and firms receiving a TARP injection tended to be riskier and more levered. In the case of TARP recipients, they appeared to finance an increase in lending (as a share of assets) with more stable financing sources such as core deposits, which lowered their liquidity risk. However, in [10] no evidence is found that bank’s capital adequacy increased after the capital injections.

In the paper [12] the authors discuss liquidity risk management for banks. In particular, their analysis under the Basel III paradigm suggests that overall liquidity risk is best measured using ratio analysis approaches such as the LCR. Their proposition is justified by numerical results which show that bank behavior related to liquidity was highly procyclical during the global financial crisis. The paper [14] provides a comprehensive analysis to calculate the Basel III liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR) of United States commercial banks. Part of their analysis include Call Report data over the period 2001 – 2011. Their finding suggests that systematic liquidity risk was a major contributor to bank failures in 2009 and 2010.

In [18], actuarial methods are used to solve a nonlinear stochastic optimal liquidity risk management problem for subprime originators with deposit inflow rates and marketable securities allocation as controls. The main objective is to minimize liquidity risk in the form of funding and credit crunch risk in an incomplete market. In order to accomplish this, they construct a stochastic model that incorporates originator mortgage and deposit reference processes. Gideon et al. [20] investigate the Net Stable Funding Ratio, which is one of the liquidity measures, under the Basel III framework. By considering the inverse net stable funding ratio as a measure to quantify the bank’s prospects for a stable funding over a one year period, the authors consider an optimal liquidity problem related to the inverse net stable funding ratio whereby optimal choices are made for the inverse net stable funding targets in order to formulate its cost. The latter was achieved by finding an analytical solution for the value function.

In [11] the authors investigate portfolio optimization problems that consist of maximizing expected terminal wealth under the constraint of an upper bound for the risk. The paper [17] considers maximizing the expected utility from consumption or terminal wealth in a market where logarithmic securities prices follow a Lévy process. The aforementioned authors derive explicit solutions for different utility functions. As a consequence of this approach, the intrinsic risk of the bank arises now not only from the reserve portfolio but also from
the deposit withdrawals.

In our contribution issues related to bank liquidity management are addressed in a jump diffusion setting. Our paper has some close connections with the aforementioned literature in the sense of establishing optimal liquidity and a rate of depository consumption that is of importance during a (random) auditing process of the reserve requirements. In particular, we investigate the interplay between a commercial bank and a central bank and how this affects the money supply between the two institutions as well as the LCR. The main motivation for studying the dynamics of LCR is to show that, in principle, banks are able to control their liquidity via an appropriate provisioning strategy. This should ensure that the said ratio does not move below an acceptable level.

The layout of the rest of our paper is as follows. In Section 2, we extend some of the modeling and optimization issues highlighted in [6] by presenting jump diffusion models for various assets (see also [11] and [17] where asset prices are generally modeled as Lévy processes). The stochastic dynamics of high-quality liquid assets and bank liabilities are presented in Section 2.1 and 2.2, respectively. In Section 2.3 we derive the dynamics of liquid assets and net cash flows. In Section 2.4, we formulate a deposit withdrawal problem and solve it via a stochastic control technique which can be found in [21]. In Section 3 we use the results obtained in Section 2.4 to generate numerical simulations of the Liquidity Coverage Ratio. In addition to the solution of the problem outlined above, we offer a few concluding remarks in Section 4.

2 The stochastic banking model

To understand the operation and management of banks, we have to study its balance sheet, the items of which are unpredictable and uncertain due to activities related to the evolution of treasuries, loan demand, risky and riskless investments, deposits, loan repayments, borrowings and bank regulatory capital.

Let \((\Omega, \mathcal{F}, \mathbb{P})\) be a complete probability space with a filtration \(\mathcal{F} := \{\mathcal{F}_t, 0 \leq t \leq T\}\) which is right continuous, for some finite \(T\) which denotes the investment time horizon. The space \(\Omega\) represents the different states of the economy in which banks operate. The information available to banks up until time \(t\) is represented by \(\mathcal{F}_t\). All random variables considered in this paper are defined on this space. Furthermore, we consider a commercial bank that enters a financial market which is subject to uncertainty, through the components of a
4-dimensional Brownian motion

\[ Z(t) \equiv (Z_0(t, \omega), Z_a(t, \omega), Z_b(t, \omega), Z_c(t, \omega))', \ t \geq 0, \ \omega \in \Omega, \]

with pairwise independent coordinates, and a 1-dimensional Poisson process

\[ N(t) = N(t, z), \ t \geq 0, \ z \geq 0. \]

For some constant \(-1 < \rho < 1\), we define \( Z_1(t) \) and \( Z_2(t) \) as

\[ Z_1(t) = \rho Z_0(t) + \sqrt{1 - \rho^2} Z_a(t) \quad \text{and} \quad Z_2(t) = \rho Z_0(t) + \sqrt{1 - \rho^2} Z_b(t), \]

respectively. The processes \( \{Z_1(t), 0 \leq t \leq T\} \) and \( \{Z_2(t), 0 \leq t \leq T\} \) are correlated. Note that \( dZ_1(t)dZ_2(t) = \rho^2 dt \).

The martingale \( \tilde{N}(t) = N(t) - \lambda t \) is called the compensated Poisson process of \( N(t) \), where \( \lambda \) is the vector of intensities of \( N(t) \). We point out that \( \lambda \) can be time-dependent. In this paper we only consider homogeneous Poisson processes, that is, with \( \lambda \) being constant over time.

Bank capital plays an important role because it balances assets and liabilities by the relation

\[
\text{Total Assets} = \text{Total Liabilities} + \text{Bank Capital}. \quad (2.3)
\]

A typical commercial bank’s asset portfolio at time \( t \) can be decomposed into many assets. A useful way of representing the balance sheet of the bank is as follows:

\[ T_0(t) + R(t) + S(t) + L(t) = D^S(t) + D^L(t) + F^U(t) + B^I(t) + E(t) \]

where we have \( T_0, R, S, L, D^S, D^L, F^U, B^I \) and \( E \) represent the market value of cash, reserves, marketable securities, loans, stable retail deposits, less stable retail deposits, unsecured wholesale funding, interbank borrowing and equities, respectively.

### 2.1 Bank Assets

Assets are considered to be high-quality liquid assets (HQLAs) if they can be easily and immediately converted into cash at little or no loss of value (for instance coins, bank notes, reserves, marketable securities and sovereign or central bank debt securities). There are two categories of assets that can be included in the stock. Assets to be included in each category are those that the bank is holding on the first day of a stress period, irrespective of their residual maturity. Level 1 assets (L1As) can be included without limit, while Level 2 assets (L2As) can only comprise up to 40% of the stock.
2.1.1 Level 1 Assets

The first component of stock of high-quality liquid assets is cash that is made up of banknotes and coins. Central Bank (CB) reserves should be able to be drawn down in times of stress ([4]). In this regard, local supervisors should discuss and agree with the relevant CB the extent to which CB reserves should count toward the stock of liquid assets.

The dynamics of the riskless asset (Cash) is given by

\[ dT_0(t) = T_0(t)r^A dt, \quad T_0(0) = T_0 > 0, \]  

\[(2.4)\]

where \( r^A > 0 \) represents the constant riskless interest rate.

Bank reserves are the deposits held in accounts with the central bank of a country (for instance, the South African Reserve Bank in the case of South Africa) plus money that is physically held by banks (vault cash). Such reserves constitute money that is not lent out but is earmarked to cater for withdrawals by depositors. Since it is uncommon for depositors to withdraw all of their funds simultaneously, only a portion of total deposits may be needed as reserves. It is the amount of money a bank sets aside and does not lend, to meet day-to-day currency withdrawals by its customers. We note that cash in the Automated Teller Machines (ATMs) network also qualifies as required reserves. The investment of bank reserves in the market via bonds and stocks is still possible in many countries. Bank reserves may actually have a stochastic nature and banks may earn a positive return on them. For instance, as reported in [19] by the end of 2013, China had over \$3.82 trillion in foreign exchange reserves. A significant proportion of these reserves was invested in U.S. treasury securities since treasuries are seen as a safe investment. Based on the above description, we may represent bank reserves as (see [6])

\[ dR(t) = \left\{ \begin{array}{l}
    [r^{R_1}(t) - f^{R_1}(t)]dt + \sigma dZ_c(t) \\
    R(t^-)\left\{ \int_{-\infty}^{\infty} x(t^-, z) \tilde{N}(dt, dz) \right\} - c(t)dt,
\end{array} \right. \]  

\[(2.5)\]

where \( r^{R_1}(t) > 0 \) is the rate of (positive) return on reserves earned by the bank, \( f^{R_1}(t) > 0 \) is the fraction of bank reserves that are available for withdrawal, and \( \sigma > 0 \) is the volatility in the level of reserves. The size of the jumps is denoted by \( x(t^-, z) \) and we assume that \( x(t^-, z) \geq -1 \), ensuring that the jumps are not too large negative. Also, in order to make provisions for deposit withdrawals, \( c(t) \), it is required of a bank to make rational decisions about the deposit taking (see [6]).
2.1.2 Level 2 Assets

In this section, the L2As consists mainly of marketable securities. Marketable securities are debt instruments that can be easily converted to cash such as government bonds, corporate bonds, common stock or certificates of deposit. These instruments are very liquid as they tend to have maturities of less than one year.

The dynamics of the marketable security price (see [7]) is assumed to be given by

\[ \frac{dS(t)}{S(t)} = (r_S(t) + \lambda_S)dt + \sigma_S dZ_1(t), \]  

(2.6)

where \( \sigma_S \) is the security volatility and \( \lambda_S \) denotes the risk premium. Under the Capital Asset Pricing Model (CAPM), \( \lambda_S \) could be quantified by the relation \( \lambda_S = \beta[E[R_m] - r_A] \) with \( E(R_m) \) representing the market expected return and \( \beta \) the sensitivity of the expected excess asset returns to the expected excess market return. The dynamics of the interest rate \( r_S(t) \) is given by

\[ dr_S(t) = \theta_{rs}(\mu_{rs} - r_S(t)))dt + \sigma_{rs} dZ_2(t), \]  

(2.7)

where \( \theta_{rs} \) is the rate of reversion, \( \mu_{rs} \) is the long-run mean and \( \sigma_{rs} \) is the volatility which are all positive constants.

2.2 Liabilities

2.2.1 Deposits

A commercial bank creates credit or makes loans, and holds reserves (to satisfy demands for withdrawals) that are less than a number of its customers’ deposits. In general, funds that are deposited at a bank, are mostly lent out to customers or financial institutions. Commercial banks keep a fraction (known as a reserve-deposit ratio) of those funds as reserves to cover its customer deposit liabilities. Central banks or other banking regulators often mandate the aforementioned reserve requirements in order to limit the amount of money creation that occurs in the commercial banking system and to ensure that banks have enough ready cash to meet normal demand for withdrawals. In our research, the term deposits include both demand and time deposits. The value of a deposit is ultimately dependent on its stability or likelihood of not being withdrawn. The longer time it will remain in the bank the longer time the bank can lend out that money and, as a long loan is worth more than a short one, charge a higher price for it. Thus, deposits have uncertainty associated with them and thus can be modeled as a stochastic process. Let \( r^{DD} : T \rightarrow \mathbb{R}_+ \) denote the rate of demand deposit which is payable on demand.
and \( r^{TD} : T \rightarrow \mathbb{R}_+ \) the rate of time deposit which is payable only after a fixed interval of time. Retail deposits are categorized into two types of deposits namely, stable and less stable deposits. Stable deposits are deposits which are fully insured by an effective deposit insurance scheme (i.e. up to the maximum coverage limit the deposit insurance scheme) or by a public guarantee. The aforementioned deposits needs to satisfy certain criteria. These requirements are as follows (see [1] or [4]):

- Depositors have one or more established relationships with the banking institution which has existed for at least 12 months; or

- Deposits are in transactional accounts, which refer to accounts which are regularly credited or debited (e.g. accounts where salaries are automatically deposited). This refers to a depositor who is contractually bounded to a bank institution for at least the next 12 months.

Retail deposits that does not satisfy the criteria outlined above are referred to as less stable deposits. The dynamics of deposits can be written as a diffusion process (see, for instance, [8]; [13]; [15] and [16]). In our article we model stable deposits as

\[
dD^S(t) = m_s dt + \sigma_s dZ_c(t), \quad D^S(0) = d_s \in \mathbb{R}_+, \tag{2.8}
\]

where \( m_s = r^{DD} + r^{TD} \) and less stable deposits as

\[
dD^L(t) = m_l dt + \sigma_l dZ_c(t) + \alpha \int_{-1}^{\infty} x(t^-, z) \tilde{N}(dt, dz), \quad D^L(0) = d_l \in \mathbb{R}_+, \tag{2.9}
\]

where \( 0 \leq \alpha \leq 1 \) is a constant. The terms \( m_s \) and \( m_l \) represents the expected rate of growth in deposits, \( \sigma_s \) and \( \sigma_l \) the level of volatility in deposits and the jump term captures the unexpected information (such as bank solvency) that will have an influence in deposit taking and deposit rates.

### 2.2.2 Unsecured wholesale funding

Banks, as well as commercial finance companies, can both be users of wholesale funding. Since both institutions are regulated differently, their daily operational activities are very distinct. Commercial finance companies solely provide business loans whereas banks provide both business and consumer loans. Wholesale funding refers to a method used by banks in addition to core demand deposits in order to finance operations and managing risk. It includes federal funds, foreign deposits and brokered deposits. Historically, banks used core demand deposits as a source of funding since they are an inexpensive source of financing. Recently, banks have turned to wholesale funding as a
way of expanding funding needs. One of the reasons why banks use whole-
sale funding is to attract new deposits, especially from brokered deposits. In
this paper, we model wholesale funding as follows (see [23] where liability is
modelled in a similar fashion):

\[ dF^U(t) = m_f dt + \sigma_f dZ_c(t), \quad F^U(0) = d_f \in \mathbb{R}_+, \tag{2.10} \]

where \( m_f \) represents the rate at which deposits are received through a broker
who takes their wealthy clients’ money and finds several different banks in
which to deposit it (known as brokered deposits). The parameter \( \sigma_f \) is the
level of volatility in these deposits.

### 2.2.3 Borrowing

Banks usually borrow money from each other in the interbank market. There
is an interest rate charged on short-term loans made between banks. It is
known as the interbank rate. Banks borrow and lend money in the interbank
market in order to manage liquidity and meet the requirements placed on them.
Thus, it is required of banks to hold an adequate amount of assets to manage
potential withdrawal. If a bank is unable to satisfy this liquidity requirement,
it will need to borrow money in the interbank market to cover the shortfall.
Banks have to hold a percentage of their deposits with the central bank every
night. In the event that a bank is short on cash at a given time, it needs to
borrow funds from the central bank at a certain rate. This rate is known as
the federal funds rate. Models of borrowing funds can be found in [18]. Here
we model borrowing funds as

\[ \frac{dB^I(t)}{B^I(t)} = m_B dt + \sigma_B dZ_c(t), \quad B^I(0) = b_I \in \mathbb{R}_+, \tag{2.11} \]

where \( m_B \in \mathbb{R} \) is the interbank rate and \( \sigma_B > 0 \) is the volatility in the funds
being borrowed.

### 2.3 Dynamics of Liquid assets and Net Cash flows

In this section we mainly focus on deriving the dynamics of L1As, L2As, the
bank’s total expected cash inflows and total expected cash inflows, respectively.
The reason for this is to numerically simulate HQLAs and NCOF given by (1.1)
and (1.2), in order to characterize the behavior of LCR.
The evolution of the $L_1$As, $A_{L_1}$, and $L_2$As, $A_{L_2}$, are given by

$$
\frac{dA_{L_1}(t)}{} = \frac{dT_0(t)}{T_0(t)} + dR(t)
= R(t^-) \left\{ [r^{R_1}(t) - f^{R_1}(t)] dt + \sigma dZ_c(t) \right\} + \int_{-1}^{\infty} x(t^-, z) \tilde{N}(dt, dz) + \left( r^A - c(t) \right) dt,
$$

(2.12)

and

$$
\frac{dA_{L_2}(t)}{} = 0.85 \times \frac{dS(t)}{S(t)} = 0.85 \times \left( (r_S(t) + \lambda_S) dt + \sigma_S dZ_1(t) \right).
$$

(2.13)

In our case, we assume that the bank’s total expected cash inflows, $Y(t)$, comprises of maturing secured lending backed by Level 1, $l_{M_1}(t)$, and Level 2, $l_{M_2}(t)$, assets as collateral, with the dynamics of the maturing secured lending backed Level 1 and Level 2 assets being described respectively by the equations ($i = 1, 2$)

$$
\frac{dl_{M_i}(t)}{dl_{M_i}(t)} = \mu_{M_i} dt + \sigma_{M_i} dZ_i(t).
$$

(2.14)

In this case the dynamics of the total expected cash inflows is expressed as

$$
dY(t) = 0 \times \frac{dl_{M_1}(t)}{dl_{M_1}(t)} + 0.15 \times \frac{dl_{M_2}(t)}{dl_{M_2}(t)}
= 0.15 \mu_{M_2} dt + 0.15 \rho \sigma_{M_2} dZ_0(t) + 0.15 \sigma_{M_2} \sqrt{1 - \rho^2} dZ_b(t)
$$

(2.15)

while the dynamics of the total expected cash outflows, $\Lambda(t)$, is given by

$$
d\Lambda(t) = \theta_1 dS^t(t) + \theta_2 dS^L(t) + \theta_3 dF^U(t) + \theta_4 dB^I(t)
= \left( \theta_1 m_s + \theta_2 m_l + \theta_3 m_f + \theta_4 m_B B^I(t) \right) dt
+ \left( \theta_1 \sigma_s + \theta_2 \sigma_l + \theta_3 \sigma_f + \theta_4 \sigma_B B^I(t) \right) dZ_c(t)
+ \theta_2 \alpha \int_{-1}^{\infty} x(t^-, z) \tilde{N}(dt, dz).
$$

(2.16)

In (2.16) $\theta_1$, $\theta_2$, $\theta_3$ and $\theta_4$ are the run-off rate for stable retail deposits, less stable retail deposits, unsecured wholesale funding and overnight interbank borrowing taken to be 7.5%, 15%, 75% and 100%, respectively (see [1]).
2.4 The deposit withdrawal problem

In our article, we suppose that the bank’s behavior towards risk is described by a logarithmic or power utility, respectively. The bank is given full access to withdraw a certain amount of money from the cash vault to satisfy daily demands by depositors. Here, our goal is to minimize deposit risk (a type of liquidity risk) arising from cash outflows of a commercial bank which is caused by changes in depositors’ behavior. In order to address the aforementioned problem, we define the performance criterion as follows:

\[
J(s, \tau, R; c) = \mathbb{E} \left[ \int_0^\tau \exp(-\delta(s + t))U(c(t)) \, dt \right],
\]

where \(U(c(t))\) is a utility function with \(\delta > 0\). The time \(\tau = \inf\{t > 0; R(t) \leq 0\}\) represents the time until bankruptcy occurs. The class \(\mathcal{A}\) of admissible control laws is defined as follows:

\[
\mathcal{A} = \left\{ c(\cdot) : c \text{ is bounded and adapted so that } R(t) > 0 \text{ for all } t \geq 0 \text{ a.s.} \right\}.
\]

We define the value function as follows

\[
V(s, R) = \sup_{c(\cdot) \in \mathcal{A}} J(s, \tau, R; c).
\]

Problem 2.1. (Optimal deposit withdrawal problem): Consider the SDE of the form (2.5). We attempt to find the supremum

\[
V(s, R) = \sup_{c(\cdot) \in \mathcal{A}} J(s, \tau, R; c)
\]

and the optimal control law, which is given by

\[
c^*(t) = \arg \sup_{c \in \mathcal{A}} J(s, \tau, R; c) \in \mathcal{A}, \text{ so that } V(s, R) = J(s, \tau, R; c^*).\]

The optimal rate of deposit flow, \(c^*(t)\), will be determined in Proposition (2.1) and Proposition (2.3) (see [6] for the infinite horizon case of a bank auditing problem). The role of \(R(t)\) should be that it is readily available to satisfy customer withdrawals or transfer to other banks as customers write checks.

It is well understood that this withdrawal should not exceed the cash reserve ratio. The required reserve ratio is sometimes used as a tool in monetary policy, influencing the country’s borrowing and interest rates by changing the amount of funds available for banks to make loans with. Commercial banks
rarely alter the reserve requirements because it would cause immediate liquidity problems for banks with low excess reserves; they generally prefer to use open market operations (buying and selling government-issued bonds) to implement their monetary policy.

We solve the optimization problem by way of the following two results. We follow an optimization method as discussed in [21]. In fact, the proof of the following Proposition is like ([21], Exercises 3.6)

**Proposition 2.1.** Suppose that the dynamics of the bank vault cash holdings is described as (2.5) and the value function is characterized by (2.19) where \( U(c(t)) = \ln(c(t)) \). Then the rate of currency outflow from the vault cash holdings is given by

\[
c^*(R(t)) = \hat{c} = \frac{1}{a} R(t).
\]  

(2.20)

**Proof.**

As a candidate for the value function \( V \), let us test a function of the form

\[
\varphi(s, R) = \exp(-\delta s) \chi(R), \quad \text{with} \quad \chi(R) = a \ln R + b \quad \text{for some} \quad a > 0, \quad b > 0
\]

\[
= \exp(-\delta s)(a \ln R + b).
\]

In this case the Hamilton-Jacobi-Bellman equation becomes

\[
\sup_{c(t) > 0} \left\{ \ln c - \delta \chi(R) + \left( R[r^{R_1} - f^{R_1}] - c \right) \chi'(R) + \frac{1}{2}(\sigma)^2 R^2 \chi''(R) \\
+ \int_{-1}^{\infty} \left\{ \chi(R + Rx(t^-, z)) - \chi(R) - Rx(t^-, z) \chi'(R) \right\} \nu(dz) \right\} = 0. \tag{2.21}
\]

In order to maximize the relevant entity above with respect to \( c(t) \), we must have its partial derivative with respect to \( c(t) \) vanishing. This gives us:

\[
\frac{1}{c} - \chi'(R) = 0,
\]

i.e.

\[
c = \frac{1}{\chi'(R)} = \frac{R}{a}. \tag{2.22}
\]

Substituting the expressions for \( c \), \( \chi(R) \) and \( \chi'(R) \) into (2.21) yields, after some simplification:

\[
(1 - \delta a) \ln R - \ln a - \delta b + [r^{R_1} - f^{R_1}] a - 1 - \frac{1}{2} \sigma^2 a \\
+ a \int_{-1}^{\infty} \{ \ln(1 + x(t^-, z)) - x(t^-, z) \} \nu(dz) = 0.
\]
The aforementioned expression is possible only if \( a = \frac{1}{\delta} \). In that case we obtain:

\[
-\ln \frac{1}{\delta} - \delta b + [r^{R_1} - f^{R_1}] \frac{1}{\delta} - 1 - \frac{1}{2}\sigma^2 \cdot \frac{1}{\delta} + \frac{1}{\delta} \int_{-1}^{\infty} \{\ln(1 + x(t^- , z)) - x(t^- , z)\} \nu(dz) = 0,
\]

and the latter yields a solution for \( b \):

\[
b = \frac{1}{\delta^2} \left[ \delta \ln \delta + [r^{R_1} - f^{R_1}] - \delta - \frac{\delta^2}{2} + \int_{-1}^{\infty} \{\ln(1 + x(t^- , z)) - x(t^- , z)\} \nu(dz) \right].
\]

With these values for \( a \) and \( b \), we can conclude that

\[
\varphi(s, R) = \exp(-\delta t)(a \ln R + b) = V(s, R)
\]

and that

\[
c^*(R(t)) = \frac{R(t)}{a}
\]

is the optimal rate of currency flow from the vault cash holdings.

**Remark 2.2.** Similarly as in [21], we can prove the following Proposition, and we omit the proof.

**Proposition 2.3.** Suppose that the dynamics of the bank vault cash holdings is described as (2.5) and the value function is characterized by (2.19) with

\[
U(c(t)) = \frac{c^2(t)}{\gamma} \quad \text{and} \quad \gamma \in (0, 1). \quad (2.24)
\]

Then the rate of currency outflow from the vault cash holdings is given by

\[
c^*(R(t)) = K \gamma - \frac{1}{\gamma - 1} \gamma - \frac{1}{R(t)},
\]

provided that

\[
K = \frac{1}{\gamma} \left[ \frac{1}{1 - \gamma} \left( \delta - \left( [r^{R_1}(t) - f^{R_1}(t)] \right) \gamma - \frac{1}{2}(\sigma)^2 \gamma(\gamma - 1) \right) - \int_{-1}^{\infty} \left\{ (1 + x(t^- , z))^\gamma - 1 - x(t^- , z) \gamma \right\} \nu(dz) \right]^{-1}.
\]
3 Numerical example involving Liquidity Coverage Ratio

In this section, we provide numerical simulations in order to characterize the behavior of the LCR. The behavior of the LCR will be influenced by the expressions of $c^*$ given by (2.20) and (2.24), respectively. These two cases can be referred to as log-utility or logarithmic case and power utility, respectively.

The instantaneous interest rate dynamics of $r^{R_1}(t)$ and $f^{R_1}(t)$ are modelled as mean reversion processes. That is,

$$dr^{R_1}(t) = (\kappa - r^{R_1}(t))dt + \sigma_r\sqrt{r^{R_1}(t)}dZ_c(t)$$  \hspace{1cm} (3.25)

and

$$df^{R_1}(t) = \beta_1(\eta - f^{R_1}(t))dt + \sigma_{f^{R_1}}dZ_c(t),$$  \hspace{1cm} (3.26)

respectively.

Here $\kappa, \beta_1, \eta, \sigma_{f^{R_1}}$ and $\sigma_r$ are all positive constants. In each case, the aforementioned parameters correspond to the degree of mean reversion, long-run mean, and volatility of the interest rate.

In the discussion that follows, the computations are done over $T = 30$ days. We denote HQLAs and NOCF at time $t$ by $H_Q(t)$ and $\Theta(t)$, respectively. The initial values for HQLAs and TNCOF are given by $H_Q(0) = 316.6667$ and $\Theta(0) = 300.2500$, respectively. Thus, $LCR(0) = H_Q(0)/\Theta(0) = 1.0547$. In order to obtain the aforementioned values, we consider the following parameters and initial conditions: $B^f(0) = 80$, $\Upsilon(0) = 6$, $\Lambda(0) = 306.25$, $A_{L_1}(0) = 190$, $A_{L_2}(0) = 150$, $r^{R_1}(0) = 0.06$, $f^{R_1}(0) = 0.02$, $\sigma_f = 0.011$, $\sigma_r = 0.05$, $\kappa = 5$, $\eta = 6$, $\sigma_r = 0.01$, $r^A = 0.03$, $r_S(0) = 0.05$, $\beta = 0.75$, $E(R) = 0.1$, $\lambda_S = 0.0525$, $\sigma_S = 0.07$, $r^{TD} = 0.06$, $r^{DD} = 0.06$, $m_s = 0.12$, $m_l = 0.04$, $m_f = 0.05$, $m_B = 0.02$, $\sigma_s = 0.03$, $\sigma_f = 0.02$, $\sigma_B = 0.01$, $a = 7$, $\mu_{M_2} = 0.04$, $\theta_{rs} = 3$, $\mu_{rs} = 4$, $\sigma_{f^{R_1}} = 0.011$, $\sigma_{r_S} = 0.05$, $\beta_1 = 2$, $\gamma = 0.4$, $\delta = 1/3$, and $\sigma_{M_2} = 0.03$. Figures (1) - (8) characterize the behavior of the LCR when the currency outflow rate is given by (2.20) and (2.24), respectively. Liquid assets (such as marketable securities) are traded on the open market. These assets are very popular amongst investors due to its high liquidity. Also, the assets are influenced by factors such as the interest rate on the open market. From Figures (1) - (8), we observe how the assets in LCR are influenced by the parameter $\rho$. For example if the interest rate increases the movement of the liquid asset prices generally increases and vice versa.
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Figure 1: Simulation of LCR with $c^*$ given by (2.20) when $\rho = -0.95$.

Figure 2: Simulation of LCR with $c^*$ given by (2.24) when $\rho = -0.95$.

Figure 3: Simulation of LCR with $c^*$ given by (2.20) when $\rho = -0.05$.

Figure 4: Simulation of LCR with $c^*$ given by (2.24) when $\rho = -0.05$. 
Figure 5: Simulation of LCR with $c^*$ given by (2.20) when $\rho = 0.05$.

Figure 6: Simulation of LCR with $c^*$ given by (2.24) when $\rho = 0.05$.

Figure 7: Simulation of LCR with $c^*$ given by (2.20) when $\rho = 0.95$.

Figure 8: Simulation of LCR with $c^*$ given by (2.24) when $\rho = 0.95$. 
Comparing the figures above we note that the graphs of LCR exhibit similar characteristics. Over certain periods of time the graphs of Figures (1) - (8) remains well above the required liquidity threshold of 100%. This is crucial because having enough liquidity to withstand a month of elevated financial stress, gives bank management and regulators additional time to respond if necessary. We also note that the graphs of LCR have a downward trend. This is due to the fact that banks reserves can be drawn down in times of stress.

From Figures (1) - (8), at some point in time, we observe that the LCR starts to drops below the liquidity threshold. In situations like these, if the shortfall continues for three consecutive business days (as can be seen in the Figures), then it will be required of the bank, who calculates their LCR on a daily basis, to provide a plan for remediation to their primary regulator. If a shortfall occurs when banks are calculating their LCR on a monthly basis, then it must promptly consult with their primary regulator if they need to provide a remediation plan of action. These plans consists of the following:

- a thorough assessment of the bank’s liquidity position;
- banks will take action to achieve full compliance with the final rule;
- they need to provide a time framework for achieving compliance;
- it will be held accountable to report to its regulator no less than weekly on progress to achieve compliance with the plan until full compliance with the final rule is achieved.

A LCR shortfall, at a minimum, would result in heightened supervisory monitoring. Also banks facing this LCR shortfall may choose term funding, since it has maturity of greater than 30 days and satisfies both LCR and reserve requirements. This could lower the demand for overnight loans, pushing down the overnight rate and reducing the effectiveness of traditional monetary policy. Determining the optimal levels for liquidity is a challenging task in itself. For instance, if regulators overestimate the cash outflows in LCR, banks could be forced to hold too much liquidity, introducing inefficiencies into the financial system.

In order to further motivate the importance of our numerical example above, we illustrate through a template of the LCR of six Canadian banks as of 31 October 2015 (see [25]) below. Note that the LCR for each bank is well above the liquidity threshold.
4 Conclusion

The global financial crises of 2007 − 09 demonstrated the importance of investments in liquid assets and its dependence on high-risk funding sources. Of course, banks exposed to liquidity risk increased their hold of liquid assets the most. Liquidity risk is part of a core function provided by banks in the sense of maturity transformation. Maturity transformation occurs when there are "mismatches" between liabilities and assets. As an example, a bank who does not have enough liquid assets to meet a sudden increase in demand on its liability side, may be forced to sell assets quickly at reduced prices or to suspend operations. This puts strain on other financial institutions since the
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bank acts as lender. Furthermore, it causes disruptions to the financial system.

After providing simulations of the liquidity coverage ratio in Section 3, we would like to point out some key similarities of the liquidity coverage ratio and the reserve requirement ratio. In a nutshell, the reserve requirement ratio refers to a regulation set by a central bank and employed by most, but not all, of the world’s central banks. It is the minimum fraction of deposits and notes that each commercial bank must hold as reserves. Historically, the reserve requirement could prevent banks from drawing on their liquidity when it was most needed as it was seen as a safety buffer. However, as the financial world became more complex and required more sophisticated tools to manage risk, especially liquidity risk, the popularity of the reserve requirement started to diminish as a means of bank regulation and monetary control. As it became less fashionable, more banks resorted to risk-based capital requirements as implemented through the international Basel Accords. Recently, the widespread practice of paying interest on bank reserves has given central banks an alternative way instead of enforcing banks to hold a certain amount of money. In the process, the liquidity coverage ratio emerged and it mimics many aspects of the reserve requirement ratio.

References


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