An Interactive Possibilistic Programming for Fuzzy Multi Objective Solid Transportation Problem

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Abstract

In this paper, we propose a fuzzy interactive possibilistic programming approach to solve a fuzzy multi objective solid transportation problem. The solid transportation problem, consider the supply, the demand and the conveyance constraints to satisfy the transportation requirement in a cost-effective manner. The main objective of the problem is to minimize the total transportation cost and total delivery time. To accommodate imprecision, the goals are defined in a fuzzy sense and linear membership function representing fuzzy goals. This procedure provides a systematic framework that facilitates the fuzzy decision making process, enabling the decision maker to interactively adjust the search direction during the solution procedure by adjusting the value of the coefficient of compensation parameter $\gamma \in [0,1]$, and obtain the decision maker preferred satisfactory solution.

Keywords: Fuzzy interactive possibilistic approach, Fuzzy solid transportation problem, Fuzzy goals, Membership function, Fuzzy multi objective programming

1. Introduction

Among different linear programming model, the one called transportation problem is very popular. Hitchcock [4] defined the transportation problems. The
Solid Transportation Problem (STP) is a generalization of the well-known Transportation Problem (TP) in which three-dimensional properties are taken into account in the objective and constraint set instead of source and destination. The STP was first stated by Shell [15]. A solid transportation problem can be converted to a classical transportation problem by considering only a single type of conveyance. In many industrial problems, a homogeneous product is delivered from an origin to a destination by means of different modes of transport called conveyance, such as trucks, cargo flights, goods trains, ships, etc. Bit et al. [2] apply fuzzy linear programming technique to multi objective solid transportation problem. The efficient solutions as well as an optimal solution are derived. There are cases that the parameters in solid transportation problem cannot be presented in a precise manner. For example, the unit shipping cost may vary in a time frame. The supplies, demands and conveyance capacities may be uncertain due to uncontrollable factors [10]. In the literature, several authors have studied the transportation problems in a fuzzy environment. Kasana [7] developed different approaches to generate the set of efficient solutions for multi objective transportation problems. The solution procedure of this method depends on determining the set of efficient solutions and, finally, the decision maker is responsible for selecting the preferred solution out of this set. Hussein [5] studies to complete set of $\alpha$-possibly efficient solutions of the multiple objective transportation problems with possibilities coefficients of the objective functions. Liang [9] proposed an interactive fuzzy multi-objective linear programming model for solving an integrated production-transportation planning problem in supply chains. In this method, author has applied the max-min approach of Zimmermann to solve the auxiliary single-objective mode. But, it is well-known that the solution yielded by max-min operator might not be unique nor efficient. Jimenez and Verdegay [6], Li and Lai [8] and Waiel [1] presented the fuzzy compromise programming approach to multi objective transportation problems. Grzegorzewski [3] approximated the fuzzy number to its nearest interval. Shaocheng [14] discussed about the interval number linear programming. In this paper, we propose a fuzzy interactive possibilistic approach for solving fuzzy multi objective STP. This method has a hybridization of the methods of Lai and Hwang [12] and Selim and Ozkarahan [13], where proposed by Torabi and Hassini [16] as a new single-phase fuzzy approach. In this paper we solve the fuzzy multi objective STP for the purpose of finding a preferred satisfactory solution for a given problem based on decision maker preferences by adjusting the value of the coefficient of compensation parameter $\gamma$.

The reminder of this paper is organized as follows: Section 2 provides the problem description. The solution methodology is presented in Section 3. Section 4 illustrated the model implementation by numerical example. Finally, the conclusions and future works are provided in Section 5.
2. Problem description

In this section, we describe the fuzzy multi objective solid transportation. Consider \( m \) sources and \( n \) destinations in a multi objective solid transportation problem. At each source, let \( s_i \) be the amount of a homogeneous product we want to transport to \( n \) destinations to satisfy the demand for \( d_j \) units of the product. Let \( e_k \) denote the units of this product can be carries by \( k \) different modes of transportation called conveyance, such as trucks, cargo flight, goods train, ships, etc. Let \( x_{ijk} \) denote the number of units to be transported from source \( i \) to destination \( j \) through conveyance \( k \). A parameter \( c_{ijk} \) represents the unit shipping cost of a product from source \( i \) to destination \( j \) by means of the \( k \)th conveyance. A parameter \( t_{ijk} \) represents transport time per unit delivered from source \( i \) to destination \( j \) by means of the \( k \)th conveyance. \( C \) denotes the total budget and \( T \) denotes the total delivery time. We want to find a feasible way of shipping the available amounts to satisfy the demand such that the total transportation cost and total transportation time are minimized.

2.1 Goals of the problem

The decision maker desires to achieve the following goals:

(i) Transportation costs: Total transportation cost must be minimized. This goal can be expressed as follows:

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} c_{ijk} x_{ijk} \leq C \quad (2.1)
\]

(ii) Transportation times: Total delivery time must be minimized. To avoid the uncertainty in availability of deliver the product to customer on time, the time goal can be expressed as follows:

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} t_{ijk} x_{ijk} \leq T \quad (2.2)
\]

where \( C \) (\( T \)) is the aspiration level of the total cost (total time) goal. "\( \leq \)" is fuzzified version of "\( \leq \)." The fuzzy goal indicates that the decision maker will be satisfied for values slightly over \( C \) (\( T \)) up to a given tolerance limit [17].

2.2 Constraints

The solid transportation problem has tree item properties. The constraints of the solid transportation problem can be expressed as follows:
(1) Constraint of supply:
\[
\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq s_i, \quad l = 1, \ldots, m.
\]

(2) Constraint of demand:
\[
\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \geq d_j, \quad j = 1, \ldots, n.
\]

(3) Constraint of conveyance:
\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq e_k, \quad k = 1, \ldots, l.
\]

To ensure the STP is feasible, it is necessary that the constraints \( \sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} d_j, \quad \sum_{k=1}^{l} e_k \geq \sum_{j=1}^{n} d_j \) are satisfied [10].

3. Solution methodology

In this section, an approach to transform the fuzzy multi objective solid transportation model into an equivalent auxiliary crisp mathematical programming model is defined. This approach solve the fuzzy multi objective STP by adopts linear membership function to represent all of the fuzzy objective functions for the decision maker and together with Torabi and Hassini fuzzy programming solution method [16]. There are many possible forms for membership functions: linear, exponential, hyperbolic, hyperbolic inverse, piece-wise linear, etc. the linear form of membership function is most common and suitable to describe impreciseness in many real world problems. Because the rate of increased membership satisfaction is considered constant and the fuzzy multi objective linear programming problem can be converted into an equivalent ordinary linear programming problem, moreover, the linear membership function has higher computational efficiency [18]. If \( b^u_k \) be the upper tolerance limit for the achievement of the desired value \( b_k \) then the corresponding linear membership functions for each fuzzy goals in the models (2.1)-(2.2), can be expressed as follows:

\[
\mu_{Z_k(x)}(x) = \begin{cases} 
1, & \text{if } Z_k(x) \leq b_k, \\
\frac{(b_k + b^u_k) - Z_k(x)}{b^u_k}, & \text{if } b_k \leq Z_k(x) \leq b_k + b^u_k, \\
0, & \text{if } Z_k(x) > b_k + b^u_k.
\end{cases}
\]

According to Torabi and Hassini [16] approach, a multi objective model could be transformed as a single objective model as follows:
**An interactive possibilistic programming...**

\[
\begin{align*}
\text{Max } & \lambda(x) = \gamma \lambda_0 + (1 - \gamma) \sum_k \theta_k \mu Z_k \\
\text{s.t. } & \lambda_0 \leq \mu Z_k, \quad k = 1, ..., K, \\
& x \in F(x), \\
& \lambda_0, \gamma \in [0,1],
\end{align*}
\]

where \( \mu Z_k \) and \( \lambda_0 = \min\{\mu Z_k(x)\} \) denote the satisfaction degree of the \( k \)th objective function and the minimum satisfaction degree of the objectives, respectively. Moreover, \( \theta_k \) and \( \gamma \) indicate the relative importance of the \( k \)th objective function and the coefficient of compensation, respectively. The \( \theta_k \) parameters are determined by the decision maker based on her/his preferences so that \( \sum_k \theta_k = 1, \theta_k > 0 \). Besides, \( \gamma \) not only controls the minimum satisfaction level of the objectives but also controls the compromise degree among the objectives implicitly. That is, the proposed formulation is capable of yielding both unbalanced and balanced compromised solutions for a problem based on the decision maker preferences by adjusting the value of parameter \( \gamma \) [12].

**3-1 Solving fuzzy multi objective solid transportation problem**

Fuzzy goals are characterized by their membership function. For each objective function, the corresponding linear membership function is defined by:

\[
\begin{align*}
\mu(z_1)(x) &= \frac{(C + C^u) - Z_1(x)}{C^u}, \\
\mu(z_2)(x) &= \frac{(T + T^u) - Z_2(x)}{T^u},
\end{align*}
\]

where, \( C^u \) is the upper tolerance limit for the achievement of the desired value \( C \) and \( T^u \) is the upper tolerance limit for the achievement of the desired value \( T \). Using the fuzzy interactive possibilistic programming, equivalent single objective nonlinear programming model for solving fuzzy multi objective STP can be formulated as follows:

\[
\begin{align*}
\text{max } & \gamma \lambda_0 + (1 - \gamma)(\theta_1 \mu Z_1 + \theta_2 \mu Z_2) \\
\text{s.t. } & \lambda_0 \leq \mu Z_1, \\
& \lambda_0 \leq \mu Z_2, \\
& \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq s_i, \quad i = 1, ..., m, \\
& \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \geq d_j, \quad j = 1, ..., n, \\
& \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq e_k, \quad k = 1, ..., l, \\
& \sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} d_j, \\
& \sum_{k=1}^{l} e_k \geq \sum_{j=1}^{n} d_j, \\
& \lambda_0, \gamma \in [0,1].
\end{align*}
\]
Solve the crisp single objective nonlinear programming problem (3.5) and obtain the initial compromise solution for the fuzzy multi objective STP. Execute and modify the interactive decision process. If the decision maker is dissatisfied with the initial solution, the model should be modified until a satisfactory solution is obtained.

4. Numerical example

In this section, we consider a fuzzy multi objective solid transportation problem. In detail we give the steps of the process of solving this problem. As an illustration of the proposed approach, consider a fuzzy multi objective solid transportation problem with two fuzzy goals, two supplies, three demands and two conveyance capacities. $C^u = 2500$ is the upper tolerance limit for the achievement of the desired value $C = 2200$ and $T^u = 660$ is the upper tolerance limit for the achievement of the desired value $T = 220$. Assume that the cost coefficients in the model are follows:

$c_{111} = 20, \quad c_{112} = 70, \quad c_{121} = 60, \quad c_{122} = 60, \quad c_{131} = 50, \quad c_{132} = 30,$
$c_{211} = 10, \quad c_{212} = 40, \quad c_{221} = 20, \quad c_{222} = 50, \quad c_{231} = 40, \quad c_{232} = 50,$

and time parameters in the model are follows:

$t_{111} = 10, \quad t_{112} = 11, \quad t_{121} = 10, \quad t_{122} = 12, \quad t_{131} = 14, \quad t_{132} = 10,$
$t_{211} = 8, \quad t_{212} = 10, \quad t_{221} = 7, \quad t_{222} = 4, \quad t_{231} = 9, \quad t_{232} = 5,$

and supplies in the model are $s_1 = 70, s_2 = 60$ and demands in model are $d_1 = 10, d_2 = 40, d_3 = 30,$

and conveyance parameters in the model are $e_1 = 50, \quad e_2 = 70.$

Because

$$\sum_{i=1}^{2} s_i = 130 \geq \sum_{j=1}^{3} d_j = 80,$$
$$\sum_{k=1}^{2} e_k = 120 \geq \sum_{j=1}^{3} d_j = 80.$$ 

Then, the STP is feasible. By using the model (3.5), solve the fuzzy multi objective STP based above data by LINGO 11, we obtain the following solutions:

By choosing $\gamma = 0.2 \text{ and } \theta_1 = \theta_2 = 0.5$, we obtain:

$x_{111} = 0, \quad x_{112} = 0, \quad x_{121} = 0, \quad x_{122} = 0, \quad x_{131} = 0, \quad x_{132} = 20,$
$x_{211} = 10, \quad x_{212} = 0, \quad x_{221} = 40, \quad x_{222} = 0, \quad x_{231} = 0, \quad x_{232} = 10,$
$\lambda_0 = 0.4, \quad Z_1 = 2000, \quad Z_2 = 610.$
Now if obtained solution is not satisfactory then he/she can modify the coefficient of compensation and $\theta_k$ parameters and execute the solution procedure. By choosing $\gamma = 0.2$ and $\theta_1 = \theta_2 = 0.5$, and resolve the model, we obtain:

$x_{111} = 10$, $x_{112} = 0$, $x_{121} = 0$, $x_{122} = 0$, $x_{131} = 0$, $x_{132} = 10$,

$x_{211} = 0$, $x_{212} = 0$, $x_{221} = 20$, $x_{222} = 20$, $x_{231} = 0$, $x_{232} = 20$,

$\lambda_0 = 0.54$, $Z_1 = 3000$, $Z_2 = 520$.

For $\gamma = 0.3$ and $\theta_1 = 0.6$, $\theta_2 = 0.4$, and resolve the model, we obtain:

$x_{111} = 0$, $x_{112} = 0$, $x_{121} = 0$, $x_{122} = 0$, $x_{131} = 0$, $x_{132} = 20$,

$x_{211} = 10$, $x_{212} = 0$, $x_{221} = 40$, $x_{222} = 0$, $x_{231} = 0$, $x_{232} = 10$,

$\lambda_0 = 0.409$, $Z_1 = 2000$, $Z_2 = 610$.

Thus, by solving the fuzzy multi-objective solid transportation problem for different value of $\gamma \in [0,1]$ and $\theta_k$ parameters based on her/his preferences the solution methodology is able to find different efficient solutions.

5. Conclusions

In this study, we proposed a fuzzy interactive possibilistic programming approach for solving the fuzzy multi-objective solid transportation problem. In the solution methodology, we control the search direction via adjusting the value of the coefficient of compensation parameter $\gamma \in [0,1]$. Besides, $\gamma$ not only controls the minimum satisfaction level of the objectives but also controls the compromise degree among the objectives implicitly. If the decision maker is dissatisfied with the initial compromise solution, we execute and modify the interactive process until a satisfactory solution is obtained. Future study may apply the solution methodology to the different problems related to supply chain management.

References

https://doi.org/10.1016/s0165-0114(98)00155-9

https://doi.org/10.1016/0165-0114(93)90158-e

https://doi.org/10.1016/s0165-0114(02)00098-2


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