Scheduling Aircraft Landing at Single Runway

Iis Setiowati
Department of Mathematics
University of Riau
Pekanbaru 28293, Indonesia

Ihda Hasbiyati
Department of Mathematics
University of Riau
Pekanbaru 28293, Indonesia

M. D. H. Gamal
Department of Mathematics
University of Riau
Pekanbaru 28293, Indonesia
Corresponding author

Abstract
This paper presents an aircraft landing scheduling problem at single runway. The focus of this discussion is to minimize the additional costs due to non-compliance of the aircraft landing target at a given time. A zero-one integer programming approach is used to solve this problem. At the end of this study a simulation is given to see the result of applying the method in this issue.

Mathematics Subject Classification: 90C05, 90C10

Keywords: Air Traffic Control, scheduling, linear programing, zero-one integer programming

1 Introduction
Aviation had begun in the late of 19th century when Wright brothers success-
fully flew their modest air plane on December 17, 1903. After seven years, the third oldest airport—Shoreham, UK—was listed as the first airport to operate commercial flight. With increasing in the number of airports, the air traffics problem is more complex. The growth of air traffic is powerful enough to be the cause of polemics severity congestion.

Air traffic congestion has significantly grown from year to year in the late 1960s and early 1970s in the Western and sharply increased after 1980s ended. Veidal [8] writes that some of the busiest airports in the world with their maximum area did not possible to expand when the arrivals increase, because of the economic, political, noise and the surrounding environment considered. Wen [7] conveys that the world’s largest airport—Atlanta, USA—handles more than 80 million departures and arrivals per year. The biggest cargo airport in the world, located in Memphis, TN, transports 2.5 million tons of goods each year. Ciesielski and Scerri [13] write the time between two planes landing are only 3 minutes in Sydney airport. Because the density of the flight, Ball et al.[15] reports that almost one of four airlines delays to their destination for 15 minutes from their target time in schedule in 2007. Additionally, Solveling and Clarke [14] say that one-third landing on latest time are caused by the direct traffic demand for exceeds the capacity of the aviation system.

Many various factors are driving the increasing of potential air track congestion, including the lack of air traffic control. Certainly, this is related to the system used by the Air Traffic Control (ATC), as a guide and the closest partner of the pilot and co-pilot when they fly. ATC is also responsible for the security and safety of every flight in the air space. One of the flight-process that requires good management and regulation is Aircraft Landing Problem (ALP).

In reference to the ALP, the plane has been scheduled and authorized land in airport by ATC local airport. Each target landing time is bounded by the earliest landing time and the latest landing time or called time windows. A plane lands at earliest time if it flies at its maximum airspeed and land in lasted time if it flies at its minimum airspeed. A many time, the plane lands in lasted time because the direct demand of ATC due to the track dense, so the plane must be holding up until it gets permission from the track control. Each plane will get the additional cost if the actual landing time is irrelevant from the scheduled target time. The aircraft is irrelevant from the target time when it lands outside the scheduled target time. The costs will continuously increase as the difference between the landing time and the target time.

With the development of aviation problems, researcher is getting interest to develop problems occurred and made it as their own research. Many studies can be used as a research topic, including air traffic congestion problems. In operations research, several studies begin to give a solution for the existing problems to inhibit congestion points, the one is the maximization of runway function. To maximize the runway function, the scheduling of landing aircraft must be better. Inefficient management processes can reduce the level of the
departure or arrival delay, especially for the airport which has only one runway. In optimization problem analyzing of the aircraft landing problem, there are two cases considered: static and dynamic cases. In static case, the scheduling aircraft landing is used to maximize the order of the flight land on a runway or more. In dynamic case, it is used to optimize the final order when there is a new aircraft (outside of the actual schedule). More than 50 articles have been published whose topic about the scheduling and planning until now. However, Bennell and Mesgarpour [17] considering the operating arrival receives the greater attention in the scheduled arrival literature.

In 1993, Abella et al. [1] considered two solutions to solve scheduling aircraft landing in statistic case with the objective function used to minimize the additional costs of the deviation from its target time. Heuristic genetic algorithm (GA) and the branch and bound are considered to solve scheduling aircraft landing problems. Abella et al. [1] research was developed by Ernst et al. [2] in 1998 with their objective function is the optimization of the aircraft order in one set landing. This research uses the space search heuristic and branch and bound to solve the ALP on a single and multiple runway.

From year to year, the ALP static problem study has continuously grown, Beasley et al. [3] developed Abella et al. [1] research too in 2000. In their paper, they introduce some additional constraints and use the same objective function, which to minimize the additional costs of deviation from the target time. The solution of ALP problems was mixed-integer zero-one programming formulation, and they also used heuristic algorithms. The result of algorithms computational was used to solve 50 aircraft and four runways problems. After one year, in 2001, Beasley et al. [4] has done an investigative research at London Heathrow airport in the UK, used a heuristic population algorithm and the result would increase the traffic control.

In 2004, Pinol and Beasley [5] only focused on the multiple runway static ALP problems in their research. They considered two heuristic techniques, Scatter Search and Binomial Algorithm, to solve problem. The computational results could solve 500 aircraft and five runways problems with effective speed computation. Three years after Pinol and Beasley [5] had published their journal, Soomer and Franx [9] focused on single runway statistics ALP problem in their research. Their paper had published in 2007 considered local heuristic to implement on computation.

There are many approaches and algorithms solution to solve runway landing problem. Xie et al. [12] used hybrid metaheuristic algorithm Kalilawar to solve problems with the objective function was to minimize the additional cost of deviation from target time. Several other studies used same objective function, such as Briskorn and Stolletz [11] who present a solution in computational studies with analyze run-time case and aircraft class when the plane landing.
In addition, in 2012 Diallo et al. [10] and the Agency for Air Navigation Safety in Africa and Madagascar (ASECNA) had made a new breakthrough to solve runway scheduling problems, and in their journal they explained that the scheduling problems for 50 aircraft would be solved by heuristic and zero-one linear programming. This paper is an implemented form of Beasley et al. [3] model, with the objective function is to minimize additional costs of deviation from target time, and this paper considers a solution of single runway scheduling aircraft used an integer zero-one programming by introducing another constraint to be an alternative solution for single runway scheduling aircraft problem.

This paper is a small contribution to the development of air traffic, especially to Sultan Syarif Kasim II (SSK II) Airport, located in city of Pekanbaru. SSK II Airport is one of the international airport which serves flight in some provinces in Indonesia and neighboring countries, such as Singapore and Malaysia. We provide a solution of landing aircraft problem which often landing outside the target time, according of the aircraft data, 100 aircraft landings outside the target time per month. Many various reason caused deviation from target time landing in SSK II airport, but we will not discuss the reason of flight arrive in time irrelevant. We try to give an aircraft landing time solution drift with provides the bounded time which has not been implemented by the Airnav and provides additional cost that must be paid by the airline to the Airnav if the landing is outside from the scheduled target time. Not only that, But also we give the design of mathematical modeling which is the adoption of Beasley et al. [3] model to minimize the additional costs of deviation of target time in single runway, and of course, it will be benefit to ATC and the airlines. An integer zero-one programming will be an alternative solution to solve single runway scheduling Aircraft landing.

In the next section, we present a model to solve single runway aircraft landing problem. The third section presents the results of the simulation computational comparison between the proposed model and Beasley et al. [3] model and the conclusions.

2 Scheduling Aircraft Landing Mathematical Model

In this section we present an integer zero-one formulation of the static single runway scheduling aircraft landing problem. This problem will be considered as static problem and we assume that there is no change in the scheduling aircraft landing or in word and we are not consider the new plane will land outside the scheduled target time. We adopt and simplify Beasley et al. [3] model to solve scheduling aircraft landing in SSK II airport. There is some assumptions to differ and make the model simpler.

Let us introduce some notations used in a single runway scheduling aircraft landing formulation as follows:

\[ n \] := the number of planes.

\[ E_i \] := the earliest landing time for plane \( i (i = 1, \ldots, n) \).
Scheduling aircraft landing at single runway

$L_i :=$ the latest landing time for plane $i \ (i:1, \ldots, n)$.

$T_i :=$ the target landing time for plane $i \ (i:1, \ldots, n)$.

$S_{ij} :=$ the required separation time ($\geq 0$) where plane $i$ lands before plane $j$ for each $i, j = 1, \ldots, n$.

The decision variables are

$x_i :=$ the landing time for plane $i$ (for each $i = 1, \ldots, n$).

$\delta_{ij} :=$ the binary variable which becomes 1 if $i$ lands before plane $j$ (for each $i, j = 1, \ldots, n$; $i \neq j$), 0 otherwise.

The constraint are given to ensure that each plane lands within its time windows, that is

\[ E_i \leq x_i \leq L_i \] (1)

To ensure that plane $i$ must land before $j$ when the pairs of plane $i$ and $j$ landing, we assume that $\delta_{ij} = 1$ and $\delta_{ji} = 0$, so the next constraint

\[ \delta_{ij} + \delta_{ji} = 1 \] (2)

Based upon this, there are three definitions of the pairs of plane landing problem, i.e.

$U :=$ the set of pairs $(i, j)$ of planes for which we are uncertain whether plane $i$ lands before plane $j$.

$V :=$ the set of pairs $(i, j)$ of planes for which $i$ definitely lands before $j$ (but for which the separation constraint is not automatically satisfied).

$W :=$ the set of pairs $(i, j)$ of planes for which $i$ definitely lands before $j$ (and for which the separation constraint is automatically satisfied).

In mathematical notations we can define

\[ U := \{(i, j) | E_j \leq E_i \leq L_j \text{ or } E_j \leq L_i \leq L_j \text{ or } E_i \leq E_j \leq L_i \right\} \]

or $E_j \leq L_j \leq L_i; \ i, j = 1, 2, \ldots, n; \ i \neq j \right\}$

\[ V := \{(i, j) | L_i < E_j \text{ and } L_i + S_{ij} > E_j; \ i, j = 1, \ldots, n; \ i \neq j \right\} \]

\[ W := \{(i, j) | L_i < E_j \text{ and } L_i + S_{ij} \leq E_j; \ i, j = 1, \ldots, n; \ i \neq j \right\} \]

The definition of $U$ means that one the end points of the time window of one plane falls within the time window of the other, so we have the constraint

\[ x_j \geq x_i + S_{ij} - M\delta_{ij}, \quad \forall (i, j) \in U. \] (3)

The value $M$ is large positive constant. There are two cases to consider here, i.e.
(i) if $\delta_{ij} = 1$, then $i$ lands before $j$, therefore $\delta_{ji} = 0$ equation (3) becomes
\[ x_j \geq x_i + S_{ij} \].
(4)

(ii) if $\delta_{ij} = 0$, then $j$ lands before $i$, therefore $\delta_{ji} = 1$ and equation (3) becomes
\[ x_j \geq x_i + S_{ij} - M \]
for $x_j \geq$ some large negative number, so it ensures that constraint is effective.

The value of $M$ would like to be as small as possible in upon constraint because $M$ can be replaced by $(L_i + S_{ij} - E_j)$, therefore equation (3) becomes
\[ x_j \geq x_i + S_{ij} - (L_i + S_{ij} - E_j)\delta_{ji} \quad \forall (i, j) \in U. \]
(6)

Using equation (2) we can rewrite constraint (3) as
\[ x_j \geq x_i + S_{ij}\delta_{ij} - (L_i - E_j)\delta_{ji} \quad \forall (i, j) \in U. \]
(7)

Then to ensure that replacing $M$ with $(L_i + S_{ij} - E_j)$ is valid, we need to recheck the case that $\delta_{ij} = 0$, therefore equation (7) becomes
\[ x_j \geq E_j + (x_i - L_i) \]
(8)

When we use equation (1), we have $(x_i - L_i \leq 0)$, so equation (8) merely says $x_j \geq E_j + [a \text{ value } \leq 0].$ We use mathematical logic for conjunction expression in this constraint to get the conclusion that constraint is always true.

The pairs of plane in $V$ need the constraint to ensure separating time satisfy this
\[ x_j \geq x_i + S_{ij} \quad (\forall i, j) \in V. \]
(9)

The constraint above ensures the actual landing time of plane $j$ bigger than plane $i$ adding separating time, and certainly the plane $i$ is the first landing plane.

In set of $W$, plane $i$ landing before plane $j$ and separation time automatically satisfied, so we have the constraint
\[ \delta_{ij} = 1 \quad (\forall i, j) \in W \cup V. \]
(10)

We need a constraint to ensure that additional cost of airline is minimized. Let $c_i$ is penalty cost per unit of plane, and then to minimize $c_i$, we need a constraint to ensure the plane irrelevant landing interval as small as possible. Let $k_i$ is time difference interval, to ensure $k_i$ as small as possible we have
\[ k_i \geq T_i - x_i, \quad \forall i = 1, \ldots, n \]
(11)
\[ 0 \leq k_i \leq T_i - E_i, \quad \forall i = 1, \ldots, n \]
(12)
\[ k_i \geq x_i - T_i, \quad \forall i = 1, \ldots, n \]
(13)
Scheduling aircraft landing at single runway

\[ 0 \leq k_i \geq L_i - T_i, \quad \forall i = 1, \ldots, n. \quad (14) \]

Hence, the definition of the landing time constraint of plane \( i \) is

\[ x_i = T_i - k_i, \quad \forall i = 1, \ldots, n. \quad (15) \]

The objective function of scheduling aircraft landing is minimizing additional cost for any plane deviation from its target, the landing time is defined by time difference interval, so the objective function define is

\[ \min \sum_{i=1}^{n} c_i k_i, \quad \forall i = 1, \ldots, n. \quad (16) \]

The complete formulation of single runway problem is therefore to minimize function (16) subject to constraints (1), (2), (7), and (9)-(15).

3 Computational Results

Airnav is a closed partner for the pilots when they are in flight, guiding the plane, caring for air traffic safety and navigating the plane landing. When a plane enters the local airport ATC radar it will directly be served and the landing time will be determined. We hope this study will help ATC to determine the landing time with the minimum cost that must be paid by the airline. Furthermore, we hope that the ATC will use additional cost of deviation from the target time.

The results of computational show that how the optimization models are suited for Airnav control, compared to their solution technique usually used. We consider scheduling for one day, because SSK II serves the same flight with the same landing time almost every day. We give the computational comparison between the Beasley et al. [3] model and our implemented model as shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Comparison of Model 1 (M₁) and Model 2 (M₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Period</td>
</tr>
<tr>
<td>( n )</td>
</tr>
<tr>
<td>( \alpha )</td>
</tr>
<tr>
<td>( \mu )</td>
</tr>
<tr>
<td>( \eta )</td>
</tr>
<tr>
<td>( \epsilon )</td>
</tr>
<tr>
<td>( \tau )</td>
</tr>
<tr>
<td>( \omega )</td>
</tr>
</tbody>
</table>
In Table 1, \( n \) is the number of planes which land every time. Period 1 (07:20-10:55), period 2 (11:25-12:55), period 3 (13:35-14:55), and period 4 (15:10-20:30). Parameter \( \alpha \) defines the optimal value from the final computation, \( \mu \) the numbers of constraints, \( \eta \) the numbers of variables, \( \epsilon \) the numbers of branches, \( \tau \) the time and \( \omega \) the number of iterations.

Based on Table 1, it concludes that the optimal value of model 1 is better then model II. From the numbers of iteration perspective, the model is quick to find the optimal value. Hence, Model 1 can be a beneficial alternative solution to aviation development, especially in single runway scheduling aircraft landing.

**Acknowledgements.** Authors thank to Director of Airnav Company who has provided data and information for this study.

**References**


Scheduling aircraft landing at single runway


Received: August 3, 2017; Published: September 11, 2017