Ranking Generalized LR Fuzzy Numbers Using

Area, Mode, Spreads and Weights

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Abstract

Ranking of fuzzy numbers plays an important role in decision making, optimization, forecasting etc. In this paper a new method is developed for ranking the generalized LR fuzzy numbers using area, mode, spreads and weights. The advantage of the proposed approach is that it provides the correct ordering of the generalized and normal LR fuzzy numbers and it can rank all types of fuzzy numbers. The computation procedure is illustrated by using some examples. The proposed ranking method overcomes the drawbacks of most of the existing methods and it is simple and easy to apply in the real life problems.

Keywords: Ranking function; centroid points; generalized LR fuzzy numbers

1. Introduction

In a classical contest real number $R$ can be linearly ordered by, however the ranking of fuzzy numbers cannot be executed in such a way. Since fuzzy numbers are represented by possibility distributions, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than the other. From the application view point, the ranking of fuzzy numbers plays a main role in real life problems involving decision-making, clustering, optimization, transportation problems, etc. [1-4]. In order to rank fuzzy numbers, one fuzzy number needs to be evaluated and compared to others, but this may not be so easy. The literature over the past few decades has proposed numerous methods for ranking fuzzy numbers. Some of them are legendary for ranking fuzzy...
numbers, such as maximizing sets and minimizing sets, centroid points, and distance minimization. Jain [5] presented a decision method for systems in which the state of the system and/or the utilities of the alternative actions are known. Yager [4] presented the centroid-index method, Dubios and Prade [2] used the maximizing set to order fuzzy numbers, and Chen [6] gave a method for calculating the ordering value of each fuzzy number with triangular, trapezoidal, and two-sided drum-like shaped membership functions, by introducing the concept of maximizing and minimizing set to decide the ordering value of each fuzzy number and uses those values to determine the order of the n fuzzy numbers. Chu and Tsao [7] ranked fuzzy numbers with an area between the centroid point and original point. Wang et al. [8] presented the centroid formulae for fuzzy numbers and justified them from the viewpoint of analytical geometry. Abbasbandy and Asady [9] suggested sign distance, while Asady and Zendehnam [10] proposed the defuzzification using minimizer of the distance between the two fuzzy numbers and obtained the nearest point with respect to a fuzzy numbers and considered the nearest point for ranking the fuzzy numbers.

Many researchers have recently employed maximizing and minimizing set and the concept of a centroid point as the basis for comparing and ranking fuzzy numbers [3,11-14]. These methods have mainly concern the correlation between LR areas and centroid point of a fuzzy number. Wang et al. [13] proposed the LR deviation degree in which the maximal and minimal reference sets were defined and the ranking index value is obtained based on the LR deviation degree and relative variation of fuzzy numbers. Asady [11] proposed a procedure based on the LR deviation degree of fuzzy number to overcome the shortcomings of Wang et al. [13]. Nejad Mashinch [12] proposed a method based on the areas on the left and the right sides of fuzzy numbers for ranking fuzzy numbers. However, most deviation degree approaches still display the same limitations due to the neglected decision maker’s attitude, the incoherent transfer coefficient formula, and the unreliable ranking index computation. More ranking literature can be found in [8, 12, 15-21]. Most of the methods presented above cannot discriminate fuzzy numbers, and some methods do not agree with human intuition, whereas some methods cannot rank crisp numbers which are a special case of fuzzy numbers.

In this paper, a new method is proposed which is based on the centroid of centroids to rank fuzzy quantities. In a trapezoidal fuzzy number, first the trapezoid is split into three parts where the first, second, and third parts are a triangle, a rectangle, and a triangle respectively. Then the centroids of these three parts are calculated followed by the calculation of the centroids of these centroids. Finally, a ranking function is defined which is the area between the centroid of centroids and the original point and also uses mode and spreads in those cases where the discrimination is not possible. Most of the ranking procedures proposed in literature, use centroid of trapezoid as a reference point, as the centroid is a balancing point of the trapezoid. But the centroid of centroids can be considered to be a more balancing point than the centroid.
The work is organized as follows: Section 2 gives preliminaries of LR fuzzy numbers. Section 3 presents the proposed ranking method. In section 4 some numerical examples are considered from the existing methods. Section 5 Comparison of the proposed ranking method with the existing methods is given. The conclusion is given in Section 6.

2. Preliminaries

Definition 1: A fuzzy number \( \tilde{A} = (m, n, \alpha, \beta)_{LR} \) is said to be an LR fuzzy number if

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{m-x}{\alpha}, & x \leq m, \alpha > 0, \\
\frac{x-n}{\beta}, & x \geq n, \beta > 0, \\
1, & \text{otherwise.}
\end{cases}
\]

If \( m = n \) then \( \tilde{A} = (m, n, \alpha, \beta)_{LR} \) will be converted into \( \tilde{A} = (m, \alpha, \beta)_{LR} \) and is said to be a triangular LR fuzzy number. L and R are called left and right reference functions, which are continuous, non-increasing functions that define the left and right shapes of \( \mu_{\tilde{A}}(x) \) respectively and \( \mu(0) = \mu(1) = 1. \)

For two generalized LR fuzzy numbers \( \tilde{A}_1 \) and \( \tilde{A}_2 \) where \( \tilde{A}_i = (m_i, n_i, \alpha_i, \beta_i; w_i)_{LR} \) and \( \tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2; w_2)_{LR} \), is defined as:

\[
\tilde{A}_1 \oplus \tilde{A}_2 = (m_1 + m_2, n_i + n_2, \alpha_i + \alpha_2, \beta_i + \beta_2; \min (w_i, w_2))_{LR}
\]

3. New ranking approach of LR fuzzy number based on area, mode and Spread

In this section, new ranking approach is presented for the ranking of LR fuzzy numbers. This method involves a procedure for ordering fuzzy sets in which a ranking approach \( R(\tilde{A}) \) is calculated for the fuzzy number \( \tilde{A} = (m, n, \alpha, \beta)_{LR} \) from centroid of centroids using area, mode and spread.

Proposed Ranking Method

Generally, the Centroid is considered as the balancing point of a trapezoid. In this new ranking method the trapezoid (APQD) is divided into the triangle (APB), the rectangle (BPQC) and the triangle (CQD), and the centroids of these three plane figures APB, BPQC and CQD be \( G_1, G_2 \) and \( G_3 \) respectively as shown in Fig. 1. The centroid of the triangle (APB), where \( A = (m-\alpha, 0), P = (m, w), C = (n, 0) \) is,

\[
G_1 = \left( \frac{3m-\alpha}{3}, \frac{w}{3} \right).
\]

The centroid of the triangle (QCD), where \( Q = (n, w), C = (n, 0) \),
D = (n+β, 0) is \( G_2 = \left( \frac{3n + \beta}{3}, \frac{w}{3} \right) \). The centroid of the rectangle (BPQC), where B = (m, 0), P = (m, w), Q = (n, w), C = (n, 0) is, \( G_3 = \left( \frac{m + n}{2}, \frac{w}{2} \right) \).

Equation of the line \( G_1G_3 \) is \( y = \frac{w}{3} \) and \( G_2 \) does not lie on the line \( G_1G_3 \).

Therefore \( G_1, G_2 \) and \( G_3 \) are non-collinear and they form a triangle.

The Centroid of the triangle \( G_1G_2G_3 \) is the intersection of the three centroid points \( G_1, G_2 \) and \( G_3 \) as shown in Fig.1. Since each Centroid point is a balancing point of each individual triangle, therefore Centroid of the triangle \( G_1G_2G_3 \) is taken as the better point of reference than the centroid point of the trapezoid, as it is much more balancing point, it is used to define the ranking of generalized LR fuzzy numbers \( \tilde{A} = (m, n, \alpha, \beta; w)_{LR} \).

For the generalized trapezoidal LR fuzzy number \( \tilde{A} = (m, n, \alpha, \beta; w)_{LR} \), The Centroid \( G_\tilde{A}(x, y) \) of the triangle \( G_1G_2G_3 \) with vertices \( G_1, G_2 \) and \( G_3 \) is

\[
G_\tilde{A}(x, y) = \left( \frac{9m + 9n - 2\alpha + 2\beta}{18}, \frac{7w}{18} \right)
\]

(1)

The Centroid \( G_\tilde{A}(x, y) \) of the triangle \( G_1G_2G_3 \) with vertices \( G_1, G_2 \) and \( G_3 \) is

\[
G_\tilde{A}(x, y) = \left( \frac{18m - 2\alpha + 2\beta}{18}, \frac{7w}{18} \right)
\]

(2)

The ranking function which maps the set of all fuzzy numbers to a set of real numbers is

\[
R(\tilde{A}) = (x \times y) = \left( \frac{9m + 9n - 2\alpha + 2\beta}{18} \times \frac{7w}{18} \right)
\]

(3)

This is the Area between the Centroid of the Centroids \( G_\tilde{A}(x, y) \) as defined in Eq. (1) and the original point.
The Mode (M) is defined as:

$$M(\tilde{A}) = \frac{1}{2} \int_0^\infty (m + n) \, dx = \frac{w}{2}(m + n)$$

(4)

The Spread (S) is defined as:

$$S(\tilde{A}) = \int_0^\infty (\alpha + \beta + n - m) \, dx = w(\alpha + \beta + n - m)$$

(5)

The Left spread (LS) is defined as:

$$LS(\tilde{A}) = \int_0^\infty \alpha \, dx = w\alpha$$

(6)

The Right spread (RS) is defined as:

$$RS(\tilde{A}) = \int_0^\infty \beta \, dx = w\beta$$

(7)

By using the above definitions the ranking procedure to compare two generalized trapezoidal LR fuzzy numbers, i.e., \( \tilde{A}_i = (m_i, n_i, \alpha_i, \beta_i; w_i) \) and \( \tilde{A}_j = (m_j, n_j, \alpha_j, \beta_j; w_j) \) is defined as follows:

**Step 1:** Find Area \( R(\tilde{A}_1) \) and \( R(\tilde{A}_2) \) : Case (i) If \( R(\tilde{A}_1) > R(\tilde{A}_2) \) then \( \tilde{A}_1 > \tilde{A}_2 \);
Case (ii) If \( R(\tilde{A}_1) < R(\tilde{A}_2) \) then \( \tilde{A}_1 < \tilde{A}_2 \).
Case (iii) If \( R(\tilde{A}_1) = R(\tilde{A}_2) \) comparison is not possible, then go to step 2.

**Step 2:** Find \( M(\tilde{A}_1) \) and \( M(\tilde{A}_2) \) : Case (i) If \( M(\tilde{A}_1) > M(\tilde{A}_2) \) then \( \tilde{A}_1 > \tilde{A}_2 \);
Case (ii) If \( M(\tilde{A}_1) < M(\tilde{A}_2) \) then \( \tilde{A}_1 < \tilde{A}_2 \).
Case (iii) If \( M(\tilde{A}_1) = M(\tilde{A}_2) \) comparison is not possible, then go to step 3.

**Step 3:** Find \( S(\tilde{A}_1) \) and \( S(\tilde{A}_2) \) : Case (i) If \( S(\tilde{A}_1) > S(\tilde{A}_2) \) then \( \tilde{A}_1 > \tilde{A}_2 \);
Case (ii) If \( S(\tilde{A}_1) < S(\tilde{A}_2) \) then \( \tilde{A}_1 < \tilde{A}_2 \).
Case (iii) If \( S(\tilde{A}_1) = S(\tilde{A}_2) \) comparison is not possible, then go to step 4.

**Step 4:** Find \( LS(\tilde{A}_1) \) and \( LS(\tilde{A}_2) \) : Case (i) If \( LS(\tilde{A}_1) > LS(\tilde{A}_2) \) then \( \tilde{A}_1 > \tilde{A}_2 \);
Case (ii) If \( LS(\tilde{A}_1) < LS(\tilde{A}_2) \) then \( \tilde{A}_1 < \tilde{A}_2 \).
Case (iii) If \( LS(\tilde{A}_1) = LS(\tilde{A}_2) \) comparison is not possible, then go to step 5.

**Step 5:** Examine \( w_1 \) and \( w_2 \) : Case (i) If \( w_1 > w_2 \) then \( \tilde{A}_1 > \tilde{A}_2 \), Case (ii) If \( w_1 < w_2 \) then \( \tilde{A}_1 < \tilde{A}_2 \), Case (iii) If \( w_1 = w_2 \) then \( \tilde{A}_1 \approx \tilde{A}_2 \).

The ranking function of centroid of centroids using the area from Eq. (3) is a linear function of the normal LR fuzzy number \( \tilde{A} = (m, n, \alpha, \beta; w) \), i.e., if \( \tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1; w_1) \) and \( \tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2; w_2) \) are two normalized LR fuzzy numbers, then \( R(k_1\tilde{A}_1 \oplus k_2\tilde{A}_2) = k_1R(\tilde{A}_1) \oplus k_2R(\tilde{A}_2) \) where \( k_1 \) and \( k_2 \) are real numbers.
4. Numerical Examples

In this section, six sets of fuzzy numbers are considered from the existing methods Chen and Sanguansat [15], Xu et al. [22], Cheng [23], Chu and Tsao [7], Yagger [4]. The proposed ranking method is explained first by ranking these fuzzy numbers and represented diagrammatically as shown in the Fig 2.

**Set1:** \( \tilde{A} = (-0.1, 0.1, 0.1, 0.1; 0.4) \) \( \tilde{B} = (0.0, 0.0, 0.1, 0.1; 0.4) \)

Step1: \( G_A(x, y) = (0, 0.15555) \) and \( G_B(x, y) = (0, 0.15555) \)

\[ R(\tilde{A}) = 0 \text{ and } R(\tilde{B}) = 0, R(\tilde{A}) = R(\tilde{B}) \]

so goto step2

Step2: \( M(\tilde{A}) = M(\tilde{B}) = 0 \), so goto step3

Step3: \( S(\tilde{A}) = 0.16 \) \( S(\tilde{B}) = 0.8 \), since \( S(\tilde{A}) < S(\tilde{B}) \) \( \Rightarrow \tilde{A} < \tilde{B} \)

**Set2:** \( \tilde{A} = (-0.2, 0.3, 0.3, 0.1; 0.6) \) \( \tilde{B} = (-0.3, 0.4, 0.3, 0.1; 0.6) \)

Step1: \( G_A(x, y) = (0.0277, 0.2333) \) and \( G_B(x, y) = (0.0277, 0.2333) \)

\[ R(\tilde{A}) = 0.0064 \text{ and } R(\tilde{B}) = 0.0064, R(\tilde{A}) = R(\tilde{B}) \]

so goto step3

Step2: \( M(\tilde{A}) = M(\tilde{B}) = 0 \), so goto step3

Step3: \( S(\tilde{A}) = 0.54 \) \( S(\tilde{B}) = 0.66 \), since \( S(\tilde{A}) < S(\tilde{B}) \) \( \Rightarrow \tilde{A} < \tilde{B} \)

**Set3:** \( \tilde{A} = (-0.2, 0.2, 0.1, 0.1; 0.7) \) \( \tilde{B} = (-0.1, 0.1, 0.2; 0.7) \)

Step1: \( G_A(x, y) = (0.02722) \) and \( G_B(x, y) = (-0.0777, 0.2722) \)

\[ R(\tilde{A}) = 0 \text{ and } R(\tilde{B}) = 0.0211, R(\tilde{A}) > R(\tilde{B}) \]

\( \Rightarrow \tilde{A} > \tilde{B}. \)

**Set4:** \( \tilde{A} = (0.0, 0.1, 0.4, 0.2; 0.8) \) \( \tilde{B} = (0.0, 0.1, 0.3, 0; 0.8) \)

Step1: \( G_A(x, y) = (0.0277, 0.3111) \) and \( G_B(x, y) = (0.0166, 0.3111) \)

\[ R(\tilde{A}) = 0.0086 \text{ and } R(\tilde{B}) = 0.0051, R(\tilde{A}) > R(\tilde{B}) \]

\( \Rightarrow \tilde{A} > \tilde{B}. \)

**Set5:** \( \tilde{A} = (-0.3, 0.3, 0.2, 0.2; 1) \) \( \tilde{B} = (0.0, 0.0, 0; 0.8) \)

Step1: \( G_A(x, y) = (-0.3, 0.3888), G_B(x, y) = (0.0, 0.3888) \) and \( G_C(x, y) = (0.3111) \)

\[ R(\tilde{A}) = -0.1166, R(\tilde{B}) = 0, R(\tilde{C}) = 0, R(\tilde{A}) < R(\tilde{B}) < R(\tilde{C}) \]

so goto step2

Step2: \( M(\tilde{B}) = M(\tilde{C}) = 0 \), so goto step3, step3: \( S(\tilde{B}) = S(\tilde{C}) = 1 \), so goto step4

Step4: \( LS(\tilde{B}) = LS(\tilde{C}) = 0 \), so goto step5, step5: since \( w_1 > w_2 \Rightarrow R(\tilde{B}) > R(\tilde{C}) \)

\( \therefore R(\tilde{B}) > R(\tilde{C}) > R(\tilde{A}) \Rightarrow \tilde{B} > \tilde{C} > \tilde{A}. \)

**Set6:** \( \tilde{A} = (0.0, 0.2, 0.2, 0.8) \) \( \tilde{B} = (0.0, 0.0, 0.2, 0.1). \)

Step1: \( G_A(x, y) = (0.0, 0.3111) \) and \( G_B(x, y) = (0.3888) \)

\[ R(\tilde{A}) = 0 \text{ and } R(\tilde{B}) = 0, R(\tilde{A}) = R(\tilde{B}) \]

so goto step2

Step2: \( M(\tilde{A}) = M(\tilde{B}) = 0 \), so goto step3

Step3: \( S(\tilde{A}) = 0.32 \text{ and } S(\tilde{B}) = 0.4 \), since \( S(\tilde{A}) < S(\tilde{B}) \) \( \Rightarrow \tilde{A} < \tilde{B} \)

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5. Comparison of the proposed ranking method with the existing methods

In this section, the results of the proposed method are compared with the ranking results of Chen and Sanguansat [15], Xu et al. [22], Cheng [23], Chu and Tsao [7], Yagger [4] and the results are given in Table 1. From Table 1, the observations of the fuzzy numbers are

Set1: \( \tilde{A} = (-0.1,0.1,0.1,0.1;0.4) \) \( \tilde{B} = (0.0,0.0,0.1,0.1;0.4) \)

By the proposed method the ranking order is \( \tilde{A} < \tilde{B} \). From Fig.2, of set1, it can be seen that the result obtained by the proposed method is reliable with human instinct. But according to Chen and Sanguansat [15], Xu et al. [22] and Yagger [4] the ranking order is \( \tilde{A} = \tilde{B} \) and Cheng [23], Chu and Tsao [7] methods are not applicable to rank these fuzzy numbers. From Fig.2, of set1, it is easy to see that the ranking results obtained by the existing methods are unreasonable and are not consistent with human instinct.

Set2: \( \tilde{A} = (-0.2,0.3,0.3,0.1;0.6) \) \( \tilde{B} = (-0.0,0.0,0.3,0.1;0.6) \)

By the proposed method the ranking order is \( \tilde{A} < \tilde{B} \). From Fig.2, of set1, it can be seen that the result obtained by the proposed method is reliable with human instinct. But according to Chen and Sanguansat [15], Xu et al. [22] and Yagger [4] the ranking order is \( \tilde{A} = \tilde{B} \) and Cheng [23], Chu and Tsao [7] methods are not
applicable to rank these fuzzy numbers. From Fig. 2, of set1, it is easy to see that the ranking results obtained by the existing methods are unreasonable and are not consistent with human instinct.

**Set3:** \( \tilde{A} = (-0.2, 0.2, 0.1, 0.1; 0.7) \quad \tilde{B} = (-0.1, -0.1, 0.2; 0.7) \)

By the proposed method the ranking order is \( \tilde{A} > \tilde{B} \). From Fig. 2, of set1, it can be seen that the result obtained by the proposed method is reliable with human instinct. But according to Chen and Sanguansat [15], Xu et al. [22] and Yager [4] the ranking order is \( \tilde{A} = \tilde{B} \) and Cheng [23], Chu and Tsao [7] methods are not applicable to rank these fuzzy numbers. From Fig. 2, of set1, it is easy to see that the ranking results obtained by the existing methods are unreasonable and are not consistent with human instinct.

**Set4:** \( \tilde{A} = (0.0, 0.1, 0.4, 0.2; 0.8) \quad \tilde{B} = (0.0, 0.1, 0.3; 0.8) \)

By the proposed method the ranking order is \( \tilde{A} > \tilde{B} \). From Fig. 2, of set1, it can be seen that the result obtained by the proposed method is reliable with human instinct. But according to Chen and Sanguansat [15], Xu et al. [22] and Yager [4] the ranking order is \( \tilde{A} = \tilde{B} \) and Cheng [23], Chu and Tsao [7] methods are not applicable to rank these fuzzy numbers. From Fig. 2, of set1, it is easy to see that the ranking results obtained by the existing methods are unreasonable and are not consistent with human instinct.

**Set5:** \( \tilde{A} = (-0.3, -0.3, 0.2, 0.2; 1) \quad \tilde{B} = (0.0, 0.0, 0; 1) \quad \tilde{C} = (0.0, 0.0, 0.8) \)

By the proposed method the ranking order is \( \tilde{B} > \tilde{C} > \tilde{A} \). From Fig. 2, of set1, it can be seen that the result obtained by the proposed method is reliable with human instinct. But according to Chen and Sanguansat [15], Xu et al. [22] and Yager [4] the ranking order is \( \tilde{A} = \tilde{B} = \tilde{C} \) and Cheng [23], Chu and Tsao [7] methods are not applicable to rank these fuzzy numbers. From Fig. 2, of set1, it is easy to see that the ranking results obtained by the existing methods are unreasonable and are not consistent with human instinct.

**Set6:** \( \tilde{A} = (0.0, 0.2, 0.2; 0.8) \quad \tilde{B} = (0.0, 0.2, 0.2; 1) \)

By the proposed method the ranking order is \( \tilde{A} < \tilde{B} \). From Fig. 2, of set1, it can be seen that the result obtained by the proposed method is reliable with human instinct. But according to Chen and Sanguansat [15], Xu et al. [22] and Yager [4] the ranking order is \( \tilde{A} = \tilde{B} \) and Cheng [23], Chu and Tsao [7] methods are not applicable to rank these fuzzy numbers. From Fig. 2, of set1, it is easy to see that the ranking results obtained by the existing methods are unreasonable and are not consistent with human instinct.
This paper proposes a new method for ranking generalized LR fuzzy numbers based on centroid of centroids using area, mode, and spread. It ranks fuzzy numbers which is simple and concrete. This method ranks trapezoidal as well as triangular LR fuzzy numbers and their images. This method also ranks crisp numbers which are a special case of fuzzy numbers and it overcame the shortcomings and limitations of the existing methods. The proposed method is simple and easier in calculation and not only gives the satisfactory results to well defined problems, but also gives a correct ranking order and agrees with the human intuition. For future research, the proposed method can be applied in real life problems involving decision-making, clustering, optimization, transportation problems, assignment problems, etc.

### References


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