Edge Detection of Medical Images

Using Markov Basis

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Abstract

Edge detection is an important task in any image processing application. An edge is a boundary between two disjoint regions. To recognize any unusual objects or growths automatically in medical images, proper edge detection is a crucial step for efficient medical image. Medical images edge detection is an important work for object recognition of the human organs. Edge is detected according to trad methods, but they are not so good for noise medical image edge detection. In this paper, a proposed filter is used to detect the edge of the medical image has been the application of the proposed filter on the actual pictures or facts to find the edge of a high-resolution and this filter has been created by Markov Basis and Laplace filter. It has been observed that the proposed edge detection method performs better than Sobel, Prewitt, Roberts and Cannys edge detection algorithms.

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### 1 Introduction

Medical images edge detection is an important work for object recognition of the human organs such as lungs and ribs, and it is an essential pre-processing step in medical image segmentation. There are many methods for edge detection; they can be divided into two categories: spatial domain detection and transformational domain detection. Classical edge detection algorithms are mostly based on spatial domain detection, such as Sobel operator, Laplace operator, Canny operator and so on. The main functions of these operators are high-pass filters. Despite having respective pertinence and characteristics, they are generally sensitive to noise because of involving the direction. So it is difficult to detect complex edge. Edge detection methods based on transformational domain can suppress noise effectively, but such methods involve a large amount of calculation, they can’t meet real-time requirement in many places[1]. The work of the edge detection decides the result of the final processed image. Conventionally, edge is detected according to algorithms like Sobel algorithm, Prewitt algorithm and Laplacian of Gaussian operator, but in theory they belong to the high-pass filtering, which are not fit for noise medical image edge detection because noise and edge belong to the scope of high frequency. [2] In this paper a proposed method is used to detect the edges of the medical images and compared with traditional methods. The application of the proposed methods on the actual pictures or facts to find the edge of a high-resolution. The proposed method based on proposed filter has been created by the Markov basis and Laplace filter.

### 2 Related Works

- **Mahesh Kumar and Sukhwinder Singh**, morphological edge detectors also have been proposed but they are based on fixed structure elements. Basic mathematical morphological theory and operations are introduced at first, and then a novel mathematical morphological edge detection algorithm based on multi shape structure, is proposed to detect the edge of lungs CT image with salt-and-pepper noise. The experimental results show that the proposed algorithm is more efficient for medical image de-noising and edge detection than the usually used template-
Based edge detection algorithms and general morphological edge detection algorithms [1].

• **K. Somasundaram and K. Ezhilarasan**, MRI head scans are used to visualize the internal structure of human head in 3-Dimension. The brain segmentation from MRI head scan is a major work in head scan analysis. In the proposed method we make use of 32 fuzzy logic for edge detection. It is less in computational complexity in searching for edges when compared to Sobel and Canny edge detectors. Application of the proposed method on several MRI scans show that it produces sharp and clear edges that can be used for segmenting brain portions in MRI of human head scans [3].

• **Mr. Shital S. Agrawal and Prof. Dr. S. R. Gupta**, although edge information is the main clue in image segmentation, it cant get a better result in analysis the content of images without combining other information. The segmentation of brain tissue in the magnetic resonance imaging (MRI) is very important for detecting the existence and outlines of tumors. An algorithm about segmentation based on the symmetry character of brain MRI image is presented. Our goal is to detect the position and boundary of tumors automatically. Experiments were conducted on real pictures, and the results show that the algorithm is flexible and convenient [4].

### 3 The Markov Basis

In this section, we describle the markov basis as in A. Takemura and S. Aoki (2004).

Let \( n \) be the number of elements in a finite set \( I \). An element \( i \in I \) is called a cell. It is often multi-index \( i_1...i_m \). A non-negative integer \( x_i \in N \) denotes the frequency of a cell \( i \), \( N = \{0, 1, 2, 3, ...\} \). The set of frequencies \( x = \{x_i\}_{i \in I} \) is called a contingency table. A contingency table \( x = \{x_i\}_{i \in I} \) can be written as \( n \)-dimensional column vector of non-negative integers in \( N^n \). Let \( Z \) be the set of integers and let \( a_j \in Z^v, j = 1, ..., v \), denote fixed column vectors consisting of integers.

The \( v \)-dimensional column vector \( t = (t_1, t_2, ..., t_v)' \) is defined as \( t_j = a_j'x \), where \(^'\) denotes the transpose of a vector or a matrix. If \( A = [a_1...a_v]' \) is \( v \times n \) matrix with jth row \( a_j' \) then \( t = Ax \). The set \( A^{-1}(t) = \{x \in N^n : Ax = t\} \)
(t-fibers) is the set of contingency tables $x$ is for agiven . is considered for performing similar tests. If $\sim$ is the relation $x_1 \sim x_2$ if and only if $x_1 \cdot x_2$ belongs to the kernel of $A$, $\text{ker}(A)$, this relation is an equivalence relation on $N^n$ and the set of t-fibers is the set of its equivalence classes.

An n-dimensional column vector $Z = \{z_i\} \in z^n$ is called amove if $z \in \text{ker}(A)$, i.e., $AZ = o$. A set of finite moves $M$ is called Markov basis if for all $t$, $A^{-1}[t]$ constitutes one $B$ equivalence class. [6]

In[5] the authors found a Markov basis $B$ for $\frac{n^2 - 3n}{3} \times 3 \times \frac{n}{3}$ contingency table, $n$ is a multiple of 3.

They proved the number of elements in $B$ equals to $\frac{n^2 - 3n}{3}$. If $n = 9$ then the number of elements in $B$ is $\frac{9^2 - 3 \times 9}{3} = 18$ these elements are

$$Z_1 = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, Z_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \end{bmatrix}, Z_3 = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Z_4 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ -1 & 0 & 1 \end{bmatrix}, Z_5 = \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, Z_6 = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$Z_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}, Z_8 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -1 & 1 & 0 \end{bmatrix}, Z_9 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$Z_{10} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, Z_{11} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix}, Z_{12} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Z_{13} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}, Z_{14} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}, Z_{15} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

$$Z_{16} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}, Z_{17} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix}, Z_{18} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

### 4 Proposed Method for Edge Detection

For $n = 3$ we will use some elements of a Markov basis $B$ and Laplace filters to generate new filter. The following steps illustrate this.
A. Filter Generation

Step 1
We use the Markov basis elements $Z_1, Z_4, Z_7$, find $Z'_1 = Z_1 + Z_4 + Z_7$

we obtain the following filter :

$$Z'_1(x, y) = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & -2 \\ -1 & -1 & 2 \end{bmatrix}$$

Step 2
Filters collect Markov included in the step (1) with Laplace filters To obtain the following filter :

$$Z''_1(x, y) = L(x, y) + Z'_1(x, y), \quad Z' \in M$$

where

$$L(x, y) = \begin{bmatrix} -2 & -3 & -2 \\ -3 & 20 & -3 \\ -2 & -3 & -2 \end{bmatrix}$$

we get on the filter following :

$$Z''_1(x, y) = \begin{bmatrix} -1 & -4 & -2 \\ -3 & 22 & -5 \\ -2 & -4 & 0 \end{bmatrix}$$

Step 3
We will add one value to the filter center ($Z''_1$) for filter ($Z'_1$):

$$Z_1^*(x, y) = Z''_1(x, y) + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then

$$Z_1^*(x, y) = \begin{bmatrix} -1 & -4 & -2 \\ -3 & 23 & -5 \\ -2 & -4 & 0 \end{bmatrix}$$

Step 4
Filters collect included in the step (3) with a special filter (F) To obtain the following filter ($Z_2^*$):

$$Z_2^*(x, y) = Z_1^*(x, y) + F(x, y)$$
Then

\[ Z_2^* = \begin{bmatrix}
-27 & -5 & -26 \\
-4 & 124 & -6 \\
-27 & -4 & -24 \\
\end{bmatrix} \]

B. Edge Detection

To detection the edges in an image, we use the following steps:

**Step 1:**
Let \( f(x, y) \) be color image with dimension \( m \times n \times 3 \) and gray image with dimension \( m \times n \).

**Step 2**
The convolution process to new filter 3 x 3. Let an image matrix \( f(x, y) \) of dimension \( m \times n \).

**Step 3:**
Smoothing image by using Gaussian or median filter.

**Step 4:**
We process gray image to get gray image with edge and each layer of color image (Red (R), Green (G) and Blue (B)) as matrix to detect the edge, and later we recombine them to get color image with edge.

**Step 5:**
The input filter \( Z_2^* \) which previously created, with dimension 3 x 3.

**Step 6:**
Combine the three image layers \( (R_1, G_1, B_1) \) to reshape the color image.

**Step 7:**
The resulting image is edge detection either color image or gray image.

5 Experimental Result

Comparisons for edge detection techniques are performed to demonstrate the superiority of proposed method in edge detection to medical image and from this the techniques are Roberts, Sobel, Prewitt edge detector, Kirsch and canny edge detector and Other from techniques the used to select edge the image medical.
Figure 1: original medical image compared with the traditional edge detection techniques are Roberts, Sobel, Prewitt, Kirsch, Robinson, Marr-Hildreth, LoG, Canny, Fuzzy, Laplacian and also compared with Suggested method that perform function edge detection for image medical.
Figure 2: Testing the original color image with of the other edge detection techniques and suggested method.
6 Conclusion

The proposed method is an improved version to edge detection for medical image. The edges generated by proposed method have less false edges. Classification of regions based on local intensity characteristics leads to prominent edges that isolate different anatomical regions as separate objects for further consideration. Presence of noise may hinder the process of edge determination. The most significant achievement of this proposed method is that it is applicable to a range of medical imaging. The edge image is devoid of any noise that is observed in other known methods of edge detection. Experimentation results showed accuracy high to edge detection to most image.

References


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