Development of Space Time Model with Exogenous Variable by Using Transfer Function Model Approach on the Rice Price Data

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Abstract

In modeling sometimes other influence variables can improve the goodness of a model and those variables are called exogenous variables. GSTARMA modeling by involving exogenous variables can use several approaches that can be done to include the influence of exogenous variables within the model. One approach to handle correlated series in exogenous variables but has dynamic model is transfer function model. In this study, the GSTARMA model with exogenous variables by using transfer function approach is called GSTARMA-X Transfer Function model.
The GSTARMA-X Transfer Function model and space-time cross-correlation function have been established and applied to the data of medium rice price on the market with the price of dry grain on milling level in six province of Java island, Indonesia. The GSTARMA-X Transfer Function model could be used well to space-time data with exogenous variables. The space-time cross-correlation function could also help in identifying order of GSTARMA-X Transfer Function model. By using the largest value of $R^2$ and the smallest value of RMSEP from forecasting 12 months ahead, the best GSTARMA-X Transfer Function model for data of medium rice price on the market with the price of dry grain on milling level in six provinces of Java Islands, Indonesia had order $b = 0$, $r = 2$, $s = 0$, $l = 1$, and $m = 1$.

**Keywords:** GSTARMA model, transfer function model, cross-correlation function

### 1 Introduction

The current data continues to evolve with the times. Data is not only containing information about a particular case, but is also associated with other data. One of which is data that contains information about location and time series. Data that contains information about location and time is called space-time data. Space-time data modeling has been widely practiced by Pfeifer & Deutsch [7, 8], Borovkova et al. [1], Ruchjana [9], Giacinto [5], and Min & Hu [6].

Pfeifer & Deutsch research [7, 8] have been modeled space-time data through STARMA (Space-Time Autoregressive Moving Average) model. STARMA modeling is considered to have deficiencies in capturing the heterogeneity of the characteristics of each location. That model has the same parameter value for same time lag but different location. In fact each location usually has different characteristics so that when the condition is ignored it will eliminate the effect of the diversity of each location within the model. Models that can overcome the shortcomings of the STARMA model are known as GSTARMA (Generalized Space-Time Autoregressive Moving Average) model. The first GSTARMA model was introduced involving only autoregressive effects so it became known as the GSTAR model by Borovkova et al. [1]. GSTAR modeling is also done by Ruchjana [9] and Giacinto [5]. Then Min and Hu [6] began to incorporate the moving average influence on the GSTAR model so it became known as the GSTARMA model. STARMA and GSTARMA modeling are space-time modeling that involves only one variable for all locations.

In modeling, other influence variables can improve the goodness of a model, because it could be an event influenced by other events. A variable affecting other variables is called an exogenous variable. Spatial modeling involving exogenous variables on the STARMA model was once performed by Stoffer [10]. GSTARMA modeling by involving exogenous variables can also be performed as STARMA modeling with exogenous variables. There are several approaches that can be done
to include the influence of exogenous variables within the model. The first is to directly insert the exogenous variables into the model. Since the exogenous variables of each location follow a time series process, the exogenous variables will affect response variables during the same period until some previous period or known as a time lag. The time lag identification of the exogenous variables influences the response variables on this technique based on the correlation value of each time lag.

If the exogenous variable is a time series process, it contains autocorrelation. The influence of autocorrelation in an exogenous variable would be confusing in the process of identifying the relationship between an exogenous variable with its response variable, since the correlation between the exogenous variables and the response variable becomes uncertain. If the relationship between the response variables and the exogenous variables is uncertain then it will also affect the value of the parameters connecting the two variables. However, such modeling can be done during proper identification that does not rely on correlation but also knowledge of the data. The second step can also be done by eliminating the effect of autocorrelation in the modeling of the two variables. This technique exists in transfer function model. Transfer function model in time series modeling is also known as ARIMAX model introduced by Box et al. [3]. The transfer function model so far is effective to dynamically model of exogenous variable with response variable.

Based on that background, this paper will discuss about the GSTARMA model with exogenous variable by using transfer function model’s approach. The transfer function model’s approach will be included model development and identification order’s model. Model performance will be applied in the case of rice price in the market in Java, Indonesia as a response variable or output series and the price of dry grain in Java, Indonesia as an exogenous variable or input series.

2 GSTARMA Model

Suppose \( z_{i(t)} \) is an observation which follow stochastic process of \( i^{th} \) location and \( T \) time periods, where \( i = 1, 2, ..., N \). The GSTAR model by Borovkova et al. [1] by adding moving average affect can be showed as follows

\[
z_{i(t)} = \sum_{k=1}^{p} \lambda_k \phi_{i,kl}^{(l)} z_{i(t-k)} - \sum_{k=1}^{q} \gamma_k \theta_{i,kl}^{(l)} a_{i(t-k)} + a_{i(t)} \tag{1}
\]

The matrix form of GSTARMA model in equation (1) can also defined as

\[
z_t = \sum_{k=1}^{p} \lambda_k \Phi_{kl}^{(l)} z_{(t-k)} - \sum_{k=1}^{q} \gamma_k \Theta_{kl}^{(l)} a_{(t-k)} + a_{(t)} \tag{2}
\]

where \( \phi_{i,kl} \) is a autoregressive parameter of \( i^{th} \) location, \( k^{th} \) time order, and \( l^{th} \) spatial order; \( \theta_{i,kl} \) is a moving average parameter of \( i^{th} \) location, \( k^{th} \) time order.
order, and $l^{th}$ spatial order. In equation (2), $\Phi_{kl}$ is a $N \times N$ diagonal matrix of autoregressive parameter for each location and $\Theta_{kl}$ is a $N \times N$ diagonal matrix of moving average parameter for each location. Model as showed in equation (1) and (2) is called GSTARMA ($p_{\lambda_1, \lambda_2, \ldots, \lambda_p}, q_{\gamma_1, \gamma_2, \ldots, \gamma_q}$) model.

3 Transfer Function Model

Suppose $Z_{it}$ and $X_{it}$ are stationary random variables which follow stochastic processes for time $t = 0, \pm 1, \pm 2, \ldots$ and $i^{th}$ location, where $i = 1, 2, \ldots, N$. We assume that $X_{it}$ influence $Z_{it}$, then the dynamic relationship of two variables can be defined as

$$Z_{it} = v_i(B)X_{it} + \epsilon_{it} \tag{3}$$

where $v_i(B)$ is the impulse response weights. Model (3) is called transfer function model by Box et al. [3]. The impulse response weights $v_i(B)$ are defined

$$v_i(B) = \frac{\omega_{is}(B)B^{b_i}}{\delta_{ir}(B)} \tag{4}$$

where $b_i$ is a delay parameter when the input series start to influence the output series for $i^{th}$ location; $B$ is a backshift operator; $\omega_{is}(B)$ are parameter for $s$ order in $i^{th}$ location which represent the length of the input series affecting the output series at the $i^{th}$ location; $\delta_{ir}(B)$ are parameter for $r$ order in $i^{th}$ location that represent autoregressive in output series for $i^{th}$ location; and $\epsilon_{it}$ is an error transfer function model for $i^{th}$ location which follows $\epsilon_{it} \sim iid N(0, \sigma_{\epsilon}^2)$.

If the observation of $Z_{it}$ is $z_{it}$ and the observation of $X_{it}$ is $x_{it}$, then equation (3) can be written as below

$$z_{it} = v_i(B)x_{it} + \epsilon_{it} \tag{4}$$

and can be also written as

$$z_{it} = \frac{\omega_{is}(B)B^{b_i}}{\delta_{ir}(B)}x_{it} + \epsilon_{it} \tag{5}$$

The linear equation of (5) is written as

$$\left(1 - \sum_{k=1}^{r} \delta_{ik}B^k\right)z_{it} = \left(\omega_{i0} - \sum_{k=1}^{s} \omega_{ik}B^k\right)x_{i(t-b)} + \left(1 - \sum_{k=1}^{r} \delta_{ik}B^k\right)\epsilon_{it} \tag{6}$$

Suppose $(1 - \sum_{k=1}^{r} \delta_{ik}B^k)\epsilon_{it} = \epsilon_{it}^*$, then equation (6) is written

$$z_{it} = \sum_{k=1}^{r} \delta_{ik}B^kz_{it} + \omega_{i0}x_{i(t-b)} - \sum_{k=1}^{s} \omega_{ik}B^kx_{i(t-b)} + \epsilon_{it}^* \tag{7}$$
4 Data and Method

Data
The data used the price of medium-grade rice on the market in January 2007 until December 2014 by as output series, and the price of dry milled grain at the milling level in January 2007 until December 2014 as exogenous variable or input series. The data was taken from six provinces of Java Island, Indonesia, i.e. DKI Jakarta, West Java, Central Java, DI Yogyakarta, East Java, and Banten.

Method
GSTARMA modeling with exogenous variables through the transfer function model could be done by several stages as follows:
Step 1 : Formation of GSTARMA model with exogenous variable through transfer function approach.
Step 2 : Formation of space-time cross correlation function.
Step 3 : Application model to real data of the price of medium-grade rice on the market and the price of dry milled grain at the milling level.

5 GSTARMA with X by Transfer Function Model

Suppose there are $N$ vary location in series $z_{it}$ and $x_{it}$, then those series need spatial lag operator with $l$ order as below:

$$L^{(l)}z_{i(t)} = \sum_{j=1}^{N} w_{ij}^{(l)} z_{i(t)}$$

and

$$L^{(l)}x_{i(t)} = \sum_{j=1}^{N} w_{ij}^{(l)} x_{i(t)}$$

with $w_{ij}^{(l)}$ is a set of spatial weight with order $l$, where

$$\sum_{j=1}^{N} w_{ij}^{(l)} = 1$$

For all $i$, $w_{ij}^{(l)}$ is not zero value if $i^{th}$ and $j^{th}$ are neighbors at $l^{th}$ order. The spatial lag operator of $z_{i(t)}$ can be defined in the matrix form as follow:

$$L^{(0)}z_t = W^{(0)}z_t = I_N z_t$$

and

$$L^{(l)}z_t = W^{(l)}z_t$$

for $l > 0$.

The input series $x_{i(t)}$ with order $l$ is defined

$$L^{(0)}x_t = W^{(0)}x_t = I_N x_t$$

and

$$L^{(l)}x_t = W^{(l)}x_t$$

for $l > 0$.  

The transfer function model (7) with spatial lag operator for \(i^{th}\) is defined as follows

\[
z_{i(t)} = \sum_{k=1}^{r} \sum_{l=0}^{r} \delta_{ikt} \Omega_{0l} \Omega_{kl} L(t) z_{i(t-k)} + \sum_{l=0}^{s} \omega_{i0l} L(t) x_{i(t-b)} - \sum_{k=1}^{s} \sum_{l=0}^{s} \omega_{ikl} L(t) x_{i(t-b-k)} + \epsilon_{i(t)}
\]  

where \(\delta_{ikt}\) is an output series autoregressive for \(i^{th}\) location, \(k^{th}\) time lag, and \(l^{th}\) spatial lag; \(L(t)\) is a \(l^{th}\) spatial lag; \(\omega_{i0l}\) is a parameter that is representing the length series \(x_{i(t-b)}\) affecting series \(z_{i(t)}\) at the same period time for \(i^{th}\) location; zero time lag, and \(l^{th}\) spatial lag; \(\omega_{ikl}\) is a parameter that is representing the length of series \(x_{i(t-b)}\) affecting series \(z_{i(t)}\) for \(i^{th}\) location, zero time lag, and \(l^{th}\) spatial lag; \(b\) represents time order when the input series start to affecting the output series; \(\xi_{k}\) is an autoregressive spatial order of \(z_{i(t)}\) at the \(k^{th}\) time lag; \(\zeta_{0}\) is a \(b\) spatial order at zero time lag; \(\zeta_{k}\) is a spatial order at \(k^{th}\) time lag; and \(\epsilon_{i(t)}\) is a noise series at the \(i^{th}\) location and \(t = 1, ..., T\). According to definition of spatial lag operator, equation (8) can be written

\[
z_{i(t)} = \sum_{k=1}^{r} \sum_{l=0}^{r} \delta_{ikt} W_{ij}^{(l)} z_{i(t-k)} + \sum_{l=0}^{s} \omega_{i0l} W_{ij}^{(l)} x_{i(t-b)} - \sum_{k=1}^{s} \sum_{l=0}^{s} \omega_{ikl} W_{ij}^{(l)} x_{i(t-b-k)} + \epsilon_{i(t)}
\]  

(9)

The matrix form of (9) can be written

\[
z_{t} = \sum_{k=1}^{r} \sum_{l=0}^{r} \Delta_{kt} W_{t}^{(l)} z_{t-k} + \sum_{l=0}^{s} \Omega_{0l} W_{t}^{(l)} x_{t-b} - \sum_{k=1}^{s} \sum_{l=0}^{s} \Omega_{kl} W_{t}^{(l)} x_{t-b-k} + \epsilon_{t}^{*}
\]  

(10)

where \(\Delta_{kt}\), \(\Omega_{0l}\), and \(\Omega_{kl}\) are \(N \times N\) diagonal matrix of parameter model; \(z_{t} = (z_{1(t)}, z_{2(t)}, ..., z_{N(t)})^{'}\); \(x_{t} = (x_{1(t)}, x_{2(t)}, ..., x_{N(t)})^{'}\); and \(\epsilon_{t}^{*} = (\epsilon_{1(t)}^{*}, \epsilon_{2(t)}^{*}, ..., \epsilon_{N(t)}^{*})^{'}\). Model in equation (9) or (10) is called GSTARMA-X Transfer Function Model with order \(b, r, s\) is determined the order \(b, r, s\) and spatial lag of GSTARMA-X Transfer Function Model by modified the cross correlation function for space-time data. Suppose the space-time data following the stochastic process \(x_{it}\) and \(z_{it}\) for \(i = 0, \pm 1, \pm 2, ...\) and \(i\) denotes the location where \(i = 1, 2, ..., N\), it says \(x_{it}\) and \(z_{it}\) are single stationary and stationary process.

**Space-Time Cross-Correlation Function**

The space-time cross-correlation function is used to measure the strength and direction of the relationship between two random variables of space-time data. In the transfer function model, the cross-correlation function is used to determine the order \(b, r, s\) of the model. It can be also determined the order \(b, r, s\) and spatial lag of GSTARMA-X Transfer Function Model by modified the cross correlation function for space-time data. Suppose the space-time data following the stochastic process \(x_{it}\) and \(z_{it}\) for \(i = 0, \pm 1, \pm 2, ...\) and \(i\) denotes the location where \(i = 1, 2, ..., N\), it says \(x_{it}\) and \(z_{it}\) are single stationary and stationary process.
variables. The cross-covariance function between \( x_{it} \) and \( z_{it} \) is expressed as \( \text{Cov}(x_{it}, z_{it}) \). If \( x_{it} \) has a lag of \( k \) on \( z_{it} \), then the cross-covariance function can be written as \( \text{Cov}(x_{it}, z_{i(t+k)}) = \gamma_{i(xz)}(k) \).

The space time cross-correlation function is a function that describes the correlation between two variables at the spatial and time lag points. If cross-correlation between two variables with different locations is involved then the results give many possible combinations. It will make it difficult to identify the relationship between the two series correctly. Restriction of points that have a delay between space and time in the same location will make it easier to know the relationship pattern between the two series. If the cross-covariance uses spatial lags then the cross-covariance between \( x_{i(t)} \) and \( z_{i(t)} \) and between the \( l^{th} \) and \( m^{th} \) spatial lag in the same location in the \( k \)-time lag are expressed as follows

\[
\gamma_{x'z^m}(k) = E \left( \sum_{i=1}^{N} \frac{L^{(l)}(x_{i(t)} - \bar{x}_i)L^{(m)}(z_{i(t+k)} - \bar{z}_i)}{N} \right)
\]

(11)

And the estimator of \( \gamma_{x'z^m}(k) \) is defined as

\[
\hat{\gamma}_{x'z^m}(k) = \begin{cases} 
\sum_{i=1}^{N} \sum_{l=1}^{T-k} \frac{L^{(l)}(x_{i(t)} - \bar{x}_i)L^{(m)}(z_{i(t+k)} - \bar{z}_i)}{N(T-k)}, & k \geq 0 \\
\sum_{i=1}^{N} \sum_{l=1}^{T-k} \frac{L^{(l)}(x_{i(t)} - \bar{x}_i)L^{(m)}(z_{i(t+k)} - \bar{z}_i)}{N(T-k)}, & k < 0
\end{cases}
\]

(12)

If we assume \( E(W^{(l)}x_t) = W^{(l)}\mu_x = 0 \) and \( E(W^{(m)}z_{t+k}) = W^{(m)}\mu_z = 0 \), the equation (11) can be defined as vector form below,

\[
\gamma_{x'z^m}(k) = E \left( \frac{[W^{(l)}x_t]'[W^{(m)}z_{t+k}]}{N} \right)
\]

(13)

Because \( \gamma_{x'z^m}(k) \) are scalar, then the space-time cross-covariance between \( x_{i(t)} \) and \( z_{i(t)} \) at the \( l^{th} \) and \( m^{th} \) spatial lag, and \( k^{th} \) time lag can be written as :

\[
\gamma_{x'z^m}(k) = \frac{1}{N} E \left( tr([W^{(l)}x_t]'[W^{(m)}z_{t+k}]) \right)
\]

(14)

\[
\gamma_{x'z^m}(k) = \frac{1}{N} tr \left( W^{(m)}W^{(l)}\Gamma_{xz}(k) \right)
\]

(15)

The estimator of matrix \( \Gamma_{x'z^m}(k) \) can be defined as

\[
\hat{\Gamma}_{xz}(k) = \frac{\sum_{t=1}^{T-k} x_t, z_{t+k}}{T-k}
\]

(16)

and the estimator values of \( \gamma_{x'z^m}(k) \) can be defined as
The properties of space-time cross-covariance are

\[ \gamma_{x'z}^m(k) = \gamma_{z'm}x(-k) \] (18)

By using definition that \((AB) = tr(BA)\) and \(tr(A) = tr(A')\), then \(\Gamma_{xz}'(k) = \Gamma_{zx}(-k)\). The properties in equation (18) can be defined as

\[ \gamma_{x'z}^m(k) = \frac{1}{N} tr(W_l^{(m)}'W_l^{(i)}\Gamma_{xz}(k)) = \frac{1}{N} tr(\Gamma_{xz}'(k)W_l^{(i)}W_l^{(m)}) \] (19)

and can be also defined as:

\[ \frac{1}{N} tr(W_l^{(i)}'W_l^{(m)}\Gamma_{xz}'(k)) = \frac{1}{N} tr(W_l^{(i)}'W_l^{(m)}\Gamma_{zx}(-k)) = \gamma_{z'm}x(-k) \] (20)

According to the space-time cross-covariance definition, then the space-time cross-correlation between \(x_{i(t)}\) and \(z_{j(t+k)}\) at the \(l^{th}\) and \(m^{th}\) spatial lag at the \(k^{th}\) time lag is defined as below:

\[ \rho_{x'z}^m(k) = \frac{\gamma_{x'z}^m(k)}{[\gamma_{x'z}^m(0)\gamma_{z'm}z(0)]^{1/2}} \] (21)

The estimator of \(\rho_{x'z}^m(k)\) is defined as below:

\[ \hat{\rho}_{x'z}^m(k) = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T-k} L_l^{(i)}x_{i(t)}L_l^{(m)}z_{i(t+k)}}{\left[\sum_{i=1}^{N} \sum_{t=1}^{T-k} (L_l^{(i)}x_{i(t)})^2 \sum_{i=1}^{N} \sum_{t=1}^{T-k} (L_l^{(m)}z_{i(t)})^2\right]^{1/2}} \] (22)

for \(k = 0, \pm 1, \pm 2, \ldots\). The properties of space-time cross-correlation is

\[ \rho_{x'z}^m(k) = \rho_{z'm}x(-k) \] (23)

6 Application Data

Analysis of the GSTARMA-X Transfer Function Model used information from modeling the transfer function for a single location. The first stage that needs to be done before starting the analysis process was making sure that the input series should be uncorrelated. In order to produce uncorrelated input series, it would require pre-whitening processes as performed on the transfer function modeling. The pre-whitening process used in this analysis is a pre-whitening process for each location as showed in table 1.
Table 1 Pre-whitening processes for input series

<table>
<thead>
<tr>
<th>No.</th>
<th>Location</th>
<th>Pre-whitening Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DKI Jakarta</td>
<td>( u_{1t} = \frac{(1 + 0.27B^2)(1 - B)}{(1 + 0.41B^{12})} x_{1t} - 16.92 )</td>
</tr>
<tr>
<td>2</td>
<td>West Java</td>
<td>( u_{2t} = \frac{(1 + 0.27B^2)(1 - B)}{(1 + 0.41B^{12})} x_{2t} - 16.92 )</td>
</tr>
<tr>
<td>3</td>
<td>Central Java</td>
<td>( u_{3t} = (1 + 0.31B^4 + 0.26B^5)(1 - B)x_{3t} - 20.18 )</td>
</tr>
<tr>
<td>4</td>
<td>D.I. Yogyakarta</td>
<td>( u_{4t} = \frac{(1 - 0.65B^{7})}{(1 - 0.83B^{10})} x_{4t} - 26.04 )</td>
</tr>
<tr>
<td>5</td>
<td>East Java</td>
<td>( u_{5t} = (1 + 0.45B^2 + 0.24B^7)(1 - B)x_{5t} - 23.18 )</td>
</tr>
<tr>
<td>6</td>
<td>Banten</td>
<td>( u_{6t} = (1 + 0.23B^2 + 0.20B^3)(1 - B)x_{6t} - 21.83 )</td>
</tr>
</tbody>
</table>

Figure 1. Six province in Java Island, Indonesia

The location weighted matrix \( W \) in this study used the location weighting of Queen Contiguity (\( W_q \)) based on figure 1. Figure 1 showed that number 1 was province of DKI Jakarta, number 2 was province of West Java, number 3 was province of Central Java, number 4 was province of D.I. Yogyakarta, number 5 was province of East Java, and number 6 was province of Banten. The map that showed on the figure 1 produced spatial relationship between locations as described in table 2.

Table 2 Spatial relationship for each spatial order

<table>
<thead>
<tr>
<th>Spatial Order</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2,6</td>
<td>3</td>
<td>4,5</td>
</tr>
<tr>
<td>2</td>
<td>1,3,6</td>
<td>4,5</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2,4,5</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1,6</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>2,4</td>
<td>1,6</td>
</tr>
<tr>
<td>6</td>
<td>1,2</td>
<td>3</td>
<td>4,5</td>
</tr>
</tbody>
</table>
According to the relationship as showed on table 2, then the location weighted matrix Queen Contiguity could be showed as:

\[
W_q^{(1)} = \begin{bmatrix}
0 & 0.5 & 0 & 0 & 0 & 0.5 \\
0.3 & 0 & 0.3 & 0 & 0 & 0.3 \\
0 & 0.3 & 0 & 0.3 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0.5 & 0.5 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Identification of order model used information of weighted matrix as showed in equation (23) to the space-time cross correlation function. The space-time cross-correlation function between input series and output series was showed on table 4.

<table>
<thead>
<tr>
<th>Spatial lag ((l))</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time lag ((k))</td>
<td>0</td>
<td>0.3486</td>
<td>0.3412</td>
<td>0.2951</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.2446</td>
<td>0.2211</td>
<td>0.2134</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.1011</td>
<td>0.1617</td>
<td>0.1282</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0208</td>
<td>-0.0279</td>
<td>0.0414</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-0.0265</td>
<td>-0.0309</td>
<td>-0.0343</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>-0.0937</td>
<td>-0.0648</td>
<td>-0.0479</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>-0.0210</td>
<td>-0.0150</td>
<td>0.0233</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>-0.1131</td>
<td>-0.0649</td>
<td>-0.0784</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-0.0970</td>
<td>-0.1176</td>
<td>-0.0823</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>-0.0665</td>
<td>-0.0308</td>
<td>-0.0954</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>-0.1101</td>
<td>-0.1438</td>
<td>-0.0849</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>0.0298</td>
<td>-0.0088</td>
<td>-0.0599</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>0.1285</td>
<td>0.0952</td>
<td>0.1279</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>0.2058</td>
<td>0.2223</td>
<td>0.1954</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>0.0644</td>
<td>0.0892</td>
<td>0.0921</td>
</tr>
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<td>0.0041</td>
<td>0.0542</td>
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<tr>
<td></td>
<td>16</td>
<td>-0.0568</td>
<td>-0.0220</td>
<td>-0.0475</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>-0.0544</td>
<td>-0.0811</td>
<td>-0.0307</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>-0.0331</td>
<td>-0.0377</td>
<td>-0.0521</td>
</tr>
<tr>
<td></td>
<td>19</td>
<td>0.0127</td>
<td>0.0040</td>
<td>0.0279</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>-0.0385</td>
<td>-0.0425</td>
<td>-0.0458</td>
</tr>
</tbody>
</table>

Figure 2 showed the space-time cross-correlation function for each spatial order \(l\) and described that the spatial lag was highest at spatial order \(l = 0\), then decreased up to \(l = 3\). At \(l = 4\) showed that it had no cross-correlation value anymore because neighborliness in the fourth order was gone. Based on the cross-correlation function pattern, then in the next analysis would use spatial order \(l = 0\) and \(l = 1\).
Development of space time model with exogenous variable

Figure 2. Space-time cross-correlation function

Time lag indicated that the tendency of cross correlation pattern had exponential or sine wave since time lag equal to zero. The pattern showed that data had order \( r = 1 \) or \( r = 2 \). The cross-correlation values tend to decrease after the 0\(^{th}\) lag. It could illustrate the order \( b = 0 \). At the 12\(^{th}\) lag, the cross-correlation function increased and decreased after the 13\(^{th}\) lag. Based on these descriptions, it could be estimated that the order \( s = 0 \) or \( s = 1 \).

Several alleged models based on space-time cross-correlation patterns were then analyzed and forecasting results of those models for 12 months ahead were taken that had largest of determination coefficient \((R^2)\) values and the smallest of root mean square error of prediction (RMSEP) values.

Table 5 The alternative best model with \( R^2 \) and RMSEP values

<table>
<thead>
<tr>
<th>Model ( b r s l m )</th>
<th>Locations</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( R^2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 2 0,1 1</td>
<td>0.9880</td>
<td>0.9870</td>
</tr>
<tr>
<td>0 (1 12) I, I I I</td>
<td>0.9867</td>
<td>0.9823</td>
</tr>
<tr>
<td>0 (12) (1 12), I I</td>
<td>0.9846</td>
<td>0.9797</td>
</tr>
<tr>
<td>( RMSEP )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 2 0,1 1</td>
<td>157.9370</td>
<td>155.4368</td>
</tr>
<tr>
<td>0 (1 12) I, I I I</td>
<td>155.3528</td>
<td>168.0399</td>
</tr>
<tr>
<td>0 (12) (1 12), I I</td>
<td>167.1233</td>
<td>179.8955</td>
</tr>
</tbody>
</table>

Based on table 5, the best order of GSTARMA-X transfer function model had \( b = 0 \), \( r = 2 \), \( s = 0 \), \( l = 1 \), and \( m = 1 \). The best model of GSTARMA-X transfer function model could be written as:
\[
\begin{bmatrix}
Z_{1t} \\
Z_{2t} \\
Z_{3t} \\
Z_{4t} \\
Z_{5t} \\
Z_{6t}
\end{bmatrix} = \begin{bmatrix}
39.333 \\
32.787 \\
25.611 \\
14.542 \\
29.481 \\
39.700
\end{bmatrix} + \begin{bmatrix}
0.093 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.080 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.211 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.011 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.036 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.007
\end{bmatrix} \begin{bmatrix}
Z_{1(t-1)} \\
Z_{2(t-1)} \\
Z_{3(t-1)} \\
Z_{4(t-1)} \\
Z_{5(t-1)} \\
Z_{6(t-1)}
\end{bmatrix} + \begin{bmatrix}
0.390 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.579 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.185 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.413 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.282 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.539
\end{bmatrix} \begin{bmatrix}
W_{q1} \\
W_{q2} \\
W_{q3} \\
W_{q4} \\
W_{q5} \\
W_{q6}
\end{bmatrix} + \begin{bmatrix}
-0.161 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.105 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.174 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.153 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.037 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.023
\end{bmatrix} \begin{bmatrix}
Z_{1(t-2)} \\
Z_{2(t-2)} \\
Z_{3(t-2)} \\
Z_{4(t-2)} \\
Z_{5(t-2)} \\
Z_{6(t-2)}
\end{bmatrix} + \begin{bmatrix}
-0.001 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.244 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.116 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.102 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.092 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.127
\end{bmatrix} \begin{bmatrix}
W_{q1} \\
W_{q2} \\
W_{q3} \\
W_{q4} \\
W_{q5} \\
W_{q6}
\end{bmatrix} \\
\begin{bmatrix}
0.115 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.155 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.070 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.394 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.203 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.134
\end{bmatrix} \begin{bmatrix}
X_{1} \\
X_{2} \\
X_{3} \\
X_{4} \\
X_{5} \\
X_{6}
\end{bmatrix} + \begin{bmatrix}
0.024 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.220 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.419 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.139 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.106 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.462
\end{bmatrix} \begin{bmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t} \\
\varepsilon_{3t} \\
\varepsilon_{4t} \\
\varepsilon_{5t} \\
\varepsilon_{6t}
\end{bmatrix}
\]

7 Conclusion

GSTARMA-X Transfer Function model could be used well to space-time data with exogenous variables. Different parameter values for each location could capture the heterogeneity of the characteristics of each location. The space-time cross-correlation function could help in identifying order of GSTARMA-X Transfer Function model. By using the largest value of $R^2$ and the smallest value
of RMSEP from forecasting 12 months ahead, the best GSTARMA-X Transfer Function model for data of medium rice price on the market with the price of dry grain on milling level in six provinces of Java Islands, Indonesia had order $b = 0$, $r = 2$, $s = 0$, $l = 1$, and $m = 1$.

References


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