Formal Design of Structure Process in Machining Parts

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Abstract

In article the theoretical aspects connected with selection of complete sets of technological bases for orientation semi-finished product at processing of details are considered. Simulation is based on a comprehensive consideration of the complete set of dimensional relationships between the surfaces of parts for all degrees of freedom simultaneously. The conditions of a formal search of technological bases for preparation of orientation, as well as the sequence of processing the workpiece surfaces are described.

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1 Introduction

The procedure of design of technological processes is quite laborious process. To date, the establishment of technological processes is not a formalized task, it is creative work for technologist. This work is based on previous experience and professionalism. Nevertheless, the domestic experience in development of
typical technological processes shows that for structurally similar parts and their elements similar processing methods used. In other words, there is a relationship between design of part and its manufacturing technology. The authors suggest that there is an unambiguous relation between the set geometric configurations of the part and set variants technology of processing of this part. In other words, the configuration of the parts is the domain for technology of forming part geometry [1], [2]. If submit specific the configuration of part \( P(S, R) \) as a collection (sets) of the surfaces \( S \) and their relative position (a set of relations) \( R \). Then the set \( P(S, R) \) is called the domain of a function \( f \) and denoted \( D(f) \). The set of all possible options for technologies \( T \) obtaining of the geometry parts, is a set of all possible values of the function \( f(P) \), which is called the set of values of the function \( f \) and denoted \( E(f) \).

\[
f : P \rightarrow T. \tag{1}
\]

The function \( f \) defines an injective mapping of the set configurations of parts into a set of technologies of their manufacture. This means that there is a finite set of manufacture technologies of configuration for specific configuration part [1]. Ensuring the accuracy of mutual location of the surfaces of the parts depends on the correctness of the formation of sets of bases for orientation the workpiece relative to the forming tool movements, when development of technological processes of machining parts [1].

2 Formulation of Problem

The most effective are schemes of handling if sets of bases correspond to the dimensional communications set on the drawing [3], [4]. It is necessary to exclude incompatible combinations basing surfaces. In order to avoid incompatible combinations basing surfaces, we must consideration of the complete set of dimensional relationships between the surfaces of parts for all degrees of freedom simultaneously. There are huge set placement schemes dimensional relations between the workpiece surfaces. Therefore, the task of finding a suitable and effective location options for machining cutting refers to the class of NP complex problems [5]. There is a finite set of options for the structure of the manufacturing process for any part. As a structural unit, in this case, considered position of the workpiece at constant installation and clamping. This structural unit has the name of ”SET” (cutter setting block). The boundaries of this set, are the most concentrated and maximally differentiated structure of the manufacturing process. In the first case, the processing of all surfaces is performed by using only one set of bases. In other words, full processing is performed on one setting and clamping). In the second case, there are the sequential processing of all surfaces, that is, amount of position are appropriate
to the number of machined surfaces. The task is to simulate processing using directional search actually existing surfaces, which can be used as the basis for orientation details for processing. Terms of the search for solutions in the using of the algorithm:
1) availability of real surfaces for use as bases;
2) the presence of surfaces that have not yet been processed;
3) availability of spatial relationships between real surfaces and surfaces that have not yet been processed, necessary and sufficient for the orientation of the processed surfaces;
4) considered only single-stage (one pass) processing of any surface details.

It should be noted that there are only ”rough” surfaces of part before processing. There is single sufficient condition for a formal search for the most concentrated structure. This is connectivity of graphs of starting coordinating sizes in each of six degrees of freedom [3]. Search differentiated structures is complicated by the fact that their existence is limited to the structure of coordinated dimensional relations [5]. The article describes the mathematical interpretation of the results of the formalization of the synthesis problem of sets bases and determine the sequence of surface treatment with sequential ”maintaining” geometric relations (ordinate dimensions) specified in the drawing.

3 Solution of Problem

Initially, the problem is solved for a two-dimensional space in which there are three degrees of freedom (two displacements and one rotation). Part is considered as closed half plane bounded lines arranged in a certain way (the sides). Before you start processing the part appears in the blank state as a set of “rough“ sides. Assume that planar and absolutely thin detail has finite set $X$ of linear sides orthogonal to coordinate axis $x$ and finite set $Y$ of linear sides orthogonal to coordinate axis $y$. Each linear side from the set $X \cup Y$ we shall consider as a node of some graph. Assume that $G'_x$ is the tree with the nodes set $X$, $A_x \in X$. Denote $A^*_x$ the node which does not include in the tree $G'_x$ and characterizes some “rough“ linear side of planar detail orthogonal to the axis $x$. Define the tree $G_{x,x}$ by a connection to $G'_x$ the edge $(A_x, A^*_x)$, as it is shown at Figure 1 (with the node $A_x$ characterizing “clean“ linear side corresponding ”rough” linear side denoted by the node $A^*_x$).
Figure 1. Description of tree $G_{x,x}$.

Analogously define the tree $G'_y$ and the tree $G_{y,y}$ by a connection to the tree $G'_y$ the edge $(A_y, A'^*_y)$, $A_y \in Y, A'^*_y \notin Y$, where the node $A'^*_y$ also characterizes “rough“ linear side of the detail, orthogonal to the axis $y$ as it is shown at Figure 2.

Fig. 2. Description of tree $G_{y,y}$.

Assume that the nodes $A'_x \in X, A'_y \in Y$. Denote $G_{x,y}$ the tree obtained by a connection of the trees $G_{x,x}, G'_y$ with the edge $(A'_x, A'_y)$ (see Figure 3).

Figure 3. Description of tree $G_{x,y}$.

In two-dimensional case the trees $G_{x,x}, G_{y,y}$ define sizes on a detail design by coordinates orthogonal to the axis $x, y$, relatively and the tree $G_{x,y}$ defines sizes by the angle $\gamma$ of a rotation around the axis $z$, orthogonal to the axis $x, y$. 
**Definition 3.1** Assume that $G$ is arbitrary tree and $A, B$ are its nodes. The node $A$ overrides the node $B$ in the tree $G$, if the nodes $A, B$ are adjacent. The node $A$ serially overrides all nodes of the tree $G$, if these nodes may be enumerated as follows $S_1 = A, S_2, \ldots, S_m$, so that for any $k, 1 < k < m$, there is $j, 1 < j < k$, so that the node $S_j$ overrides the node $S_k$.

**Lemma 3.2** Each node $A$ of arbitrary tree $G$ serially overrides all this tree nodes.

Proof. Take all nodes of the tree $G$ adjacent with the node $A$, and denote them $S_1 = A, S_2, \ldots, S_k$. By Definition 3.1 the nodes $S_2, \ldots, S_k$ are overridden by the node $S_1$. Take the node $S_2$ and designated $S_{k+1}, \ldots, S_n$ the nodes of the tree $G$ (besides of $S_1$), which are connected with the node $S_2$. These nodes are overridden by the node $S_2$. Continuing serially add new nodes to such constructing sequence it is possible to obtain a sequence of all nodes of the tree $G$ which satisfy Lemma 3.2 condition. Lemma 3.2 is proved.

**Definition 3.3** Nodes $A^*_x, A^*_y$ serially override all nodes of the set $X \cup Y$, if the nodes of the set $\{A^*_x, A^*_y\} \cup X \cup Y$ may be enumerated as $S_1, S_2, \ldots, S_n$, so that $S_1 = A^*_x, S_2 = A^*_y$ and for any $k, 2 < k < n$, the following condition is true.

If $S_k \in X$, then there is the node $S \in \{S_1, \ldots, S_{k-1}\}$, which overrides in the trees $G_{x,x}, G_{x,y}$ the node $S_k$.

If $S_k \in Y$, then there is the node $S \in \{S_1, \ldots, S_{k-1}\}$, which overrides $S_k$ in the trees $G_{y,y}, G_{x,y}$, or there is the node $S$, which overrides $S_k$ in the tree $G_{y,y}$ and the node $S' \in \{S_1, \ldots, S_{k-1}\} \cap X$, which overrides $S_k$ in the tree $G_{x,y}$.

The sequence $S_1, \ldots, S_n$ allows to define a sequence of operations to process sides of planar detail with linear sides, orthogonal to coordinate axis $[1]$, because it characterizes an arrangement of sizes on detail design. Necessary and sufficient condition of this serial overriding is in the following statement.

**Theorem 3.4** 1) If $A'_y = A_y$, then for any $A'_x \in X$ the nodes pair $\{A^*_x, A^*_y\}$ serially overrides all nodes of the set $X \cup Y$.

2) If $A'_y \neq A_y$, then for any $A'_x \in X$ the nodes pair $\{A^*_x, A^*_y\}$ does not override serially all nodes of the set $X \cup Y$.

Proof. The node $A^*_x$ overrides the node $A_x$ in the tree $G_{x,x}$. Using Lemma 3.2, it is simple to prove that all nodes of the set $X$ may be overridden serially by the node $A^*_x$. The node $A_y$ is overridden by the pair of nodes $\{A'_y, A'_x\}$, so all nodes of the set $Y$ also may be overridden serially by the node $A_y$. If $A'_y \neq A_y$, then any node $A_y \in Y$ may not be overridden by the nodes pair $\{A^*_x, A^*_y\}$. Theorem 3.4 is proved.
Using these statements it is possible to obtain a solution of maximally differentiated structure of technological process of three-dimensional detail working. Assume that three dimensional detail with planar surfaces orthogonal to coordinate axis has finite set \( X \) of planar surfaces orthogonal to the axis \( x \), finite set \( Y \) of planar surfaces orthogonal to the axis \( y \) and finite set \( Z \) of planar surfaces orthogonal to the axis \( z \). Further planar surfaces will be nodes of the trees \( G'_x, G'_y, G'_z \), relatively.

Introduce the nodes

\[ A_x, A'_x \in X, \ A_y, A'_y, A''_y \in Y, \ A_z, A'_z \in Z, \ A^*_x \notin X, \ A^*_y \notin Y, \ A^*_z \notin Z, \]

where the nodes \( A^*_x, A^*_y, A^*_z \) characterize "rough" surfaces orthogonal to the axis \( x, y, z \), and correspond "clean" surfaces \( A_x, A_y, A_z \) relatively.

Construct now the tree \( G_{x,x} \) by connection the edge \((A_x, A^*_x)\) to the tree \( G'_x \), the tree \( G_{y,y} \) by connection the edge \((A_y, A^*_y)\) to the tree \( G'_y \), the tree \( G_{z,z} \) by connection the edge \((A_z, A^*_z)\) to the tree \( G'_z \).

Introduce the tree \( G_{x,y} \) by connection the trees \( G_{x,x}, G''_y \) with the edge \((A'_x, A'_y)\), the tree \( G_{x,z} \) by connection the trees \( G_{x,x}, G''_z \) with the edge \((A'_x, A'_z)\), the tree \( G_{y,z} \) by connection the trees \( G_y, G'_z \) with the edge \((A''_y, A'_z)\).

**Definition 3.5** The nodes \( A^*_x, A^*_y, A^*_z \) serially override all nodes of the set \( X \cup Y \cup Z \), if the nodes of the set \( \{A^*_x, A^*_y, A^*_z\} \cup X \cup Y \cup Z \) may be enumerated as follows \( S_1, S_2, \ldots, S_n \) so that \( S_1 = A^*_x \), \( S_2 = A^*_y \), \( S_3 = A^*_z \) and for any \( k, 3 < k < n \), we have the following condition.

If \( S_k \in X \), then there is the node \( S \in \{S_1, \ldots, S_{k-1}\} \), which overrides in the trees \( G_{x,x}, G_{x,y}, G_{x,z} \) the node \( S_k \).

If \( S_k \in Y \), then there is the node \( S \in \{S_1, \ldots, S_{k-1}\} \), which overrides \( S_k \) in the trees \( G_{y,y}, G_{y,z}, G_{x,y} \) or there is the node \( S \in \{S_1, \ldots, S_{k-1}\} \), which overrides \( S_k \) in the trees \( G_{y,y}, G_{y,z} \) and the node \( S' \in \{S_1, \ldots, S_{k-1}\} \cap X \), which overrides \( S_k \) in the tree \( G_{x,y} \).

If \( S_k \in Z \), then there is the node \( S \in \{S_1, \ldots, S_{k-1}\} \), which overrides \( S_k \) in the trees \( G_{z,z}, G_{y,z}, G_{x,z} \) or there is the node \( S \in \{S_1, \ldots, S_{k-1}\} \), which overrides \( S_k \) in the tree \( G_{z,z} \) and the node \( S' \in \{S_1, \ldots, S_{k-1}\} \cap X \), which overrides \( S_k \) in the tree \( G_{y,z} \), and the node \( S'' \in \{S_1, \ldots, S_{k-1} \cap Y \), which overrides \( S_k \) in the tree \( G_{y,z} \).

In three dimensional case the trees \( G_{x,x}, G_{y,y}, G_{z,z} \) define sizes on the detail design by coordinates orthogonal to the axis \( x, y, z \), relatively, and the trees \( G_{y,z}, G_{z,z}, G_{x,y} \) define sizes of turning by the angles \( \alpha, \beta, \gamma \) around the axis \( x, y, z \), relatively.

**Theorem 3.6** 1) If \( A'_y = A_y, A'_z = A_z \), then for any \( A'_y \in X \), \( A''_y \in Y \) three nodes \( A^*_x, A^*_y, A^*_z \) serially override all nodes of the set \( X \cup Y \cup Z \).

2) If \( A'_y \neq A_y \) or \( A'_z \neq A_z \), then for any \( A'_y \in X \), \( A''_y \in Y \) three nodes \( A^*_x, A^*_y, A^*_z \) do not override all nodes in the set \( X \cup Y \cup Z \).
Proof. Theorem 2 is proved analogously with Theorem 1. It is enough to remark that the node $A_x$ is overridden by the node $A_x^*$, and then using Lemma 1 to obtain that all other nodes of the set $X$ are overridden also. Further the set $A_y$ is overridden by the nodes $A_y^*$, $A_y'$, and then all other nodes of the set $Y$ are overridden also. At last the node $A_z^*$ is overridden by the nodes $A_z^*$, $A_z'$, $A_z''$, then all other nodes of the set $Z$ are overridden also. Theorem 3.6 is proved.

Theorem 3.6 allows to define a sequence of processing surfaces for three dimensional detail when these surfaces are orthogonal to coordinate axis even in a case when there are cylinder holes with axis parallel to coordinate axis.

It is convenient to represent procedures of machine parts processing in two-dimensional and in three-dimensional cases by the following matrices (2), (3):

\[
\begin{pmatrix}
G_{x,x} & G_{x,y} \\
G_{x,y} & G_{y,y}
\end{pmatrix} \quad (2)
\]

\[
\begin{pmatrix}
G_{x,x} & G_{x,y} & G_{x,z} \\
G_{x,y} & G_{y,y} & G_{y,z} \\
G_{x,z} & G_{y,z} & G_{z,z}
\end{pmatrix} \quad (3)
\]

Then linear sides corresponding nodes of the set $X$ are processed accordingly the first line of the appropriate matrix, linear sides corresponding nodes of the sets $Y$ are processed accordingly the second line of the appropriate matrix, linear sides corresponding nodes of the sets $Z$ are processed accordingly the third line of the appropriate matrix.

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