A Empirical Study on Annuity Pricing with Minimum Guarantees

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Abstract

Minimum guaranteed benefits are the features of variable annuities which protect the annuitants against unfavorable changes in the economic conditions. Pricing such minimum guarantees pose a challenge to the annuity issuers, as many factors need to be taken into account for pricing the product. This study focuses on the pricing of guaranteed minimum death and accounts benefits which are embedded in flexible premium variable annuity. The study wishes to evaluate the impact of various factors: mortality improvements, volatility models, initial contribution, subsequent contributions, interest rate, guaranteed rate and accumulation period on the annuity prices. Using simulation results, this research provides useful information about the impact of these factors on the annuity pricing to both annuity providers/buyers and academia.

Mathematics Subject Classification: 91G80

Keywords: minimum guarantees, annuity pricing

1 Introduction

The need for alternative schemes to retirement plans formally organized by the government and employers to protect the pensioners upon retirement ([1], [2]). This phenomena has been contributed by mortality improvements, decrease in state pensions and among other factors. The decreasing of mortality rates
and increasing life expectancy over time [3] have led to pricing problems in annuities and pensions products. For example, the fact on average that women live longer and have lower mortality rates than men ([4], [5]) has made the pricing of the annuities and their embedded guarantees to be gender sensitive.

Variable annuity (VA) is among the alternatives for retirement plans which have gained popularity in Northern America, Japan, Europe and other countries. It is an agreement between the insurance company (writer) and the annuitant (policy holder) where the writer is to make periodical payments to the holder in the future time. The premiums paid by the holder are invested in different sub-accounts with different characteristics and investment strategies. Flexible premium variable annuity (FPVA) is a variable annuity with periodical premium payment during the accumulation period while Single premium variable annuity (SPVA) is a VA with only one upfront payment of premium. These products are designed to meet long term objectives of providing retirement income which offer protection against outliving one’s accumulation of assets for life as shown by [6].

Changes of volatility and interest rates affect the performances of the financial markets [7], which directly affects the VA providers since their sales and profits are driven by the performance of financial markets [8]. VA have embedded options like guaranteed minimum death benefits (GMDB) [9] and guaranteed minimum accounts benefits (GMAB) which provide guaranteed minimum benefits for a fee known as mortality and expense fee (M&E fee) ([10], [11]). In case the financial market performs poorly, the holder can get the minimum guaranteed benefits even if the return of his/her investment in a VA policy is far below the guaranteed value. If the market performs well above the guaranteed value, the holder will get the return of the investment. The M&E fee covers the cost of providing and administering guaranteed minimum death benefits (mortality fee) and living benefits (expense fee). The costs includes commissions by the variable annuity issuers and are charged as a percentage of the sub-account value. The policy holder can choose to terminate the policy if the fees are too high compared to the fund performance; or withdraw part of his investment prior to expiration date. Researches on determining the fair M&E fee have been mentioned by [12] and [13]. Many VAs provide more than one guarantees for the purpose of increasing sales [14] but only one will be exercised during the policy period. The providers collect fees from every guarantee but pay for only one guarantee. Bundling of many guarantees on a single VA contract decreases the account value (many fees are deducted). This in turn increases the claims costs in the worst scenario case of the financial market.

Many literature review can be found on the pricing of variable annuity with minimum guarantees under constant volatility assumptions ([15], [16], [17]). One of the drawback of constant volatility assumptions is that it can lead to
mis-pricing of the guarantees in VA ([18] and [19]). Various volatility models such as constant elasticity of variance (CEV), jump-diffusion, local volatility models, exponential Levy and stochastic model have been used to improve the pricing framework ([20] and [21]). A recent work done by [22] has incorporated the Heston model in evaluating the VA embedded with GMDB and GMLB. Similar works such as [23], [24] and [25] has also been considered by other.

Besides the volatility, the price of the GMDB and GMAB embedded in a FPVA is affected by other many factors. This study evaluates the impact of mortality improvements, volatility models, initial contribution, subsequent contributions, interest rate, guarantee rate and accumulation period on the price of the guarantees. Other factors affecting the guarantees price, like age and gender have been discussed in [22]. The simulation results from our study provides useful insights to the VA providers and academia about these factors. The remainder of the study is structured as follow: the methodology section describes the methods and materials used in the study. The findings are discussed and presented in results and discussions section while we conclude our research in the conclusion section.

2 Methodology

This section describes the methodology of this study, which includes the details of the models and data used.

2.1 Account value dynamics

An annuitant is assumed to pay an initial amount of money $A_0 \geq 0$ and a subsequent annual payment amount of $k \geq 0$, payable continuously within the accumulation phase, $T$. During the accumulation phase, the assumption is that there will be no withdrawal or surrender. If $c$ and $A_t$ are denoted as the M&E fee payable continuously and the value of the sub account at time $t$ respectively, then under constant volatility model assumptions, the sub account $(A_t)$ as stated in [23] follows a SDE given by:

\[
\begin{align*}
    dA_t &= (r - c)A_t dt + \sigma A_t dW_t + k dt, \quad A(0) = A_0 \\
    A_t &= A_0 e^{(r - c - \frac{\sigma^2}{2})t + \sigma W_t} + k \int_0^t e^{(r - c - \frac{\sigma^2}{2})(t-s)} + \sigma (W_t - W_s) ds, \quad t \geq 0
\end{align*}
\]

Here, we also employed the pricing framework developed by [22] which incorporate the stochastic volatility. The SDE for $A_t$ is then is given by:

\[
\begin{align*}
    dA_t &= (r - c)A_t dt + A_t \sqrt{V_t} dW_t^s + k dt, \quad A(0) = A_0
\end{align*}
\]
where $V_t$ is the instantaneous variance, which follows a CIR process \[26\] given by the following SDE:

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V\sqrt{V_t}dW_t^V, \quad V(0) = V_0$$

(4)

where, $\theta$ is long-term variance, $\kappa$ is the speed of reversion, $\sigma_V$ is volatility of variance, $dt$ is time interval, $W_t, t \geq 0$ is a standard Brownian motion, $dW_t^V$ and $dW_s^V$ are correlated Wiener processes with correlation coefficient, $\rho_{V,s}$ of the Wiener processes of $V_t$ and $S_t$ ($dW_t^VdW_s^V = \rho_{V,s}dt$). The process $V_t$ is strictly positive as given by Feller condition \[27\] \((2\kappa\theta > \sigma_V^2)\). For more details on this stochastic pricing framework, please refer to \[22\].

### 2.2 Minimum guarantees dynamics

This study considers two embedded options: guaranteed minimum death benefits (GMDB) and guaranteed minimum accounts benefits (GMAB). They take the form of premiums roll up with a pre-agreed guaranteed interest rate $g \geq 0$ chosen such that $g < r$. The minimum guarantee, $G_t$ at time $t$ is given by the following ordinary differential equation (ODE):

$$dG(t) = gG(t)dt + kdt, \quad 0 \leq t \leq T \quad \text{with} \quad G(0) = A_0$$

(5)

or

$$G_t = \begin{cases} A_0e^{gt} + \frac{k(e^{gt} - 1)}{g} & \text{for } g > 0, \\ A_0 + kt & \text{for } g = 0 \end{cases}$$

(6)

For the GMDB option, if the annuitant dies at time $t, 0 < t \leq T$, the beneficiary will receive a death benefit equivalent to $\max\{A_t, G(t)\}$. For the GMAB, the annuitant will receive a $\max\{A_t, G(t)\}$ upon survival to time $t, t = T$. For a combined guarantee, the first event to occur determines whether GMDB or GMAB will be exercised. Hence, the payoff $P(t)$, of either guarantee at any time $t$ is given by:

$$P(t) = [G(t) - A_t]_+ = \max\{G(t) - A_t, 0\} \quad \text{for } t \leq T$$

(7)

where $P(t)$ is the payoff of an arithmetic Asian put option with the underlying asset $A_t$. Methods on how to evaluate such options can be found in \[28\] and \[29\].

### 2.3 Minimum guarantees evaluation

The loss function associated with the minimum guarantees is denoted as a process of cash outflows minus cash inflows. The cash outflows comprises of the embedded options and cash inflows consist of the charges deducted from the
Annuity pricing with minimum guarantees

The expected present values of the loss functions \( (c = \text{M&E fees}) \) are given by:

\[
L_0(c) = \mathbb{E}[L_0]
\]

and the loss function with embedded GMDB and GMAB can be written as:

\[
L_0(c) = \int_0^T \prod (0, t) t p_x \mu_{x+t} + \prod (0, T) T p_x - c A_0 \bar{a}_{x:T} - \frac{k c}{r - c} (\bar{a}_{x:T} - \bar{a}_{x:T})
\]

(8)

For annuity with GMDB only, the loss function is given by:

\[
L_0(c) = \int_0^T \prod (0, t) t p_x \mu_{x+t} - c A_0 \bar{a}_{x:T} - \frac{k c}{r - c} (\bar{a}_{x:T} - \bar{a}_{x:T})
\]

(9)

For annuity with GMAB only, the loss function is given by:

\[
L_0(c) = \prod (0, T) T p_x - c A_0 \bar{a}_{x:T} - \frac{k c}{r - c} (\bar{a}_{x:T} - \bar{a}_{x:T})
\]

(10)

where

\[
\prod (0, t) = e^{-rt} \mathbb{E}(t) = e^{-rt} \mathbb{E}([G(t) - A_t]), \quad t \leq T
\]

(12)

and

\[
\bar{a}_{x:T} = \int_0^T e^{rt} t p_x dt, \quad \bar{a}_{x:T} = \int_0^T e^{ct} t p_x dt
\]

is the T-year temporary life annuity payment payable continuously for a life aged x with rate r and c. \( t p_x \) and \( u_{x+t} \) are the survival rate and the force of mortality respectively from age x to x + t. The fair M&E fee, \( c^* \) is determined using Secant method ( [30], [31]) and such that the expected present value of loss is zero, i.e.

\[
L_0(c^*) = 0
\]

(14)

2.4 Mortality tables

The mortality tables used in this study is the Canadian Institute of Actuaries (CIA) 1986–1992 and 1997–2004 insurance mortality tables, as shown in Table 1. The CIA is the national organization of the actuarial profession in Canada; and is responsible to prepare mortality tables based on the experience of certain years, to be used by actuaries in their work. The CIA 1986–1992 and CIA 1997–2004 insurance mortality tables are prepared using data of deaths in the years of 1986 to 1992 and 1997 through 2004 respectively. For complete data, please refer to http://www.mae.umontreal.ca/en/medico-legal-expertise-insurance-medicine-publications-tables/mortality-tables.
Table 1: CIA 86-92 and CIA 97-04 Mortality tables

<table>
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<tr>
<th>Attained age</th>
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<th></th>
<th>CIA 97-04</th>
<th></th>
</tr>
</thead>
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<td>Male</td>
<td>Female</td>
</tr>
<tr>
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<td>24</td>
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<td>25</td>
<td>0.00076</td>
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<td>26</td>
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<td>26</td>
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</tr>
<tr>
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<td>0.00080</td>
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<td>28</td>
<td>0.00081</td>
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<td>...</td>
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<td>0.0496</td>
<td>80</td>
<td>0.05169</td>
</tr>
</tbody>
</table>

2.5 Parameters of interest

Apart from the volatility and different embedded options, this study also focusing on various parameters that are affecting the annuity pricing. These parameters include initial contributions ($A_0$), subsequent contributions ($k$), interest rate ($r$), guaranteed rate ($g$) and accumulation period ($T$). Sensitivity analysis are done on these parameters to determine how much they are affecting the pricing for male and female annuitants aged 50. The parameters such as age is not being considered in this study as similar work has been done by [22]. The details of such analysis can be found in the next section.

3 Results and Discussions

3.1 Mortality improvements

Table 2 shows the M&E fees for GMDB and GMAB under two different mortality tables (two periods) for male and female annuitants aged 25-69. The results show that the price for male GMDB and GMAB at all ages decrease and increase respectively from 1986-1992 period to 1997-2004 period. It further shows that female GMDB prices decrease in all ages except ages 61-69 (where price increases, in the table ages 65 and 69) between the two periods while the female GMAB prices increase in all ages except some ages before 61 and part of 61-69 (age 55 and 60 in the table). The decrease (improvement) in mortality
rates over a period leads to low probability of dying and hence low fees for GMDB but high fees for GMAB. On the other hand, the increase (worsening) of mortality over a period of time leads to high probability of dying thereby increasing the fees for GMDB but decreasing the fees for GMAB.

<table>
<thead>
<tr>
<th>Age</th>
<th>GMDB CIA8692</th>
<th>GMDB CIA0704</th>
<th>GMAB CIA8692</th>
<th>GMAB CIA0704</th>
<th>GMDB CIA8692</th>
<th>GMDB CIA0704</th>
<th>GMAB CIA8692</th>
<th>GMAB CIA0704</th>
</tr>
</thead>
<tbody>
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<td>25</td>
<td>0.54</td>
<td>0.42</td>
<td>175.33</td>
<td>176.03</td>
<td>0.19</td>
<td>0.17</td>
<td>176.21</td>
<td>176.77</td>
</tr>
<tr>
<td>30</td>
<td>0.62</td>
<td>0.46</td>
<td>175.13</td>
<td>175.96</td>
<td>0.32</td>
<td>0.21</td>
<td>175.79</td>
<td>176.50</td>
</tr>
<tr>
<td>35</td>
<td>0.70</td>
<td>0.48</td>
<td>174.39</td>
<td>175.36</td>
<td>0.42</td>
<td>0.27</td>
<td>174.48</td>
<td>175.93</td>
</tr>
<tr>
<td>40</td>
<td>0.80</td>
<td>0.57</td>
<td>171.61</td>
<td>173.90</td>
<td>0.70</td>
<td>0.38</td>
<td>173.01</td>
<td>174.84</td>
</tr>
<tr>
<td>45</td>
<td>1.29</td>
<td>0.83</td>
<td>167.54</td>
<td>171.33</td>
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<td>0.59</td>
<td>170.79</td>
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</tr>
<tr>
<td>50</td>
<td>2.20</td>
<td>1.34</td>
<td>161.98</td>
<td>167.28</td>
<td>1.56</td>
<td>1.02</td>
<td>167.45</td>
<td>169.50</td>
</tr>
<tr>
<td>55</td>
<td>3.84</td>
<td>2.29</td>
<td>152.22</td>
<td>160.11</td>
<td>2.39</td>
<td>1.87</td>
<td>162.31</td>
<td>161.81</td>
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<td>6.72</td>
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<td>139.16</td>
<td>147.50</td>
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<td>3.46</td>
<td>153.58</td>
<td>151.11</td>
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<td>122.61</td>
<td>133.52</td>
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<td>7.01</td>
<td>140.54</td>
<td>144.33</td>
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<tr>
<td>69</td>
<td>19.11</td>
<td>13.19</td>
<td>106.62</td>
<td>121.22</td>
<td>9.95</td>
<td>10.12</td>
<td>126.73</td>
<td>139.48</td>
</tr>
</tbody>
</table>

Table 2: M&E fee for GMDB and GMAB using CIA8692 and CIA9704 mortality rates

Fig 1 shows the male mortality rates for both 1986-1992 and 1997-2004 periods. It indicates that the mortality rates for 1997-2004 period are lower than that for 1986-1992 period in all ages [32]. This implies a decrease or an improvement in mortality at all ages from 1986-1992 period to 1997-2004 period. This situation decreases the price of GMDB but increasing the price of GMAB for all ages (as shown in Table 2).

Figure 1: CIA8692 and CIA9704 male mortality rates
Fig 2 elucidates female mortality rates for both 1986-1992 and 1997-2004 periods. It shows that the mortality rates for 1997-2004 period are lower than that for 1986-1992 period in all ages except ages 61-69 where mortality rates for 1986-1992 period are higher than 1997-2004 period [32]. The implied increase in female mortality for ages 61-69 from 1986-1992 period to 1997-2004 period leads to the increase in the price for GMDB for the respective ages and a decrease in price for GMAB for the ages before and part of 61-69 (as illustrated in Table 2).

![Figure 2: CIA8692 and CIA9704 female mortality rates](image1)

Fig 3 illustrates the decrease in mortality rates calculated as the difference between 1986-1992 and 1997-2004 mortality rates at every age for male and female. It depicts more decrease in mortality rates for male than female. For female aged 61-69 it shows an increase in mortality rates between the two periods (a negative decrease in mortality rates). The decrease in mortality rates decrease the price of GMDB while increasing the price of GMAB and vise versa.

![Figure 3: Decrease in mortality rates from 1986-1992 to 1997-2004](image2)
3.2 Volatility models

Fig 4 shows GMDB M&E fees for male aged 24-69 under constant and stochastic volatility model assumptions. It indicates that M&E fees for the guarantee in stochastic volatility model assumptions are higher than that for deterministic volatility model assumptions. Unlike [24] and [33] who found overpricing of GMDB, this study found that GMDB is under priced in the deterministic volatility assumptions.

Figure 4: Male M&E fees for GMDB.

Fig 5 shows GMAB M&E fees for male aged 24-69 under constant and stochastic volatility model assumptions. It illustrates that M&E fees for GMAB in stochastic volatility model assumptions are higher than that for constant volatility model assumptions.

Figure 5: Male M&E fees for GMAB.
Fig 6 shows GMDB M&E fees for female aged 24-69 under constant and stochastic volatility model assumptions. It indicates that M&E fees for the guarantee in stochastic volatility model assumptions are higher than that for a constant volatility model assumptions. Unlike [24] and [33] who found overpricing of GMDB under constant volatility model assumptions, this study found that GMDB is under priced.

![Figure 6: Female M&E fees for GMDB.](image)

Fig 7 shows GMAB M&E fees for female aged 24-69 under constant and stochastic volatility model assumptions. The results show that M&E fees for GMAB in stochastic volatility model assumptions are higher than that for deterministic volatility model assumptions.

![Figure 7: Female M&E fees for GMAB.](image)
3.3 Initial contribution, subsequent contributions and accumulation period

Table 3 depicts GMDB and GMAB M&E fees for male and female aged 50 for various values of initial contribution, $A_0$. It indicates that the increase of initial contribution, $A_0$ leads to the increase in GMDB M&E fee but decrease in GMAB M&E fee. The major reason is that the increase in $A_0$ increases the value of GMDB while decreasing the future claims for GMAB. This makes the holder to charge higher fee for GMDB and lower fee for GMAB.

<table>
<thead>
<tr>
<th>$A_0$</th>
<th>Male GMDB</th>
<th>GMAB</th>
<th>Female GMDB</th>
<th>GMAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.17</td>
<td>203.91</td>
<td>0.88</td>
<td>205.56</td>
</tr>
<tr>
<td>100</td>
<td>1.34</td>
<td>167.28</td>
<td>1.02</td>
<td>169.50</td>
</tr>
<tr>
<td>150</td>
<td>1.40</td>
<td>162.65</td>
<td>1.06</td>
<td>164.17</td>
</tr>
<tr>
<td>200</td>
<td>1.43</td>
<td>160.26</td>
<td>1.09</td>
<td>161.40</td>
</tr>
</tbody>
</table>

Table 3: Initial contribution

Table 4 shows GMDB and GMAB M&E fees for male and female aged 50 for various values of subsequent contributions, $k$. The results indicates that M&E fee for GMDB and GMAB decreases and increases respectively as $k$ increases. The major reason is that the increase in $k$ leads to the increase of the future claims of GMAB while decreasing that of GMDB. This makes the holder to charge higher fee for GMAB and lower for GMDB. When $k = 0$, the VA becomes a single premium variable annuity (SPVA).

<table>
<thead>
<tr>
<th>$k$</th>
<th>Male GMDB</th>
<th>GMAB</th>
<th>Female GMDB</th>
<th>GMAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
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<td>189.22</td>
<td>0.92</td>
<td>190.68</td>
</tr>
</tbody>
</table>

Table 4: Subsequent contribution
Table 5 illustrates GMDB and GMAB M&E fees for male and female aged 50 for various values of accumulation periods, $T$. It demonstrates that as the accumulation period, $T$ increases, M&E for GMDB increases while for GMAB decreases. The reason is that as the accumulation period, $T$ increases, the number of policyholders who are likely to die increase and therefore less are likely to survive. This translates into high claims for GMDB and less for GMAB. Hence higher fees for GMDB and lower for GMAB. $T$ cannot be zero as FPVA is a deferred annuity which must have both accumulation and decumulation phases.

<table>
<thead>
<tr>
<th>$T$</th>
<th>Male GMDB</th>
<th>Female GMDB</th>
<th>Male GMAB</th>
<th>Female GMAB</th>
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<td>10</td>
<td>1.34</td>
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</table>

Table 5: Accumulation period

3.4 Interest rate and guaranteed rate

Table 6 expresses GMDB and GMAB M&E fees for male and female aged 50 for various values of interest rates, $r$. It indicates that as interest rate, $r$ increases, the M&E fee for both GMDB and GMAB decreases. When the discounting factor increases, the amount being discounted has to decrease and vise versa. So the guarantees fees decreases as interest rate increases. For $r \geq g$, the fees are not fair and are very high for the guarantee to be offered at that rate. Hence for $r \geq 0.01$ the product is not fair. Obviously $r$ can not be zero because the equity market is a risky market. Individuals undertaking risks need to be rewarded or compensated by $r > 0$. Table 7 displays GMDB and GMAB M&E

<table>
<thead>
<tr>
<th>$r$</th>
<th>Male GMDB</th>
<th>Male GMAB</th>
<th>Female GMDB</th>
<th>Female GMAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>1.84</td>
<td>496.65</td>
<td>1.39</td>
<td>516.98</td>
</tr>
<tr>
<td>0.02</td>
<td>1.34</td>
<td>167.28</td>
<td>1.02</td>
<td>169.50</td>
</tr>
<tr>
<td>0.03</td>
<td>0.97</td>
<td>82.61</td>
<td>0.73</td>
<td>83.33</td>
</tr>
<tr>
<td>0.04</td>
<td>0.68</td>
<td>42.64</td>
<td>0.52</td>
<td>42.85</td>
</tr>
</tbody>
</table>

Table 6: Interest rate

fees for male and female aged 50 for various values of guaranteed rate, $g$. It elucidates that the increase of the guaranteed rate, $g$ leads to the increase of GMDB and GMAB M&E fees. This is because it increases the future claims costs for both guarantees. The provider has to charge higher fee for both
guarantees for compensation of taking high risk. For $g = 0$, the guarantee is a money-back which pays out at maturity the amount equal to the premium paid by the policyholder. If $g \geq r$, the risk neutral value of the contract (future cash outflow) will generate high fees (compared to the performance of the fund) to equate to cash inflow.

<table>
<thead>
<tr>
<th>$g$</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GMDB</td>
<td>GMAB</td>
</tr>
<tr>
<td>0.00</td>
<td>0.96</td>
<td>83.29</td>
</tr>
<tr>
<td>0.005</td>
<td>1.14</td>
<td>116.00</td>
</tr>
<tr>
<td>0.01</td>
<td>1.34</td>
<td>167.28</td>
</tr>
<tr>
<td>0.02</td>
<td>1.85</td>
<td>495.18</td>
</tr>
</tbody>
</table>

Table 7: Guarantee rate

4 Conclusion

This study evaluated the impact of mortality improvements and volatility models on the M&E fees for GMDB and GMAB embedded in FPVA. It further performed sensitivity analysis on initial contribution, subsequent contributions, accumulation period, interest rate and guaranteed rate. The study found that there is a relationship between mortality change and the M&E fees for the guarantees. It also found that the guarantees are under-priced in the deterministic volatility assumptions. In the future work, we intend to study the impact of stochastic interest rate and stochastic mortality rates on annuity pricing with minimum guarantees. Other volatility models such as [34], [35], [36] and [37] can also be considered in the pricing framework.

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