A Study of Impact of Stochastic Volatility on Variable Annuity Pricing

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Abstract

Variable annuities play an important role in protecting one’s future income. The pricing of such annuities remain a big challenge to the annuities’ issuers due to the presence of embedded options. Many variable annuity pricing methodologies employ constant volatility assumption for the movement of the underlying asset, which is not the case in real market. In this study, we propose a new pricing model that can account for stochastic volatility, by incorporating the Heston model into our pricing framework. Heston model is one of the commonly used methods to model volatility. Using this new pricing framework, we evaluate the mortality and expenses (M&E) fee charged by the issuers for a group of male and female aged 50-69. The M&E fee evaluated based on the constant volatility pricing model will be compared to the results obtained using our framework.

Mathematics Subject Classification: 91G80

Keywords: heston model, minimum guarantees, annuity pricing
1 Introduction

An annuity contract involves two parties: the annuitant and the annuity provider. The annuity provider agrees to make periodical payments to the annuitant for a specified time period or for life. Meanwhile the annuitant agrees to make payment of premium either in single premium payment during inception or by installments. Variable annuities (VA) are deferred annuities where the premium paid is held in a sub-accounts, It is then used to purchase units of investment assets (fund) which earn returns over the period of the annuity contract. This makes the benefits paid during the annuitization period to the annuitant to be linked to the capital market performance. Flexible premium variable annuity (FPVA) is a variable annuity with periodical premium payment during the deferred period or accumulation period.

Variable annuities (VA) are very common in personal retirement plans, individual retirement plans (IRA), employer sponsored plans, tax-advantaged investment plans or generally they compete with individual and group pensions plans. It is one way to replace part of the social security with private retirement savings account. They are contracts designed to provide a periodic payment to the annuitant for a specific period of time or for life which starts in a future time although the payment can also start immediately. The risks faced by annuities’ issuers are mostly due to the unpredictable financial market fluctuations [1], [2]. When there is a market fall, many annuity holders are likely to make a claim at the same time from the issuers. Such risks are not easily hedged. The changing economic and market conditions, e.g. interest rates, asset volatility, inflation and longevity have brought some difficulties to the issuers to price the product ([3], [4], [5]).

Due to the volatility of asset returns, investors are concerned with their investment value and wish to protect against the downside risk of the sub-accounts (the possibility of the value of the investment being lower than the average due to fluctuations of market factors)[6]. Different embedded options can be bundled together in the annuities products to protect the annuitant. The embedded options includes guarantee minimum death benefits (GMDB) and guarantee minimum living benefits (GMLB). GMLB consists of guarantee minimum account benefits (GMAB), guarantee minimum income benefits (GMIB), guarantee minimum withdrawal benefits (GMWB), guarantee lifetime withdrawal benefits (GLWB), guarantee annuity option (GAO), guarantee minimum maturity benefits (GMMB). Details on these embedded options can be found in [7]. When the investment value falls below the minimum guaranteed amount, the investor is entitled to that minimum guaranteed amount rather than the actual amount of the investment. For example, GMDB guarantees a minimum death benefits if the annuitant dies during the accumulation period; the beneficiary receives from the issuer the greater of the account value
or the guaranteed minimum amount. In a GMMB, the annuitant is guaranteed a lump sum of the greater of the account value and the guaranteed amount at the end of the accumulation period or at some pre-agreed time. These embedded options also help increasing the annuity sales volume and profits for the annuity provider.

The variable annuities issuer will charge a fee to cover the costs of the embedded options. In this study, we consider an annuity product with GMDB and GMLB as the embedded options. Hence, the fee charged is known as mortality and expenses (M&E) fee, which is also the price of the variable annuities. Many literature on the evaluation of M&E fees concentrates on the options embedded in a single premium variable annuity and constant volatility which implies that the market conditions are complete ([8], [9] and [10]). Research on how to evaluate the M&E fee collectively for annuities with more than one embedded options in one policy can be found on [9] and [11].

Pricing framework using stochastic volatility models provide a more realistic M&E fee as explained in [12]. For long term products like variable annuity, stochastic models is a better fit for the short term volatility (market fluctuations) than the constant volatility model ([13], [14], [15]). We are using the Heston model in our study because it can incorporate the relationship between asset returns and its volatility; and it is easily tractable compared to other stochastic volatility models such as the Cox-Ingersoll-Ross (CIR) model [16].

Using our pricing framework, we evaluate the M&E fee which assumes stochastic volatility. Our model is different from Chi and Lin [9], as it takes into account the stochastic volatility, by incorporating the Heston model. Other parameters such as correlation between the equity returns and volatility [5] and magnitude of volatility can also be considered in our pricing framework. For our numerical illustrations, we first evaluate the M&E fee for male and female aged 50-69 based on the model described in Chi and Lin [9] paper, which works on constant volatility assumptions. We then evaluate the M&E fee based on our framework, and compare both these results.

The remainder of the study is organized as follow: The methods and materials section describes the methodology of our model as well as the simulation studies conducted to evaluate the M&E fee. We discuss our findings in results and discussion section and conclude our study in the conclusion section.

2 Methodology

In this section we describe the methodology and the simulation studies on how we evaluate the M&E fee for a variable annuity with embedded GMDB and GMLB options.
2.1 Asset dynamics

The value of an unit asset, $S_t$ at time $t$ is assumed to follow the Heston model process given by the following stochastic differential equation (SDE):

$$dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_t^s, \quad S(0) = S_0$$  \hfill (1)

where $\mu$ is the rate of return from an asset and $S(0) = S_0$ is the initial boundary condition. In our study, we assume the risk-free interest rate $r$ as the rate of return, hence we rewrite Eq (1) as:

$$dS_t = r S_t dt + \sqrt{V_t} S_t dW_t^s, \quad S(0) = S_0$$  \hfill (2)

where $V_t$ is the instantaneous variance, which follows a CIR process given by the following SDE:

$$dV_t = \kappa (\theta - V_t) dt + \sigma V_t dW_t^V, \quad V(0) = V_0$$  \hfill (3)

If the parameters obey Feller condition

$$2\kappa \theta > \sigma^2_V$$  \hfill (4)

then the process $V_t$ is strictly positive.

The long-term variance is denoted by $\theta$ and the parameter $\kappa$ is the speed of reversion: the rate at which the short term variance reverts to long-term variance. The volatility of variance and time interval are represented by $\sigma_V$ and $dt$ respectively. $W_t$, $t \geq 0$ is a standard Brownian motion, where $dW_t^s$ and $dW_t^V$ are Wiener processes with correlation coefficient $\rho$ which is the association between asset returns and its volatility.

2.1.1 How to simulate correlated normally distributed variates

There are three steps involved to generate correlated normally distributed variates from correlated $X_1 \sim N(\mu_1, \sigma_1)$, $X_2 \sim N(\mu_2, \sigma_2)$, with coefficient $\rho$ and covariance matrix given by:

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho \sigma_1 \sigma_2 \\ \rho \sigma_1 \sigma_2 & \sigma_2^2 \end{bmatrix}$$  \hfill (5)

1. Generate $Z_1 \sim N(0, 1)$ and $Z_2 \sim N(0, 1)$

2. Decompose $\Sigma$ into $LL^T$, $\Sigma = LL^T$ (Cholesky)

3. $X_1 = \mu_1 + LZ_1$, $X_2 = \mu_2 + LZ_2$
Alternatively, 
\[ X_1 = \sqrt{1 - \rho^2} \sigma_1 Z_1 + \rho \sigma_1 Z_2 + \mu_1, \]  
\[ X_2 = \sigma_2 Z_2 + \mu_2 \]  
(6)  
(7)

By Cholesky decomposition, \( \Sigma \) is given by 
\[
\Sigma = \begin{bmatrix}
\sigma_1^2 & \rho \sigma_1 \sigma_2 \\
\rho \sigma_1 \sigma_2 & \sigma_2^2
\end{bmatrix} = \begin{bmatrix}
\sqrt{1 - \rho^2} \sigma_1 & \sigma_1 \\
0 & \sigma_2
\end{bmatrix} \begin{bmatrix}
\sqrt{1 - \rho^2} \sigma_1 & 0 \\
\sigma_1 & \sigma_2
\end{bmatrix}
\]  
(8)

### 2.1.2 How to generate correlated Wiener processes

In Heston model, \( dW^s_t \) and \( dW^V_t \) are correlated with correlation coefficient \( \rho \). It is assumed that 
\[ dW^s_t \sim N(0, \sigma_s^2 dt) \]
and 
\[ dW^V_t \sim N(0, \sigma_V^2 dt) \]

Therefore:
\[
dW^s_t = \sqrt{1 - \rho^2} \sigma_s dB_{1,t} + \rho \sigma_s dB_{2,t} \]  
(9)
\[
dW^V_t = \sigma_V dB_{2,t} \]  
(10)

where \( dB_{1,t} \) and \( dB_{2,t} \) are two independent Wiener process

\[
Var[dW^V_t] = E[(dW^V_t - 0)(dW^V_t - 0)] = E[\sigma_V^2 dB_{2,t}^2] = \sigma_V^2 dt
\]
\[
Var[dW^s_t] = E[(1 - \rho^2)\sigma_s^2 dB_{1,t}^2 + 2 \sqrt{1 - \rho^2} \sigma_s^2 \rho dB_{1,t} + dB_{2,t} + \rho^2 \sigma_s^2 dB_{2,t}^2]
\]
\[= (1 - \rho^2)\sigma_s^2 dt + 0 + \rho^2 \sigma_s^2 dt \]
\[= \sigma_s^2 dt \]  
(12)

\[
E[(dW^V_t - 0)(dW^s_t - 0)] = E[\sqrt{1 - \rho^2} \sigma_s \sigma_V dB_{1,t} dB_{2,t} + \rho \sigma_s \sigma_V dB_{2,t}^2]
\]
\[= \rho \sigma_s \sigma_V dt \]  
(13)

Therefore
\[
\Sigma = \begin{bmatrix}
\sigma_s^2 dt & \rho \sigma_s \sigma_V dt \\
\rho \sigma_s \sigma_V dt & \sigma_V^2 dt
\end{bmatrix}
\]  
(14)

### 2.2 Account value dynamics

An annuitant is assumed to contribute an initial amount of \( A_0 \) units of money and a subsequent annual contribution amount of \( k \geq 0 \) payable continuously within the accumulation period, \( T \). We also assume that during the accumulation period there is no lapses, withdrawal or surrender. We denote \( c \) as the
M&E fee payable continuously and $A_t$ is the value of the sub account at time $t$. Then the sub account follows a SDE:

$$dA_t = (r - c)A_t dt + A_t \sqrt{V_t} dW_t^s + kdt$$  \hspace{1cm} (15)$$

A special case of the above framework is when volatility of variance $\sigma_v$ is equal to zero and initial variance is equal to the long run variance ($V_0 = \theta_0$). In this case the framework is reduced to a constant volatility model. Under this reduced constant volatility model, $S_t$ follows a geometric Brownian motion (GBM) and is given by the following SDE:

$$dS_t = rS_t dt + \sigma S_t dW_t, \quad S(0) = S_0$$  \hspace{1cm} (16)$$

and SDE for the sub account $A_t$ is given by:

$$dA_t = (r - c)A_t dt + \sigma A_t dW_t + kdt, \quad A(0) = A_0$$  \hspace{1cm} (17)$$

Using Itô’s lemma, we can write $A_t$ as

$$A_t = A_0 e^{(r-c-\frac{\sigma^2}{2})t + \sigma W_t} + k \int_0^t e^{(r-c-\frac{\sigma^2}{2})(t-s)} \sigma (W_t - W_s) ds, \quad t \geq 0$$  \hspace{1cm} (18)$$

as stated in Chi and Lin [9].

### 2.3 Minimum guarantees dynamics

Our embedded option is the combined GMDB and GMMB guarantee. It takes the form of roll up of premiums with a pre-agreed guaranteed interest rate $g \geq 0$ chosen such that $g < r$. The minimum guarantee, $G_t$ at time $t$ is given by the following the differential equation:

$$dG(t) = gG(t) dt + k dt, \quad 0 \leq t \leq T \quad \text{with} \ G(0) = A_0$$  \hspace{1cm} (19)$$

or

$$G_t = \begin{cases} A_0 e^{gt} + \frac{k(e^{gt} - 1)}{g} & \text{for} \ g > 0, \ G(0) = A_0 \\ A_0 + kt & \text{for} \ g = 0 \end{cases}$$  \hspace{1cm} (20)$$

For the GMDB option, if the annuitant dies during the contract period and before the maturity $T$, the beneficiary will receive a death benefit equivalent to the greater amount of the sub account $A_t$ or the minimum guarantee $G_t$. Similarly for the GMLB case, the annuitant will receive a lump sum upon maturity of the contract. Hence, the payoff $P(t)$, of the combined GMDB and GMMB at any time $t$ is given by:

$$P(t) = \left[G(t) - A_t\right]^+ = \max\{G(t)A_t, 0\}$$  \hspace{1cm} (21)$$

where $P(t)$ is the payoff of an arithmetic Asian put option with the underlying asset $A_t$. Methods on how to evaluate such options can be found in [17], [18], [19].
### 2.4 Evaluation of the GMDB and GMLB

We now describe how to evaluate the fair M&E fee for our minimum guarantees. We first denote a loss function associated with the guarantee as a process of cash outflows minus cash inflows. The cash outflows comprises of the GMDB and GMMB payout and cash inflows consists of the charges deducted from the sub account. Hence, the present value of the loss function (with discounted factor $e^{-rt}$) is given by:

\[
L_0 = \int_0^T e^{-rt}P(t)\mathbb{E}[G(t)] - c \int_0^T e^{-rt}A(t)\mathbb{E}[A(t)] - \int_0^T e^{-rt}u_{x+t}x^t \, dt
\]

where $t_p_x$ is survival probability from age $x$ to $x+t$ and $u_{x+t}$ is the hazard rate from age $x$ to $x+t$. The expected present value of the loss as a function of the M&E fee is given by:

\[
L_0(c) = \mathbb{E}[L_0] = \int_0^T \prod (0, t) t_p_x \mu_{x+t} + \prod (0, T) T_p_x - cA_0a_{x:T|c} - \frac{kc}{r-c}(\bar{a}_{x:T|c} - \bar{a}_{x:T|c})
\]

where

\[
\prod (0, t) = e^{-rt}\mathbb{E}[P(t)] = e^{-rt}\mathbb{E}([G(t) - A(t)]_+)
\]

is the price of the put option and

\[
\bar{a}_{x:T|c} = \int_0^T e^{rt}t_p_x \, dt, \quad \bar{a}_{x:T|c} = \int_0^T e^{ct}t_p_x \, dt
\]

is the T-year temporary life annuity payment payable continuously for a life aged $x$ with rate $r$ and $c$. The fair M&E fee, $c^*$ is determined using Secant method \cite{20}, \cite{21} and such that the expected present value of loss is zero. i.e.

\[
L_0(c^*) = 0
\]

### 2.5 Mortality table

The mortality table we use in this study is the Canadian Institute of Actuaries (CIA) 1997–2004 insurance mortality table shown in Table 1. The CIA is the national organization of the actuarial profession in Canada; and is responsible to prepare mortality tables based on the experience of certain years, to be used by actuaries in their work. The CIA 1997–2004 insurance mortality table is the mortality table prepared from data of deaths in the years of 1997 through 2004.
Table 1: CIA 97-04 Mortality table

<table>
<thead>
<tr>
<th>Attained age</th>
<th>CIA 97-04</th>
<th>Attained age</th>
<th>CIA 97-04</th>
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<td>Male</td>
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<tr>
<td>64</td>
<td>0.01160</td>
<td>0.01040</td>
<td>79</td>
</tr>
</tbody>
</table>

2.6 Simulation Studies

Here we describe the simulations conducted to evaluate the M&E fee for the variable annuity. We consider the following 4 different cases:

- **Case 1**: We evaluate the M&E fees under constant volatility assumptions as stated in [9].

- **Case 2**: We evaluate the M&E fees using our pricing framework, which incorporates the Heston model.

- **Case 3**: We evaluate the M&E fees using different correlation values $\rho$.

- **Case 4**: We evaluate the M&E fees using different asset’s volatilities $\sigma_s$

The parameters we use in our case studies are tabulated in Table 2, where

<table>
<thead>
<tr>
<th>Cases</th>
<th>$A_0$</th>
<th>$k$</th>
<th>$T$</th>
<th>$r$</th>
<th>$g$</th>
<th>$\sigma_s$</th>
<th>$\sigma_V$</th>
<th>$\theta$</th>
<th>$V_0$</th>
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<tr>
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</table>

Table 2: Parameters used in the case studies
To evaluate the M&E fees, we simulate numerically the coupled SDEs for the sub account values $A_t$ (Eq. 15) and the stochastic volatilities given by Eq. 3. We use a secant method to find the root, $c^*$, for the equation $L_0(c) = 0$, where $c^*$ gives the fair value for the M&E fee. For each iterations in the secant method, the loss function $L_0(c)$ is re-calculated using Eq. 23, where the expected payoff of the GMDB and GMLB, $\mathbb{E}[P(t)]$, is averaged over 50,000 simulated paths of $A_t$.

The above steps are repeated using those parameters mentioned in Table 1 and Table 2 to evaluate the M&E fees for Case 1 to Case 4. The simulations and M&E fees evaluation processes are implemented using Python. The codes of are available upon request. M&E fees are calculated as basis points (bsp).

3 Main Results

Case 1

In Case 1, we evaluate M&E fees for the VA under the constant volatility assumptions. The parameters used are shown in Table 2. Fig 1 shows the M&E fees for male and female annuitants aged 50-69. We noticed that the M&E fees for female are generally higher than male. The main reason in that the female mortality rate is slightly lower than male (higher probability of surviving the accumulation period), hence having a higher possibility of receiving the GMLB compared to male. The VA issuer will charge a higher fee to compensate for the higher payout.
We also noticed that M&E fee decreases with age of the annuitant (for male and female). The downward trend will reach a turning point for male (around age 67) and female (around age 60), and it will starts to increase again. We investigate this trend pattern and we noticed that this is related to the change in mortality rate (in %). That means for female aged 61-79 there is rapid mortality improvements compared to male due to decrease in heart and cerebrovascular diseases.

Fig 2 shows the percentage change in mortality rates (calculated yearly) for male and female aged 50-69. For the female case, the percentage change in mortality rate increases up to age 60, then start to decrease again. Interestingly this coincides with the pattern of the M&E fees shown in Fig 1. However, this properties is not too obvious in the male case.
Case 2

We evaluate the M&E fees for the VA using our pricing framework. In this case, we introduce the stochastic volatility into our model, by setting $\sigma_v = 0.2$. Fig 3 shows the M&E fees for the male and female annuitants aged 50-69 years old, which shown similar pattern as Fig 1. We are also interested to know how different values of $\sigma_v$ will affect the M&E fees. The M&E fees for male with $\sigma_v = 0.0, 0.1, 0.15, 0.20$ are evaluated and shown in Table 3.

![Figure 3: M&E fees (bsp) under stochastic volatility assumptions, $\sigma_v = 0.2$.](image1)

We noticed that the results are not significantly different from Case 1 ($\sigma_v = 0$). The results are similar for female’s case. However, we can’t conclude that $\sigma_v$ has no impact on M&E fees since the other parameters still remain the same as in Case 1. Using the parameters in Case 2, we perform sensitivity analysis on $\theta$, to study the effects of different $\theta$ values on evaluating M&E fees. Fig 4 and Fig 5 show the M&E fees for different $\theta$ values for male and female annuitants aged 50-69 respectively.

![Figure 4: Male M&E fee (bsp) for different values of $\theta$.](image2)
We noticed that M&E fees increase (decreases) when $\theta$ increases (decreases). The main reason being that, VA issuers are concerned with the increase of volatility leads to a lower return which consequently causes losses to them. Hence they charge higher fees for compensation of such risk. Of course increase in volatility can lead to higher returns or lower returns. Various studies [22], [23], [24] have also shown this inverse relationship between equity returns and its volatility.

<table>
<thead>
<tr>
<th>Age</th>
<th>$\sigma_v = 0$</th>
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Table 3: Male M&E fees (bsp) for various $\sigma_v$. 
Case 3

In this case we conduct a sensitivity analysis on $\rho$ to study the effects of different $\rho$ values on M&E fees for a male aged 50. Fig 6 shows M&E fees for different values of $\rho$ while holding other parameters the same as in Case 3. We noticed that M&E fees decreases as $\rho$ increases and there seems to be a symmetrical results on M&E fees for negative and positive $\rho$ (e.g. M&E fees for $\rho = 1$ and $\rho = -1$ are equal).

Fig 7 shows M&E fees for different values of $\sigma$ while holding other parameters the same as in Case 3. Due to Feller condition, $2\kappa\theta > \sigma_v^2$, we can increase $\sigma_v$ up to 0.30. We noticed that the increase in $\sigma_v$ causes M&E fees to decrease.
Case 4

In case 4, we wish to study the effect of $\sigma_s$ on M&E fees. Table 4 shows M&E fee for a male aged 50 for different values of $\sigma_s$. We noticed that M&E fee increases when $\sigma_s$ increases. The main reason is that increased return’s volatility caused by increased equity price volatility is likely to cause increased losses to the VA issuers although it is likely also to cause more profits. As a result VA issuers charge higher M&E fee to protect themselves against the risk of falling returns due to increased volatility. We also found that $\kappa$, the mean reversion rate has no significant influence on the M&E fees.

<table>
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<th>Mean reversion rate</th>
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Table 4: A 50 year old male M&E fee (bsp) for different values of $\sigma_s$ and $\kappa$

4 Conclusions

In this study, we have evaluated the M&E fees for the VA with GMDB and GMLB options. We evaluate the M&E fees using our pricing framework, which incorporates the Heston model that can take into account the asset’s volatility, correlation between asset returns and its volatility. The result is then compared to those obtained using the Chi and Lin 2012 model, which assumes constant
volatility. We found that M&E fees at all ages is generally higher for female than for male. This is due to the mortality rate of female is lower than male. Also, by just introducing stochastic volatility, the M&E fees do not differ significantly from the one using constant volatility model. However, by taking into account the long term variance, asset volatility and correlation, the M&E fees will differ significant as discussed in results.

Assuming that asset volatility at any future time is constant misleads the M&E fees. This produces fees which are far from the reality of the financial market. This study suggests the use of stochastic volatility in the estimation of M&E fee of these guaranteed minimum benefits to reflect the realistic financial market. Future work can include different volatility models such as a time series model mentioned by [25] and [26]. Also, we can consider incorporating the stochastic interest rate for the evaluation of M&E fee ([27] and [28]).

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References


VA pricing using stochastic volatility


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