Investment Management Using Portfolio Optimization with Stock Price Forecasting

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Abstract

Management of investors’s capital in portfolio investment can be regarded as a dynamic optimal control problem. In order to optimize the investors’s capital, the dynamic of stock price in portfolio needs to be forecasted by investors. In this paper, ARIMA-Kalman filter is applied to forecast the daily stock price in portfolio. Subsequently, the stock return value is calculated based on that stock price forecasting. Furthermore, we try to solve the portfolio optimization problem using Model Predictive Control (MPC). MPC is utilized to get the optimal controller. Thus, we can obtain the best strategy in regulating the optimal portfolio management.

Keywords: ARIMA-Kalman filter, forecasting, Model Predictive Control (MPC), portfolio optimization

1 Introduction

In a stock investment, there are two things that should be considered by investors. Those are the rate of return and the rate of risk. Stock investment is expected to provide maximum return on a certain level of risk or to give the minimum risk on a certain level of return. In general, to reduce the level of risk, investors diversify or spread investment in several companies to build a stock portfolio [1]. If investors make incorrect decision, then the additional capital will not be gained, but losses. Therefore, it is necessary for investors to manage their stock portfolio investment to obtain the optimal capital in portfolio.

In literature, some problems of portfolio optimal control have been discussed. In 1952, Markowitz observed the portfolio selection and introduced the optimal portfolio.
So, it is able to produce the maximum expected return and minimize the variance [2]. Another research is conducted by Bielecki and Pliska using a method of risk-sensitive control theory in portfolio optimization [3]. Moreover, the research regarding portfolio optimization problem is growing up. In 2004, Dombrovskiy et al. used Model Predictive Control (MPC) for solving portfolio optimization problem. MPC method produces a good strategy in portfolio optimization with the ideal transaction costs [4].

MPC is a controller category based on the process model which can be used explicitly to design the controller by minimizing an objective function. Beside that, MPC has several advantages i.e combine all purposes into a single objective function and able to overcome the constraints on the state and control variable with a very effective optimization result [5]. In Indonesia, Syaifuddin applied MPC in stock portfolio optimization problem and shown that MPC is well-applied [6].

In stock investment management, portfolio optimization problem also involves the movement or change in the stock price from time to time. So, it is necessary for investors to know the best strategy to optimize every capital in their portfolio. Therefore, the forecasting of stock price is important for investors to provide future information. One of the method that can be applied in forecasting is ARIMA Box-Jenkins. In 1970, Box and Jenkins introduced the forecasting model of ARIMA (Autoregressive Integrated Moving Average) as the basis of fundamental idea in the analysis of time series [7]. ARIMA method analyzes data technically by observing the patterns of history data to obtain forecasting model. Some research discussing about forecasting, especially ARIMA method can be seen in [8], [9], [10], and [11].

However, to obtain more accurate forecasting result, ARIMA model should be updated time to time. Nevertheless, this step will bring more complicated and timely problem. One of the most successful estimation procedure of dynamic system is Kalman filter. Kalman filter is an estimation method of state variable from linear discrete dynamical systems which can be used to predict one step ahead. Moreover, this method provides an updating process in the model. Therefore, by utilizing Kalman filter in ARIMA model, the accuracy of forecasting has been improved. In [12], Guo et al. used Kalman filter to predict the traffic flow level. In addition, the implementation of Kalman filter for improving the previous prediction result has been applied by Galanis et al. to predict the weather [13] and by Cassola and Burlando to predict wind speed and wind energy [14].

Based on the work by [13] and [14], this paper studied about Kalman filter modified ARIMA in MPC to obtain an optimal portfolio. There are two steps which are discussed, i.e. forecasting stock price and solving optimal control problem in portfolio optimization. In forecasting, we applied ARIMA method. Then, we combine ARIMA and Kalman filter method to be an ARIMA-Kalman filter. In this case, Kalman filter algorithm is utilized to improve forecasting result based on error correction of ARIMA. Subsequently, Model Predictive Control (MPC) method is applied to solve the optimal control problem in stock investment portfolio optimization. In our research, the portfolio consists of three stock assets, one risk-free asset, and one asset from capital loan. The mathematical model of investment portfolio that is used in this paper mostly refers to [4].
2 Arima-Kalman Filter Implementation in Stock Price Forecasting

In this paper, we used daily closing price of three companies. Those companies are PT. Unilever Indonesia Tbk., Perusahaan Gas Negara (PGN) and PT. Kalbe Farma Indonesia Tbk (KFI). The data is from May 2014 to June 2015. In this case, we only forecast the stock price in short of time during one month (23 days) on July 2015. Firstly, we used ARIMA method, and then the forecasting result of ARIMA model is improved by implementing the Kalman filter algorithm. The time series plot of each company's stock price is given in Figure 1.

![Time Series Plot of Unilever](image1)
![Time Series Plot of KPN](image2)
![Time Series Plot of KFI](image3)

**Figure 1.** Time Series Plot of Stock Price From (a) PT. Unilever Indonesia Tbk (b) Perusahaan Gas Negara and (c) PT. Kalbe Farma Indonesia Tbk

Figure 1 shows trends in all company’s stock price. It indicates that the data is nonstationary tested using unit root test Dickey-Fuller. Therefore, the transformation by differencing is applied to the data, so that it is stationary. Subsequently, to obtain the alternatives of ARIMA model, the ACF and PACF plots were presented. The parameters of each model were estimated using Least Square method and their significance were tested. Furthermore, the residual of each model was tested to know its white noise using Ljung-Box test and its normal distribution using Kolmogorov-Smirnov test. The best model used in forecasting should meet all of tests and has the smallest value of AIC and SC criterion, see [9]. In this case, we obtain the best ARIMA model to predict the stock price of PT. Unilever Indonesia Tbk., Perusahaan Gas Negara, and PT. Kalbe Farma Indonesia Tbk., each of those are ARIMA(1,1,0), ARIMA (1,1,1) and ARIMA (2,1,1).

The forecasting model of ARIMA is obtained based on historical data. Therefore, to forecast the stock price, that model should be improved to get better forecasting result. Nevertheless, to improve the model, we need to do the steps explained before recursively. Certainly, this process will bring difficulty and time consuming. Because of that, Kalman filter algorithm is applied to improve the forecasting result based on error correction from the direct output of ARIMA model. That error can be expressed as polynomial function given as follows [13]:

\[ y_i = a_{0,i} + a_{1,i} m_i + a_{2,i} m_i^2 + \cdots + a_{n-1,i} m_i^{n-1} + \epsilon_i \]

with \( y_i \) is error of forecasting, \( a_{j,i} \) is the parameter that has to be estimated by Kalman filter, with \( j = 0, 1, 2, ..., n - 1 \), \( m_i \) is direct output of ARIMA model, and \( \epsilon_i \)
is noise process. The detail process of implementing Kalman filter algorithm can be seen in [13]. In this case, we used third polynomial degree and the initial value for \([a_0, a_1, a_2, 0]\) is \([0.1, 0.2, 0.05]\), covariance matrices \(Q(t_i) = l_{3 \times 3} Q\) with \(Q = 0.1\), and \(R(t_i) = 0.01\). Figure 2 shows the forecasting result and Table 1 shows the comparison of MAPE between ARIMA and ARIMA-Kalman filter. The forecasting using ARIMA-Kalman filter has smaller error than ARIMA model. Based on Table 1, the accuracy of forecasting result using ARIMA-Kalman filter increase 44.7% for PT. Unilever Indonesia Tbk., 40.7% for Perusahaan Gas Negara, and 68.9% for Kalbe Farma Indonesia. It means that Kalman filter is able to improve the forecasting result of ARIMA.

![Figure 2: Stock Price Forecasting Result](image1)

**Figure 2.** Stock Price Forecasting Result of (a) PT. Unilever Indonesia Tbk. (b) Perusahaan Gas Negara and (c) PT. Kalbe Farma Indonesia Tbk

**Table 1.** MAPE(%) ARIMA Versus ARIMA-Kalman filter

<table>
<thead>
<tr>
<th>Stock</th>
<th>ARIMA</th>
<th>ARIMA-Kalman filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unilever</td>
<td>1.732790</td>
<td>0.957566</td>
</tr>
<tr>
<td>PGN</td>
<td>1.432664</td>
<td>0.849598</td>
</tr>
<tr>
<td>KFI</td>
<td>4.830665</td>
<td>1.502456</td>
</tr>
</tbody>
</table>
Based on that result, the performance of ARIMA-Kalman filter is better than ARIMA. Therefore, ARIMA-Kalman filter can be used exceptionally proper to forecast the stock price of PT. Unilever Indonesia Tbk., Perusahaan Gas Negara, and PT. Kalbe Farma Indonesia Tbk.. After we gained the stock price forecasting, the return from each stock is calculated to be used in portfolio optimization problem.

3 Implementation of Model Predictive Control (MPC) in Portfolio Optimization

The aim of portfolio optimization is to find the optimal portfolio, so that investors’s capital increase approaching the capital reference or the capital target. In this case, the formulation of optimal control in portfolio optimization problem, consists of the dynamic system of portfolio model, the constraints, and the objective function. In this paper, we follow [4] with $i = 1, 2, 3$ as the dynamic system of portfolio model, so $\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T$ as the state variable and $\mathbf{u} = [p_1 \ p_2 \ p_3 \ q_1 \ q_2 \ q_3 \ v]^T$ as the control variable. For more information detail regarding the model of investment portfolio can be seen in [4]. In this research, we assumed that short-selling is prohibited. Therefore, we rewrite the constraints for this problem as follow:

$$x_i(k) + p_i(k) - q_i(k) \geq 0$$

$$x_{n+1}(k) + \vartheta(k) - (1 + \alpha) \sum_{i=1}^{n} p_i(k) + (1 - \beta) \sum_{i=1}^{n} q_i(k) \geq 0$$

$$0 \leq x_{n+2}(k) + \vartheta(k) \leq d$$

$$0 \leq p_i(k) \leq p_{i\text{max}}$$

$$0 \leq q_i(k) \leq q_{i\text{max}}$$

$$\vartheta_{\text{min}}(k) \leq \vartheta(k) \leq \vartheta_{\text{max}}(k)$$

with $d$ is maximum amount of capital loan. Then, we define the objective function below:

$$j(\mathbf{u}(k), \mathbf{e}(k)) = \sum_{j=1}^{N_p} \mathbf{e}^T(k + j)Q\mathbf{e}(k + j) + \sum_{j=1}^{N_p} \mathbf{u}^T(k + j)R\mathbf{u}(k + j)$$

with $\mathbf{e}(k + j) = \mathbf{y}(k + j) - r(k + j)$, and $\mathbf{y}$ is the total capital of investors. It is an accumulation of investors’s capital in stock assets and in risk-free asset, but it is reduced by the number of capital loan asset. $r$ is the reference trajectory (the capital target of investors), $N_p$ is the prediction horizon, matrix $Q$ and $R$ are a positive semidefinit matrix. The goal of solving an optimal control problem is to find the optimal controller $\mathbf{u}$ that can minimize the objective function. The Optimal control method which is used in this MPC has a quadratic programming (QP) form. Therefore, the objective function can be reexpressed as QP below:
\begin{equation}
J(\breve{u}(k)) = \breve{u}^T(k)H\breve{u}(k) + 2f^T\breve{u}(k).
\end{equation}

The control optimal solution of the optimization problem in equation (1) is
\begin{equation}
\{\breve{u}^*(k), \breve{u}^*(k+1), \cdots, \breve{u}^*(k+N_p-1)\}.
\end{equation}

Based on principle of receding horizon in MPC, the optimal control value that is entered in the system is the first vector of its optimal solution in (2). Thus, the optimal control value is given by \(\breve{u}(k) = \breve{u}^*(k)\). Based on MPC algorithm, after inputting the optimal control value in the state, we obtain the state prediction value in the next step and the output value of the system \(\tilde{y}\). Moreover, the value of the output is included into optimization calculation to get the next optimal control value. This process is repeated until the output value of the system follows the reference trajectory given.

4 Simulation Result of Portfolio Optimization Problem

In the this simulation, the interest rate of bank is given by \(r_1 = 3 \times 10^{-5}\), the interest rate of the capital loan \(r_2 = 3.1 \times 10^{-4}\), the transaction cost of buying and selling stock is assumed same, \(\alpha = \beta = 2 \times 10^{-3}\), initial value of state variable \(x(0) = [0 \ 0 \ 0 \ 10^8 \ 0]^T\), \(d = 5 \times 10^8\), \(r(k) = 1.5 \times 10^8\), \(p_{imax} = q_{imax} = 1 \times 10^8\), and \(|\theta_{max}(k)| \leq 1 \times 10^8\).

The control variables of this problem are shown in Figure 3. Figure 3(a)-(c) show the value of \(p_t\) and \(q_t\), the fund transfer which is used for buying and selling stock respectively are in the constraint boundary. Another control variable, the fund transfer between risk-free asset and capital loan asset (\(\theta\)) is presented on Figure 3(d). It shown that the value of \(\theta\) is in its constraint boundary. If \(\theta > 0\), then it means that the investors get the capital loan to buy the stock. Whereas, if \(\theta < 0\), it means that the investors pay out the capital loan. MPC attempt to give the best decision related to the appropriate timing for investors to buy or sell the stock. Not only that, but also MPC give the best strategy regarding when the investors must borrow and give back the capital loan.
Figure 3. Control Variable in Portfolio Optimization: (a) \( p \) and \( q \) Value of PT. Unilever Indonesia Tbk. Stock, (b) \( p \) and \( q \) Value of Perusahaan Gas Negara Stock, (c) \( p \) and \( q \) Value of PT. Kalbe Farma Indonesia Tbk. Stock, (d) Transfer Value Between Risk-Free Asset and Capital Loan Asset.

Figure 4. Evolution of investors’s capital in stock assets and risk-free asset.

Figure 5. Total Capital of Investors

Figure 4 shows the comparison between investors’s capital in portfolio asset and in risk-free asset. From Figure 4, we know that MPC consider the investors to invest their capital in portfolio more than in the risk-free asset. The investors’s capital in portfolio is increase rapidly. It shows that invest in portfolio can provide more profit.
for investors than only save their capital in the bank. Moreover, based on Figure 5, we can see that total capital of investors as the output is increase. In this result, the optimal total capital given by MPC only can achieve $1.2057 \times 10^9$ at the end of the time period. It is reasonable because the time period in this case is really short, only one month (23 days). Thus, to meet the target, MPC still need a longer time. However, it has been adequate in order to show that MPC can give the best strategy into the stock investment management. Therefore, investors’s capital can increase at the end of time.

5 Conclusion

In this paper, it had been shown that Kalman filter can be used to improve the stock price forecasting result of ARIMA. From the MAPE of forecasting result, we can conclude that the performance of ARIMA-Kalman filter method is better than ARIMA method to forecast the stock price. Based on the stock price prediction, we calculated the return from each stock to be used in portfolio optimization problem. Moreover, we used MPC method in order to solve this problem. From the simulation result, we know that MPC is well-applied to solve portfolio optimization problem. In this case, the MPC is able to design the best controller that aims to control the wealth or capital of investors in stock portfolio investment. Moreover, the MPC acts as decision maker to determine the best strategy in stock investment. Therefore, the total capital of investors can increase at the end of time. In our advance research, we will solve the portfolio optimization problem to find the optimal time and to see the performance of MPC in order to meet the target.

References


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