Common Fixed Point Using Weakly Compatible and Property(E.A.) Maps in IFMS

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Abstract

In this paper, we prove common fixed point theorems using weakly compatible and property(E.A.) maps in IFMS. We improve and generalize the results of [1].

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1 Introduction

The notion of fuzzy sets was introduced by Zadeh[10] in 1965. The fuzzy metric space with the concept of fuzzy sets was introduced Kramosil and Michalek[4], Kaleva and Seikkala[3]. We consider the definition of fuzzy metric space suggested by George and Veeramani[2] which is a modification of the definition given in [4], and Pant[5] obtained the common fixed point for weakly compatible maps in fuzzy metric space.

Recently, Park et.al.([7], [8]) introduced the IFMS, and studied some results using property(E.A.) maps in IFMS. Also, Park[8] obtained fixed point theorem for common property(E.A.) and weakly compatible functions in IFMS.
In this paper, we prove common fixed point using weakly compatible and property (E.A.) maps with distinct conditions from Park [8] in IFMS. We improve and generalize the results of [1].

2 Preliminaries

Let us recall (see [9]) that a continuous $t$–norm is a operation $\ast : [0, 1] \times [0, 1] \rightarrow [0, 1]$ which satisfies the following conditions: (a) $\ast$ is commutative and associative, (b) $\ast$ is continuous, (c) $a \ast 1 = a$ for all $a \in [0, 1]$, (d) $a \ast b \leq c \ast d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0, 1])$. Also, a continuous $t$–conorm is a operation $\diamond : [0, 1] \times [0, 1] \rightarrow [0, 1]$ which satisfies the following conditions: (a) $\diamond$ is commutative and associative, (b) $\diamond$ is continuous, (c) $a \diamond 0 = a$ for all $a \in [0, 1]$, (d) $a \diamond b \geq c \diamond d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0, 1])$.

**Definition 2.1.** ([7]) The 5–tuple $(X, M, N, \ast, \diamond)$ is said to be an intuitionistic fuzzy metric space (Shortly, IFMS) if $X$ is an arbitrary set, $\ast$ is a continuous $t$–norm, $\diamond$ is a continuous $t$–conorm and $M, N$ are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions; for all $x, y, z \in X$, such that

(a) $M(x, y, t) > 0$,
(b) $M(x, y, t) = 1$ if and only if $x = y$,
(c) $M(x, y, t) = M(y, x, t)$,
(d) $M(x, y, t) \ast M(y, z, s) \leq M(x, z, t + s)$,
(e) $M(x, y, \cdot) : (0, \infty) \rightarrow (0, 1]$ is continuous,
(f) $N(x, y, t) > 0$,
(g) $N(x, y, t) = 0$ if and only if $x = y$,
(h) $N(x, y, t) = N(y, x, t)$,
(i) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$,
(j) $N(x, y, \cdot) : (0, \infty) \rightarrow (0, 1]$ is continuous.

Note that $(M, N)$ is called an IFM on $X$. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between $x$ and $y$ with respect to $t$, respectively.

Let $X$ be an IFMS. For $t > 0$, the open ball $B(x, r, t)$ with center $x \in X$ and radius 0 < $r < 1$ is defined by

$$B(x, r, t) = \{ y \in X : M(x, y, t) > 1 - r, \ N(x, y, t) < r \}.$$  

A subset $A \subset X$ is called open if for each $x \in A$, there exist $t > 0$ and 0 < $r < 1$ such that $B(x, r, t) \subset A$.

**Definition 2.2.** ([8]) Let $\{x_n\}$ be a sequence of IFMS $X$.

(a) $\{x_n\}$ is convergent to a point $x \in X$, if for all $t > 0$,

$$\lim_{n \to \infty} M(x_n, x, t) = 1, \ \lim_{n \to \infty} N(x_n, x, t) = 0.$$
(b) \( \{x_n\} \) is called a Cauchy sequence if for \( t > 0 \) and \( p > 0 \),

\[
\lim_{n \to \infty} M(x_n, x_{n+p}, t) = 1, \quad \lim_{n \to \infty} N(x_n, x_{n+p}, t) = 0.
\]

(c) \( X \) is complete if every Cauchy sequence converges in \( X \).

**Definition 2.3.** ([8]) Let \( (A, B) \) be a pair of self-maps on IFMS \( X \). (a) \( (A, B) \)
is compatible (or asymptotically commuting), if for all \( t > 0 \),

\[
\lim_{n \to \infty} M(ABx_n, BAx_n, t) = 1, \quad \lim_{n \to \infty} N(ABx_n, BAx_n, t) = 0
\]

whenever \( \{x_n\} \subset X \) such that \( \lim_{n \to \infty} Ax_n = \lim_{n \to \infty} Bx_n = x \) for some \( x \in X \).

(b) \( (A, B) \) is non-compatible, if \( (A, B) \) is not compatible, that is, if there exists a sequence \( \{x_n\} \) in \( X \) such that \( \lim_{n \to \infty} Ax_n = \lim_{n \to \infty} Bx_n = x \) for some \( x \in X \), and

\[
\lim_{n \to \infty} M(ABx_n, BAx_n, t) \neq 1, \quad \lim_{n \to \infty} N(ABx_n, BAx_n, t) \neq 0
\]

or non-existent for all \( t > 0 \).

(c) \( (A, B) \) is said to be weakly compatible, if for \( x \in X \), \( Ax = Bx \) implies that \( ABx = BAx \).

(d) \( (A, B) \) satisfy the property (E.A.), if there exists \( \{x_n\} \subset X \) such that for some \( u \in X \) and all \( t > 0 \),

\[
\lim_{n \to \infty} M(Ax_n, u, t) = \lim_{n \to \infty} M(Bx_n, u, t) = 1,
\]
\[
\lim_{n \to \infty} N(Ax_n, u, t) = \lim_{n \to \infty} N(Bx_n, u, t) = 0.
\]

**Example 2.4.** Let \( X = R \) and for every \( x, y \in X \), \( t > 0 \),

\[
M(x, y, t) = \frac{t}{t + d(x, y)}, \quad N(x, y, t) = \frac{d(x, y)}{t + d(x, y)}.
\]

Define the maps \( A \) and \( B \) by \( Ax = 2x + 1 \), \( Bx = x + 2 \). Also, define the sequence \( \{x_n\} \subset X \) by \( x_n = 1 + \frac{1}{n}, \ n = 1, 2, \ldots \). We have for every \( t > 0 \),

\[
\lim_{n \to \infty} M(Ax_n, 3, t) = \lim_{n \to \infty} M(Bx_n, 3, t) = 1,
\]
\[
\lim_{n \to \infty} N(Ax_n, 3, t) = \lim_{n \to \infty} N(Bx_n, 3, t) = 0.
\]

Hence the pair \( (A, B) \) satisfies the property (E.A.). Since \( ABu \neq BAu \), \( (A, B) \) is not weakly compatible.
3 Main results

Let $F$ be the set of all fuzzy set on $X^2 \times [0, \infty)$. That is,

$$F = \{f, g : X^2 \times [0, \infty) \to [0, 1]\}.$$ 

**Definition 3.1.** Let $f, g \in F$. The algebraic sum and difference $f \oplus f$, $g \ominus g$ of $f$ and $g$ is defined by

$$f(x, y, t) \oplus f(x', y', t') = \sup_{t_1 + t_2 = t} \min\{f(x, y, t_1), f(x', y', t_2)\},$$

$$g(x, y, t) \ominus g(x', y', t') = \inf_{t_1 + t_2 = t} \max\{g(x, y, t_1), g(x', y', t_2)\}.$$

Note that for every $x, y \in X$ and $t > 0$,

(a) $f(x, y, 2t) \oplus f(x, y, 2t) \geq \min\{f(x, y, t), f(x, y, t)\} = f(x, y, t)$,

(b) $g(x, y, 2t) \ominus g(x, y, 2t) \leq \max\{g(x, y, t), g(x, y, t)\} = g(x, y, t)$,

(c) $f(x, y, t) \oplus 1 \geq \min\{f(x, y, t - \epsilon), f(x, x, \epsilon)\} = f(x, y, t - \epsilon)$,

(d) $g(x, y, t) \ominus 0 \leq \max\{g(x, y, t - \epsilon), g(x, x, \epsilon)\} = g(x, y, t - \epsilon)$. Let $\epsilon \to 0$,

we have

$$f(x, y, t) \oplus 1 \geq f(x, y, t), \quad g(x, y, t) \ominus 0 \leq g(x, y, t).$$

In this section, $\Phi$ denotes a family of maps such that for each $\phi, \psi \in \Phi$, $\phi, \psi : [0, 1]^3 \to [0, 1]$, $\phi$ is continuous and increasing in each co-ordinate variable and $\psi$ is continuous and decreasing in each co-ordinate variable. Also, $\phi(t, t, t) \geq t$ and $\psi(t, t, t) \leq t$ for every $t \in [0, 1]$.

**Theorem 3.2.** Let $A, B, S$ and $T$ be the self maps from an IFMS $X$ satisfying for all $x, y \in X$, $t > 0$, $\phi, \psi \in \Phi$ and some $0 \leq k < 2$,

$$A(X) \subseteq T(X), \quad B(X) \subseteq S(X), \quad (3.1)$$

$$M(Ax, By, t) \geq \phi\{M(Sx, Ty, \frac{2t}{k}), M(Ax, Sx, \frac{2t}{k}) \oplus M(By, Ty, \frac{2t}{k})\},$$

$$M(Ax, Ty, \frac{4t}{k}) \oplus M(Sx, By, \frac{4t}{k})\}, \quad (3.2)$$

$$N(Ax, By, t) \leq \psi\{N(Sx, Ty, \frac{2t}{k}), N(Ax, Sx, \frac{2t}{k}) \ominus N(By, Ty, \frac{2t}{k})\},$$

$$N(Ax, Ty, \frac{4t}{k}) \ominus N(Sx, By, \frac{4t}{k})\}.\quad (3.3)$$

Suppose that

(a) one of the pairs $(A, S)$ and $(B, T)$ satisfies the property(E.A.),

(b) $(A, S)$ and $(B, T)$ are weakly compatible,

(c) one of $A(X)$, $B(X)$, $S(X)$ and $T(X)$ is a complete subspace of $X$.

Then $A, B, S$ and $T$ have a unique common fixed point in IFMS $X$. 

Proof. Suppose that from (a), \((B,T)\) satisfies the property(E.A.), then there exists a sequence \(\{x_n\} \subset X\) such that for some \(u \in X\) and all \(t > 0\),

\[
\lim_{n \to \infty} M(Bx_n, u, t) = \lim_{n \to \infty} M(Tx_n, u, t) = 1,
\]

\[
\lim_{n \to \infty} N(Bx_n, u, t) = \lim_{n \to \infty} N(Tx_n, u, t) = 0.
\]

From the above equation,

\[
\lim_{n \to \infty} M(Bx_n, Tx_n, t) = 1, \quad \lim_{n \to \infty} N(Bx_n, Tx_n, t) = 0.
\]

Since \(B(X) \subseteq S(X)\), there exists a sequence \(\{y_n\} \subset X\) such that \(Bx_n = Sy_n\). Thus

\[
\lim_{n \to \infty} M(Sy_n, u, t) = 1, \quad \lim_{n \to \infty} N(Sy_n, u, t) = 0.
\]

Now, we prove that \(\lim_{n \to \infty} M(Ay_n, u, t) = 1\), \(\lim_{n \to \infty} N(Ay_n, u, t) = 0\).

From (3.2), we obtain

\[
M(Ay_n, Bx_n, t) \geq \phi\{M(Sy_n, Tx_n, \frac{2t}{k}), M(Ay_n, Sy_n, \frac{2t}{k}) \oplus M(Bx_n, Tx_n, \frac{2t}{k})\},
\]

\[
M(Ay_n, Tx_n, \frac{4t}{k}) \oplus M(Sy_n, Bx_n, \frac{4t}{k})\}.
\]

\[
= \phi\{M(Bx_n, Tx_n, \frac{2t}{k}), M(Ay_n, Bx_n, \frac{2t}{k}) \oplus M(Bx_n, Tx_n, \frac{2t}{k})\},
\]

\[
M(Ay_n, Tx_n, \frac{4t}{k}) \oplus 1\}, \quad (3.3)
\]

\[
N(Ay_n, Bx_n, t) \leq \psi\{N(Sy_n, Tx_n, \frac{2t}{k}), N(Ay_n, Sy_n, \frac{2t}{k}) \ominus N(Bx_n, Tx_n, \frac{2t}{k})\},
\]

\[
N(Ay_n, Tx_n, \frac{4t}{k}) \ominus N(Sy_n, Bx_n, \frac{4t}{k})\}.
\]

\[
= \psi\{N(Bx_n, Tx_n, \frac{2t}{k}), N(Ay_n, Bx_n, \frac{2t}{k}) \ominus N(Bx_n, Tx_n, \frac{2t}{k})\},
\]

\[
N(Ay_n, Tx_n, \frac{4t}{k}) \ominus 0\}.
\]

Since

\[
\lim_{n \to \infty} \inf M(Ay_n, Bx_n, \frac{2t}{k}) \oplus M(Bx_n, Tx_n, \frac{2t}{k})
\]

\[
\geq \lim_{n \to \infty} \inf \min\{M(Ay_n, Bx_n, \frac{2t}{k} - \epsilon), M(Bx_n, Tx_n, \epsilon)\}
\]

\[
= \lim_{n \to \infty} \inf M(Ay_n, Bx_n, \frac{2t}{k} - \epsilon),
\]
we have as $\epsilon \to 0,$

$$\liminf_{n \to \infty} M(A_{y_n}, B_{x_n}, \frac{2t}{k}) \oplus M(B_{x_n}, T_{x_n}, \frac{2t}{k}) \geq \liminf_{n \to \infty} M(A_{y_n}, B_{x_n}, \frac{2t}{k}),$$

$$\limsup_{n \to \infty} N(A_{y_n}, B_{x_n}, \frac{2t}{k}) \ominus N(B_{x_n}, T_{x_n}, \frac{2t}{k}) \leq \limsup_{n \to \infty} N(A_{y_n}, B_{x_n}, \frac{2t}{k}).$$

Furthermore, we have, from given conditions

$$\lim\inf_{n \to \infty} M(A_{y_n}, u, t) = \lim\inf_{n \to \infty} M(A_{y_n}, B_{x_n}, t) \geq \phi\{\lim\inf_{n \to \infty} M(B_{x_n}, T_{x_n}, \frac{2t}{k}), \lim\inf_{n \to \infty}\{M(A_{y_n}, B_{x_n}, \frac{2t}{k}) \oplus M(B_{x_n}, T_{x_n}, \frac{2t}{k})\},$$

$$\lim\inf_{n \to \infty}\{M(A_{y_n}, T_{x_n}, 2\frac{t}{k}) \oplus 1\}\}

\geq \phi\{M(\lim\inf_{n \to \infty} A_{y_n}, u, \frac{2t}{k}), M(\lim\inf_{n \to \infty} A_{y_n}, u, \frac{2t}{k}), M(\lim\inf_{n \to \infty} A_{y_n}, u, \frac{2t}{k})\}

\geq M(\lim\inf_{n \to \infty} A_{y_n}, u, \frac{2t}{k}) \geq \cdots \geq M(\lim\inf_{n \to \infty} A_{y_n}, u, (\frac{2}{k})^n t) \to 1,$$

$$\lim\sup_{n \to \infty} N(A_{y_n}, u, t) = \lim\sup_{n \to \infty} N(A_{y_n}, B_{x_n}, t) \leq \lim\sup_{n \to \infty}\{N(B_{x_n}, T_{x_n}, 2\frac{t}{k}) \ominus N(B_{x_n}, T_{x_n}, \frac{2t}{k})\},$$

$$\lim\sup_{n \to \infty}\{N(A_{y_n}, T_{x_n}, 4\frac{t}{k}) \oplus 0\}\}

\leq \psi\{N(\lim\sup_{n \to \infty} A_{y_n}, u, \frac{2t}{k}), N(\lim\sup_{n \to \infty} A_{y_n}, u, \frac{2t}{k}), N(\lim\sup_{n \to \infty} A_{y_n}, u, \frac{2t}{k})\}

\leq N(\lim\sup_{n \to \infty} A_{y_n}, u, \frac{2t}{k}) \leq \cdots \leq N(\lim\sup_{n \to \infty} A_{y_n}, u, (\frac{2}{k})^n t) \to 0,
By same method,
\[
\limsup_{n \to \infty} M(Ay_n, u, t) = M(\limsup_{n \to \infty} Ay_n, u, t) = 1,
\]
\[
\liminf_{n \to \infty} N(Ay_n, u, t) = N(\liminf_{n \to \infty} Ay_n, u, t) = 0.
\]
Thus, \( \lim_{n \to \infty} M(Ay_n, u, t) = 1 \), \( \lim_{n \to \infty} N(Ay_n, u, t) = 0 \). Suppose that \( S(X) \) is complete subspace of \( X \), then, there exists \( v \in X \) such that \( Sv = u \). From (3.2), we get
\[
M(Av, Bx_n, t)
\geq \phi\{M(Sv, Tx_n, \frac{2t}{k}), M(Av, Sv, \frac{2t}{k}) \oplus M(Bx_n, Tx_n, \frac{2t}{k})\}
\]
\[
= \phi\{M(u, Tx_n, \frac{2t}{k}), M(Av, u, \frac{2t}{k}) \oplus M(Bx_n, Tx_n, \frac{2t}{k})\}
\]
\[
= \phi\{M(u, Tx_n, \frac{2t}{k}), M(Av, u, \frac{2t}{k}) \oplus M(Bx_n, Tx_n, \frac{2t}{k})\}
\]
\[
N(Av, Bx_n, t)
\leq \psi\{N(Sv, Tx_n, \frac{2t}{k}), N(Av, Sv, \frac{2t}{k}) \oplus N(Bx_n, Tx_n, \frac{2t}{k})\}
\]
\[
= \psi\{N(u, Tx_n, \frac{2t}{k}), N(Av, u, \frac{2t}{k}) \oplus N(Bx_n, Tx_n, \frac{2t}{k})\}
\]
Since \( \lim_{n \to \infty} Bx_n = \lim_{n \to \infty} Tx_n = Sv \) and for all \( \epsilon \in (0, \frac{t}{k}) \),
\[
M(Av, Sv, \frac{2t}{k}) \oplus M(Bx_n, Tx_n, \frac{2t}{k}) \geq M(Av, Sv, \frac{2t}{k} - \epsilon),
\]
\[
N(Av, Sv, \frac{2t}{k}) \oplus N(Bx_n, Tx_n, \frac{2t}{k}) \leq N(Av, Sv, \frac{2t}{k} - \epsilon).
\]
Hence we have, as \( \epsilon \to 0 \),
\[
\liminf_{n \to \infty} \{M(Av, Sv, \frac{2t}{k}) \oplus M(Bx_n, Tx_n, \frac{2t}{k})\} \geq M(Av, Sv, \frac{2t}{k}),
\]
\[
\limsup_{n \to \infty} \{N(Av, Sv, \frac{2t}{k}) \oplus N(Bx_n, Tx_n, \frac{2t}{k})\} \leq N(Av, Sv, \frac{2t}{k})
\]
Also,
\[
\liminf_{n \to \infty} \{M(Av, Tx_n, \frac{4t}{k}) \oplus M(Sv, Bx_n, \frac{4t}{k})\} \geq M(Av, u, \frac{2t}{k}),
\]
\[
\limsup_{n \to \infty} \{N(Av, Tx_n, \frac{4t}{k}) \oplus N(Sv, Bx_n, \frac{4t}{k})\} \leq N(Av, u, \frac{2t}{k}).
\]
Therefore, letting $n \rightarrow \infty$ in (3.4),

$$M(Av, u, t) \geq \phi\left\{1, M(Av, u, \frac{2t}{k})\right\} \geq \frac{M(Av, u, 2t)}{k}$$

$$\cdots \cdots \geq \frac{M(Av, u, (\frac{2}{k})^n t)}{k} \rightarrow 1,$$

$$N(Av, u, t) \leq \psi\left\{0, N(Av, u, \frac{2t}{k})\right\} \leq \frac{N(Av, u, 2t)}{k}$$

$$\cdots \cdots \leq \frac{N(Av, u, (\frac{2}{k})^n t)}{k} \rightarrow 0.$$

Hence, $M(Av, u, t) = 1$ and $N(Av, u, t) = 0$. That is, $Av = Sv = u$.

Also, since $A(X) \subseteq T(X)$, there exists $z \in X$ such that $u = Tz$. By same method with the above pattern and using (3.2), we get $u = Bz = Tz$. Since $(A, S)$ and $(B, T)$ are weakly compatible, we have $Au = Su$ and $Bu = Tu$.

From (3.2), we have

$$M(Au, u, t) = M(Au, Bz, t)$$

$$\geq \phi\left\{M(Su, Tz, \frac{2t}{k}), M(Au, Su, \frac{2t}{k}) \oplus M(Bz, Tz, \frac{2t}{k})\right\}$$

$$\geq \frac{M(Au, u, 2t)}{k} \geq \cdots \geq \frac{M(Au, u, (\frac{2}{k})^n t)}{k} \rightarrow 1,$$

$$N(Au, u, t) = N(Au, Bz, t)$$

$$\leq \psi\left\{N(Su, Tz, \frac{2t}{k}), N(Au, Su, \frac{2t}{k}) \oplus N(Bz, Tz, \frac{2t}{k})\right\}$$

$$\leq \frac{N(Au, u, 2t)}{k} \leq \cdots \leq \frac{N(Au, u, (\frac{2}{k})^n t)}{k} \rightarrow 0.$$

Thus $Au = Su = u$. Also, we can prove that $Bu = Tu = u$. Hence $u$ is a common fixed point of $A, B, S$ and $T$.

Furthermore, if $y(y \neq u)$ be another common fixed point of $A, B, S$ and $T$. Then from (3.2), we get

$$M(u, y, t) = M(Au, By, t)$$

$$\geq \phi\left\{M(Su, Ty, \frac{2t}{k}), M(Au, Su, \frac{2t}{k}) \oplus M(By, Ty, \frac{2t}{k})\right\}$$

$$\geq \frac{M(u, y, 2t)}{k} \geq \cdots \geq \frac{M(u, y, (\frac{2}{k})^n t)}{k} \rightarrow 1,$$
\[ N(u, y, t) = N(Au, By, t) \]
\[ \leq \psi \{ N(Su, Ty, \frac{2t}{k}), N(Au, Su, \frac{2t}{k}) \ominus N(By, Ty, \frac{2t}{k}), \]
\[ N(Au, Ty, \frac{4t}{k}) \ominus N(Su, By, \frac{4t}{k}) \} \]
\[ \leq N(u, y, \frac{2t}{k}) \leq \cdots \leq N(u, y, (\frac{2}{k})^n t) \to 0, \]

which is a contradiction. Hence \( u = y \). That is, \( u \) is unique common fixed point of \( A, B, S \) and \( T \). this complete the proof. \qed

**Corollary 3.3.** Let \( A \) and \( S \) be the self maps from an IFMS \( X \) satisfying for all \( x, y \in X, t > 0, \phi, \psi \in \Phi \) and some \( 0 \leq k < 2, \)
\[ A(X) \subseteq S(X), \]
\[ M(Ax, Ay, t) \]
\[ \geq \phi \{ M(Sx, Sy, \frac{2t}{k}), M(Ax, Sx, \frac{2t}{k}) \oplus M(Ay, Sy, \frac{2t}{k}), \]
\[ M(Ax, Sy, \frac{4t}{k}) \oplus M(Sx, Ay, \frac{4t}{k}) \} \}, \]
\[ N(Ax, Ay, t) \]
\[ \leq \psi \{ N(Sx, Sy, \frac{2t}{k}), N(Ax, Sx, \frac{2t}{k}) \ominus N(Ay, Sy, \frac{2t}{k}), \]
\[ N(Ax, Sy, \frac{4t}{k}) \ominus N(Sx, Ay, \frac{4t}{k}) \} \}.

Suppose that
(a) \((A, S)\) satisfy the property(E.A.),
(b) \((A, S)\) is weakly compatible,
(c) one of \( A(X) \) and \( S(X) \) is a complete subspace of \( X \).
Then \( A \) and \( S \) have a unique common fixed point in IFMS \( X \).

**Proof.** From Theorem 3.2, we obtain the result of Corollary 3.3 as \( B = A \) and \( T = S \). \qed

**References**

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