On the Modification of Lattice Boltzmann Method

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Abstract

Modification of the operator–splitting–based lattice Boltzmann method for the case of incompressible fluid flows is proposed. The method is based on the splitting of the operator in Navier — Stokes equations. Main feature of the method proposed is the usage of explicit finite-difference schemes. The efficiency of the method proposed is demonstrated by the solution of 2D lid-driven cavity flow problem.

Mathematics Subject Classification: 76-04; 76M10; 65N30

Keywords: lattice Boltzmann method, operator splitting

1 Introduction

The lattice Boltzmann method (LBM) is a useful approach for computational fluid dynamics (CFD). Instead of solving of Navier — Stokes equations, in the mesoscopic LBM-based approach a set of equations derived from kinetic theory is considered [7],[8]. Simple implementation of boundary conditions together with inherent grid generation makes the method geometrically very versatile, and it is thus convenient for simulation of complex hydrodynamical systems.

In this study, a modification of operator–splitting–based LBM proposed by C. Shu et al in [14] is considered. The modification is based on the LBM scheme for computations of incompressible flows proposed by X. He and L. S. Luo in [6]. Algorithm of the modified method include three stages of the computation on every time step. The main feature of the modification proposed is the usage of explicit computational rules. The efficiency of the modified method is demonstrated by the solution of 2D lid-driven cavity flow problem.
The paper is organized as follows. In Section 2 the variant of LBM proposed by He and Luo is discussed. In Section 3 the modification of operator–splitting–based LBM is considered. In Section 4 2D lid-driven cavity flow problem is solved and results of computations are discussed. Concluding remarks are made in Section 5.

2 Lattice Boltzmann scheme for incompressible flows

In this study isothermal incompressible flows of viscous Newtonian fluids are considered. The attention is focused on flows in 2D domains. In LBM the fluid is represented as an ensemble of mesoscopic particles that can move on a regular spatial lattice and undergo collisions at its nodes. Velocities of the particles are formed a discrete set of vectors $V_i = V v_i, i = 1, \ldots, n$, where $V = l/\delta t$ is a typical velocity, $\delta t$ is a time step, $l$ is a typical spacing. In the case of planar flows so-called D2Q9 lattice can be used. This lattice is defined by the following dimensionless vectors:

$v_1 = (0,0), \ v_2 = (1,0), \ v_3 = (0,1), \ v_4 = (-1,0), \ v_5 = (0,-1), \ v_6 = (1,1), \ v_7 = (-1,1), \ v_8 = (-1,-1), \ v_9 = (1,-1)$.

The distribution functions $f_i$ of the particles with velocities $V_i$ are used as main variables. The dynamics of the particle ensemble is described by the following system of lattice Boltzmann equations (LBEs):

$$f_i(t_j + \delta t, r_{kl} + V_i \delta t) - f_i(t_j, r_{kl}) = -\frac{1}{\tau} \left( f_i(t_j, r_{kl}) - f_i^{(eq)}(t_j, r_{kl}) \right),$$

where $r_{kl} = (x_k, y_l)$ is a lattice node, $t_j$ is a time node, $\tau$ is a dimensionless relaxation time, $f_i^{(eq)}$ are equilibrium distribution functions.

The fluid density $\rho$ and the velocity $u$ at a lattice node are calculated as:

$$\rho(t_j, r_{kl}) = \sum_{i=1}^{9} f_i(t_j, r_{kl}), \ \rho(t_j, r_{kl})u(t_j, r_{kl}) = \sum_{i=1}^{9} V_i f_i(t_j, r_{kl}).$$

In applications of LBM to incompressible flows, the case of small values of Mach number $M = |u|/V$ is considered. In [1], [13] links of LBM with artificial compressibility method proposed in [2] are demonstrated.

Special type of LBM for the computations of incompressible flows is proposed by X. He and L. S. Luo in [6]. The scheme is constructed for the fluids with constant density $\rho_0$. The object of their study is a weakly compressible
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fluid with density $\rho(t, r) = \rho_0 + \delta \rho(t, r)$, where $|\delta \rho| << 1$. According to this assumption, the following expressions for $f_i^{(eq)}$ are obtained:

$$f_i^{(eq)} = W_i \left( \rho + \rho_0 \left( 3 \frac{(V_i \cdot u)}{V^2} + \frac{9}{2} \frac{(V_i \cdot u)^2}{V^4} - \frac{3}{2} \frac{u^2}{V^2} \right) \right),$$

where $W_i = \frac{4}{9}; i = 1, \frac{1}{9}; i = 2, 3, 4, 5, \frac{1}{36}; i = 6, 7, 8, 9$.

For the case of incompressible flows one of the main problems is the computation of pressure $p(t, r)$. For the calculation of $p$, in [6] with the usage of the state equation $p = c_s^2 \rho$ (where $c_s^2 = V/3$ for D2Q9 lattice) new variables $p_i = c_s^2 f_i$ are introduced. The evolution of $p_i$ is described by:

$$p_i(t_j + \delta t, r_{kl} + V_i \delta t) - p_i(t_j, r_{kl}) = -\frac{1}{\tau} \left( p_i(t_j, r_{kl}) - p_i^{(eq)}(t_j, r_{kl}) \right), \quad (3)$$

where

$$p_i^{(eq)} = W_i \left( p + p_0 \left( 3 \frac{(V_i \cdot u)}{V^2} + \frac{9}{2} \frac{(V_i \cdot u)^2}{V^4} - \frac{3}{2} \frac{u^2}{V^2} \right) \right),$$

where $p_0 = c_s^2 \rho_0$.

The values of $p$ and $u$ at lattice nodes are calculated from the values of $p_i$ by the following formulas:

$$p(t_j, r_{kl}) = \sum_{i=1}^{9} p_i(t_j, r_{kl}), \quad p_0 u(t_j, r_{kl}) = \sum_{i=1}^{9} V_i p_i(t_j, r_{kl}), \quad (4)$$

which are obtained from (2) with the usage of assumption $|\delta \rho| << 1$.

In [6] by the method of Chapman — Enskog expansion the following macroscopic equations for $p$ and $u$ are obtained:

$$\frac{1}{\rho_0 c_s^2} \frac{\partial p}{\partial t} + \nabla \cdot u + O(M^3) = 0, \quad (5)$$

$$\frac{\partial u}{\partial t} + (\nabla u) \cdot u = -\frac{1}{\rho_0} \nabla p + \nu \Delta u + O(M^3), \quad (6)$$

where $\nu$ is the kinematic viscosity, that is calculated from relaxation time $\tau$ by the following formulae:

$$\nu = \left( \tau - \frac{1}{2} \right) \frac{l^2}{3\delta t}. \quad (7)$$

Term $1/(\rho_0 c_s^2) \partial p / \partial t$ in (5) is of order $O(M^2)$ [6]. Thus, the terms of order $O(M^2)$ and $O(M^3)$ may be considered as negligible at $M << 1$ and system (5) – (6) takes the form of standard Navier — Stokes system. In this study the terms of order $O(M^3)$ are considered as negligible, while the term of order $O(M^2)$ plays important role in explicit calculation of $p(t, r)$. 
3 Operator–splitting–based lattice Boltzmann method

Operator–splitting–based approach to LBM is introduced in [14]. The approach is based on the decoupling of differential operator in Navier — Stokes system. This operation provide the possibility to subdivide computation process on two stages — on predictor and corrector steps. On the predictor stage LBEs with fictitious relaxation time are solved. On the corrector step the system of linear diffusion equations is solved. In [3] fractional step LBM is proposed to the simulation of thermal flows for the case of high Peclet number. In [4] the computations of two-phase systems are performed. In [15] the simulations of swirling and rotating axissymmetric flows are performed. In this study the modification of operator–splitting–based approach is introduced. The basis idea is to use different LBEs on different time substeps. The presented method provide the possibility to use explicit schemes.

Operator splitting is performed by inclusion of fictitious viscosity $\nu_1$ into eq. (6):

$$\frac{\partial \mathbf{u}}{\partial t} + (\nabla \mathbf{u}) \cdot \mathbf{u} = -\frac{1}{\rho_0} \nabla p + (\nu - \nu_1) \Delta \mathbf{u} + \nu_1 \Delta \mathbf{u}. \quad (8)$$

By denoting of operators $A_1(\mathbf{u}, p) = - (\nabla \mathbf{u}) \cdot \mathbf{u} - \frac{1}{\rho_0} \nabla p + \nu_1 \Delta \mathbf{u}$ and $A_2(\mathbf{u}) = (\nu - \nu_1) \Delta \mathbf{u}$ system (5),(8) could be rewritten as:

$$\frac{1}{\rho_0 c_s^2} \frac{\partial p}{\partial t} + \nabla \cdot \mathbf{u} = 0, \quad \frac{\partial \mathbf{u}}{\partial t} = A_1(\mathbf{u}, p) + A_2(\mathbf{u}). \quad (9)$$

According to fractional step method, for the solution of system (9) on time interval $[t_j, t_{j+1}]$, where $t_{j+1} = t_j + \delta t$, the fractional time node $t_{j+1/2} = t_j + \delta t/2$ is introduced.

On the first stage of the method the system:

$$\frac{1}{\rho_0 c_s^2} \frac{\partial p}{\partial t} + \nabla \cdot \mathbf{u} = 0, \quad \frac{\partial \mathbf{u}}{\partial t} = A_1(\mathbf{u}, p), \quad (10)$$

is solved on time interval $[t_j, t_{j+1/2}]$.

On the second stage the system:

$$\frac{\partial \mathbf{u}}{\partial t} = A_2(\mathbf{u}), \quad (11)$$

is solved on interval $[t_{j+1/2}, t_{j+1}]$. Solution of (10) at $t = t_{j+1/2}$ $\mathbf{u}(t_{j+1/2}, \mathbf{r})$ is considered as initial condition for (11).

On the third stage the computation of $p$ at $t = t_{j+1}$ is performed by finite difference (FD) scheme presented below.
For the solution of (10) LBM with fictitious relaxation time \( \tau_1 \) correspond to \( \nu_1 \) is used. The value of \( \tau_1 \) can be varied for the stabilization of LBM. In [14] the following value of \( \tau_1 \) is used: \( \tau_1 = 1 \), that satisfy the stability condition of LBM. In this study we use this value of \( \tau_1 \) for the stabilization of the method proposed.

Thus, on the first stage of the method on the time interval \([t_j, t_{j+1/2}]\) the following system of LBEs obtained from (3) at \( \tau = 1 \) is solved:

\[
p_i \left( t_j + \frac{\delta t}{2}, r_{kl} + V_{i} \frac{\delta t}{2} \right) = p_i^{(eq)}(t_j, r_{kl}).
\] (12)

System (11) is formed by two independent linear diffusion equations. By the appropriate choice of FD scheme another possibility for the stabilization of the method is realized. In the present study the lattice Boltzmann scheme for linear diffusion equation which is based on the usage of D2Q4 lattice [9], defined by vectors \( V_i = V v_i, i = 1, \ldots, 4 \), where:

\[
V_1 = (1, 0), \quad V_2 = (0, 1), \quad V_3 = (-1, 0), \quad V_4 = (0, -1).
\]

For the solution of linear diffusion equation (LDE)

\[
\frac{\partial c}{\partial t} = D \Delta c,
\]

where \( D \) is a diffusion coefficient, the system of LBEs

\[
g_i(t_j + \delta t, r_{kl}) - g_i(t_{j+1/2}, r_{kl} - V_i \frac{\delta t}{2}) = \\
= -\frac{1}{\chi} \left( g_i(t_{j+1/2}, r_{kl} - V_i \frac{\delta t}{2}) - g_i^{(eq)}(t_{j+1/2}, r_{kl} - V_i \frac{\delta t}{2}) \right),
\] (13)

is solved. In (13) \( g_i \) are the fictitious distribution functions. Solution of LDE is calculated by the following formulae:

\[
c(t_{j+1}, r_{kl}) = \sum_{i=1}^{4} g_i(t_{j+1}, r_{kl}).
\] (14)

Formulas for \( g_i^{(eq)} \) are written as [9]:

\[
g_i^{(eq)} = \frac{c}{4}.
\] (15)

Diffusion coefficient is calculated as:

\[
D = \left( \chi - \frac{1}{2} \right) \frac{l^2}{2 \delta t}.
\]
Stability condition for the solutions of LDE has the following form: $D \geq 0$. In equations of system (11) $D$ is equal to $\nu - \nu_1$. According to (7) and to $\tau_1 = 1$, the stability condition of the method proposed has the following form:

$$
\nu - \nu_1 \geq 0 \Rightarrow (\tau - 1) \frac{l^2}{3 \delta t} \geq 0 \Rightarrow \tau \geq 1.
$$

This inequality restricts the possibilities of the method applications only to the cases of small and moderate Reynolds numbers.

On the third stage of the method the calculation of pressure $p$ at the moment $t = t_j + 1$ is performed. After the multiplication on $p_0 = \rho_0c_s^2$ the following equation is obtained from (5):

$$
\frac{\partial p}{\partial t} + p_0 \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) = 0. \tag{16}
$$

Explicit FD scheme for the solution of (16) can be constructed by the standard technique. Time derivative $\partial p/\partial t$ is approximated by backward difference:

$$
\frac{\partial p}{\partial t}(t_{j+1}, x_k, y_l) \approx \frac{p(t_{j+1}, x_k, y_l) - p(t_{j+1/2}, x_k, y_l)}{\delta t},
$$

where the value of $p$ at the node $(x_k, y_l)$ could be calculated by linear spline interpolation of the values of $p$ at nodes $(x_k - \delta t/2V_i, y_l - \delta t/2V_i)$.

Derivatives of $u_x$ and $u_y$ are approximated by central differences with the second order:

$$
\frac{\partial u_x}{\partial x}(t_{j+1}, x_k, y_l) \approx \frac{u_x(t_{j+1}, x_{k+1}, y_l) - u_x(t_{j+1}, x_{k-1}, y_l)}{2l},
$$

$$
\frac{\partial u_y}{\partial y}(t_{j+1}, x_k, y_l) \approx \frac{u_y(t_{j+1}, x_k, y_{l+1}) - u_y(t_{j+1}, x_k, y_{l-1})}{2l},
$$

and FD scheme for (16) has the following form:

$$
p(t_{j+1, r_{kl}}) = p(t_{j+1/2, r_{kl}}) - p_0 \frac{\delta t}{4l} (u_x(t_{j+1, r_{k+1l}}) + u_y(t_{j+1, r_{kl+1}}) - u_x(t_{j+1, r_{k-1l}}) - u_y(t_{j+1, r_{kl-1}})). \tag{17}
$$

It must be noted, that (17) is an explicit FD scheme, — the values of $u_x$ and $u_y$ are already known after the solution of (11).

The overall computational procedure of presented LBM modification for simulation of incompressible flows is summarized as follows:

1. The values of $p$ at $t = t_{j+1/2}$ in fractional nodes $(x_k - \delta t/2V_i, y_l - \delta t/2V_i)$ are obtained after the solution of (12) from the values of $p^{(eq)}_i$ at $t = t_j$.
2. The values of $p$ and $u$ at $t = t_{j+1/2}$ are calculated by (4).
3. Linear spline interpolation procedure for the obtaining $p$ and $u$ values at nodes $(x_k, y_l)$ at $t = t_{j+1/2}$ is performed.
4. System (11) is solved for the calculation of $u(t_{j+1}, x_k, y_l)$.
5. The values of $p$ at $t = t_{j+1}$ are calculated by FD scheme (17).
4 Numerical solution of 2D lid-driven cavity flow problem

The problem is formulated in square domain \( \{(x, y) | x \in [0, P], y \in [0, P]\} \) \cite{5}. Boundary conditions has a form \cite{5}:

\[
\begin{align*}
    u_x(t, x, 0) &= u_y(t, x, 0) = 0, \\
    u_x(t, x, P) &= U_0, u_y(t, x, P) = 0, \\
    u_x(t, 0, y) &= u_y(t, 0, y) = u_x(t, P, y) = u_y(t, P, y) = 0, \\
    y &\in [0, P),
\end{align*}
\]

where \( U_0 = \text{const} \neq 0 \) is the velocity of the top lid.

All numerical results obtained by LBM proposed are compared with the results, obtained after the solution of hydrodynamical equations in "vorticity — stream functions" variables. The criterion for the comparison is calculated by the following formulae:

\[
I = \sqrt{I_x^2 + I_y^2},
\]

where \( I_x \) and \( I_y \) are calculated as:

\[
\begin{align*}
    I_x &= \frac{1}{N} \sum_{i=1}^{N} (u_x(T, 0.5P, y_i) - U_x(T, 0.5P, y_i))^2, \\
    I_y &= \frac{1}{N} \sum_{i=1}^{N} (u_y(T, x_i, 0.5P) - U_y(T, x_i, 0.5P))^2,
\end{align*}
\]

where \( T \) is the length of time interval; \( U_x, U_y \) — velocity components, obtained after the hydrodynamical equations solution; \( N = 100 \) is the number of the nodes of the grid, constructed in interval \([0, P]\) for comparison of the results.

To show the efficiency of the modified method, computations by LBM scheme from \cite{6} are realized for comparison with the results, obtained by modified LBM.

5.1. The choice of the parameters

The main input parameter of the program is Reynolds number \( Re \):

\[
Re = \frac{UL}{\nu},
\]

where \( L \) is typical length, \( U \) is typical velocity. For lid-driven cavity flow problem \( L = P, U = U_0 \). The following values are used: \( P = 1 \text{ m}, U_0 = 0.01 \text{ m/sec} \). The time interval from 0 to 1000 sec is considered. Uniform grids of \( 50 \times 50, 100 \times 100 \) and \( 200 \times 200 \) nodes are used. Kinematic viscosity \( \nu \) is calculated from (19) from the values of \( Re, L \) and \( U \). Parameter \( \tau \) is calculated from the value of \( \nu \) by formulae \cite{7}.

The cases of small and moderate values of \( Re \) are considered: \( Re = 10, 1, 0.1, 0.01 \). The choice of such values is defined by stability condition \( \tau \geq 1 \) formulated above.
5.2. Initial and boundary conditions

The values of \( f_i \) at the initial moment \( t = 0 \) are assumed to be equal to the values of \( f_i^{(eq)} \). Velocity \( \mathbf{u} \) at this moment is chosen as zero vector at all internal lattice nodes and initial pressure is chosen to be equal to unity.

One of the main problems of LBM consist with the fact, that boundary conditions (BC) have to be imposed to the distribution functions \( f_i \) rather than on hydrodynamical variables. At the same time, BC for physical problems are formulated for these variables. There are several ways of incorporation of the hydrodynamical conditions to the conditions for \( f_i \) with different orders of accuracy and various characteristics on stability [10].

In this study different types of BCs are used on first and second stages of computational process. Due to (12) equilibrium distribution functions are advected along the characteristics of kinetic equations. This fact allow us to formulate equilibrium boundary conditions on all boundaries. The possibilities of the equilibrium BCs application are investigated in [7], [12].

On the second stage LDEs with Dirichlet BCs (25) are solved by the system of LBEs (13). The computations by He and Luo LBEs are realized with Le Coupene et al [11] BCs realization for (18).

5.3. Results and discussion

As it is mentioned in [6], LBM scheme for incompressible flows is valid for computations when Mach number \( M \) is less than 0.15. For both variants of LBM values of \( l \) and \( \delta t \) are chosen to satisfy this restriction on \( M \). For \( Re = 10 \) all computations are realized with \( M = 0.1442 \), for \( Re = 1 \) — for \( M = 1.531 \cdot 10^{-2} \), for \( Re = 0.1 \) — for \( M = 1.493 \cdot 10^{-3} \), for \( Re = 0.01 \) for \( M = 1.472 \cdot 10^{-4} \).

In table 1 values of time step \( \delta t \) for both LBM schemes are presented. In table 2 the values of \( I \) for the LBM scheme from [6] are presented. The values of \( I \) for modified LBM proposed in this study are presented in table 3. The plots of dimensionless velocity vector components in comparison with the solution of hydrodynamic equations in "vorticity – stream function” formulation are presented at fig. 1.
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Fig. 1. Plots of dimensionless $u_x$ and $u_y$ at the case of $Re = 1$. 1 — results obtained by modified LBM; 2 — results obtained by solution of hydrodynamical equations in “vorticity–stream function” variables.

<table>
<thead>
<tr>
<th>Number of grid nodes</th>
<th>$Re = 10$</th>
<th>$Re = 1$</th>
<th>$Re = 0.1$</th>
<th>$Re = 0.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 $\times$ 50</td>
<td>$2.942 \cdot 10^{-1}$</td>
<td>$3.125 \cdot 10^{-2}$</td>
<td>$3.048 \cdot 10^{-3}$</td>
<td>$3.004 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>100 $\times$ 100</td>
<td>$1.471 \cdot 10^{-1}$</td>
<td>$1.562 \cdot 10^{-2}$</td>
<td>$1.524 \cdot 10^{-3}$</td>
<td>$1.502 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>200 $\times$ 200</td>
<td>$7.354 \cdot 10^{-2}$</td>
<td>$7.812 \cdot 10^{-3}$</td>
<td>$7.621 \cdot 10^{-4}$</td>
<td>$7.512 \cdot 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 1. Values of $\delta t$ for different $Re$ and different grid nodes number

<table>
<thead>
<tr>
<th>Number of grid nodes</th>
<th>$Re = 10$</th>
<th>$Re = 1$</th>
<th>$Re = 0.1$</th>
<th>$Re = 0.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 $\times$ 50</td>
<td>$1.939 \cdot 10^{-4}$</td>
<td>$2.384 \cdot 10^{-4}$</td>
<td>$2.186 \cdot 10^{-4}$</td>
<td>$2.079 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>100 $\times$ 100</td>
<td>$1.763 \cdot 10^{-4}$</td>
<td>$2.246 \cdot 10^{-4}$</td>
<td>$2.025 \cdot 10^{-4}$</td>
<td>$1.906 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>200 $\times$ 200</td>
<td>$1.683 \cdot 10^{-4}$</td>
<td>$2.185 \cdot 10^{-4}$</td>
<td>$1.955 \cdot 10^{-4}$</td>
<td>$1.832 \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 2. Values of $I$ for LBM scheme from [6]

<table>
<thead>
<tr>
<th>Number of grid nodes</th>
<th>$Re = 10$</th>
<th>$Re = 1$</th>
<th>$Re = 0.1$</th>
<th>$Re = 0.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 $\times$ 50</td>
<td>$7.656 \cdot 10^{-6}$</td>
<td>$5.058 \cdot 10^{-7}$</td>
<td>$5.009 \cdot 10^{-7}$</td>
<td>$1.827 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>100 $\times$ 100</td>
<td>$1.865 \cdot 10^{-9}$</td>
<td>$2.216 \cdot 10^{-9}$</td>
<td>$2.067 \cdot 10^{-9}$</td>
<td>$1.981 \cdot 10^{-9}$</td>
</tr>
<tr>
<td>200 $\times$ 200</td>
<td>$2.322 \cdot 10^{-9}$</td>
<td>$2.602 \cdot 10^{-9}$</td>
<td>$2.468 \cdot 10^{-9}$</td>
<td>$2.393 \cdot 10^{-9}$</td>
</tr>
</tbody>
</table>

Table 3. Values of $I$ for modified LBM

As it can be seen from tables 2 and 3, values of $I$ for modified LBM are less than values for LBM scheme from [6] for all considered values of $Re$ on all grids. This suggest to make a proposition on higher numerical tolerance of the method proposed for the considered problem. This fact can be explained
by the application of second order scheme for the solution of (16). It must be noted, that tolerance of the method may be improved by application of high order schemes to the solution of (11) and (16). Value of $I$ for modified method may be decreased by the usage of smaller value of $\delta t$. For example, at $Re = 10$ with grid resolution $100 \times 100$ with computations with $\delta t/2$, value of $I$ is equal to $7.085 \cdot 10^{-6}$, for grid resolution $200 \times 200$ such value is equal to $4.447 \cdot 10^{-6}$. For $Re = 1$ such values of $I$ are equal to $8.826 \cdot 10^{-8}$ and $5.387 \cdot 10^{-6}$, for $Re = 0.1$ — to $6.246 \cdot 10^{-8}$ and $5.035 \cdot 10^{-6}$, for $Re = 0.01$ — to $1.884 \cdot 10^{-9}$ and $4.833 \cdot 10^{-6}$.

5 Conclusion

The modified LBM for the simulation of incompressible viscous Newtonian fluids at small and moderate values of Reynolds number is proposed. The method is based on the splitting of differential operator in Navier — Stokes equations.

Despite of well-known versions of LBM, the modified method provide possibilities for increasing of stability and tolerance by an appropriate choices of FD schemes for the solution of LDE and equation for pressure.

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