Applied Mathematical Sciences, Vol. 10, 2016, no. 1, 13 - 22
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ams.2016.510608

Preprocessing and Cross Border Traffic

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Abstract

Cross border traffic is studied to determine the effect of a preprocessing on wait times and traffic flow. Since it is desirable to minimize wait times at border crossings, preprocessing at another location may reduce the wait at the border but may incur other costs. Analytical work is presented and a simulation study obtains some graphical and numerical results.

Mathematics Subject Classification: 90B22

Keywords: preprocessing, queueing, cross border, traffic

1 Introduction

Preprocessing is a useful method to improve efficiency. In computer science, there are many applications. For example, Nawi et al. [11] consider preprocessing for training neural networks. Kotsiantis et al. [8] use preprocessing to generate the final training set with less time in supervised learning.

Almost all types of data analysis can use preprocessing to speed the analysis. Schmieder and Edwards [12] use data preprocessing of genomic and metagenomic datasets for rapid quality control. Kavraki and Latombe [5] use a preprocessing approach to properly and quickly select the best choice in path planning for robots. Egge et al. [3] study preprocessing for production decisions. Ahmad et al. [1] use preprocessing for health care monitoring. Zhang and Li [13] use preprocessing to help with scheduling.

Cross border issues include taxation (e.g. Mishra and Tatti [10]), international trade (e.g. Meltzer [9]), and vehicular traffic (e.g. Knowles and Matthiessen [7]).

In this paper we are concerned with truck traffic crossing an international border. We are particularly interested in a situation which has cities at the border in one or both of two countries, and the use of preprocessing to lower the time that trucks spend at the border crossing.

As an example, almost 25% of trade carried by trucks between Canada and the USA crosses the Ambassador Bridge between Windsor, Ontario and Detroit, Michigan. These two cities are affected by trucks using city streets causing traffic congestion and pollution. One possibility to lower the time that trucks spend on city streets is the use of [more] preprocessing.

2 Model and Results

Assume that we have a system with all processing for trucks taking place at the border, with one or many servers, as below.

$$\longrightarrow \boxed{\text{Processor}} \longrightarrow$$

In contrast, consider another system with a preprocessor for trucks, far from the border, in a nonurban setting, plus a processor at the border.

$$\longrightarrow \boxed{\text{Preprocessor}} \longrightarrow \boxed{\text{Processor}} \longrightarrow$$

In the second system, the service time by the processor at the border is less, since the preprocessor already completed some of the service. We also assume that there is a threshold level at the border processor. If the queue of trucks is below the threshold, then their pollution, noise, and interference with local traffic, is fairly insignificant. Beyond the threshold level, each additional truck in the queue adds to the pollution and noise and traffic interence to nearby homes and businesses and roads.

In an $M/M/1$ queueing system, interarrival times are exponential with rate λ and service times are exponential with rate μ. The infinitesimal generator (rate matrix) is:

$$Q = \begin{bmatrix} -\lambda & \lambda & 0 & 0 & \cdots \\ \mu & -(\lambda+\mu) & \lambda & 0 & \cdots \\ 0 & \mu & -(\lambda+\mu) & \lambda & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Let N represent the length of the queueing system (customers in service or waiting), in steady state. It is well known ([4]) that $P(N = n) = \rho^n(1-\rho)$, where $\rho = \frac{\lambda}{\mu}$, for $n = 0, 1, 2, \ldots$, and $E(N) = \dfrac{\lambda}{\mu - \lambda}$. Define k to be a threshold level. That is, below k, the queue of customers has no serious undesirable effect to the surroundings, while above k, each additional customer has an undesirable effect. Note that

$$P(N \geq k) = (\rho^k + \rho^{k+1} + \cdots)(1-\rho) = \rho^k,$$

$$P(N = n | N \geq k) = \frac{P(N = n)}{P(N \geq k)} = \frac{\rho^n(1-\rho)}{\rho^k} = \rho^{n-k}(1-\rho), n = k, k+1, \ldots.$$

Hence,

$$\begin{aligned} E(N|N \geq k) &= \sum_{n=k}^{\infty} nP(N = n|N \geq k) = \sum_{n=k}^{\infty} n\rho^{n-k}(1-\rho) \\ &= k + \frac{\rho}{1-\rho} = k + \frac{\lambda}{\mu - \lambda}. \end{aligned}$$

Thus, the expected undesirabililty excess above k is :

$$P(N \geq k)E(N - k|N \geq k) = \rho^k(\frac{\rho}{1-\rho}).$$

This expected undesirability excess is one possible measure of the overall damage due to the truck traffic within the city, and can be a useful measure in other queueing models as well (for example, in health settings).

A model using preprocessing before the border crossing will have a higher service rate μ (and lower service times) at the border crossing. The preprocessing occurs outside the city limits and so the noise and pollution of trucks waiting at the preprocessing station (in a non residential area) do not affect the city population. If we fix the arrival rate $\lambda = 0.5$ and threshold $k = 3$, then the expected excess above k is shown as a function of μ, in Figure 1. The rapid drop in the curve emphasized the value of increasing the service rate by preprocessing.

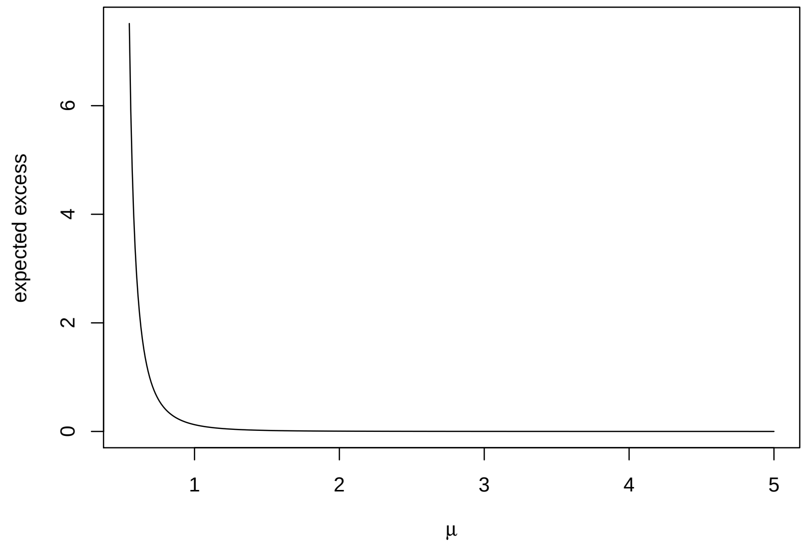

Figure 1: Expected Excess over k

In an $M/M/c$ queueing system, there are c servers. Then (see [4])

$$P(N=n)=\begin{cases}\frac{(\lambda/\mu)^n}{n!}P_0 & \text{if } 0\le n\le c\\ \frac{(\lambda/\mu)^n}{c!c^{n-c}}P_0 & \text{if } n>c\end{cases},$$

where $P_0=\left(\sum_{i=0}^{c-1}\frac{(\lambda/\mu)^i}{i!}+\frac{(\lambda/\mu)^c}{c!}\sum_{i=c}^{\infty}\rho^{i-c}\right)^{-1}$, $\rho=\frac{\lambda}{c\mu}$.

If k (the threshold) is larger than c (the number of servers), then

$$P(N=n|N\ge k)=\frac{P(N=n)}{P(N\ge k)}=\frac{\rho^n(1-\rho)}{\rho^k}=\rho^{n-k}(1-\rho), n=k,k+1,...$$

and

$$E(N|N\ge k)=k+\frac{\rho}{1-\rho}k+\frac{\lambda}{c\mu-\lambda}.$$

The expected undesirability excess over k is:

$$P(N\ge k)E(N-k|N\ge k)=\rho^k(\frac{\rho}{1-\rho}).$$

It is interesting that this expression exactly matches that for the M/M/1 case. Also, the graph of the expected excess over k as a function of μ, can be shown to have a similar shape, but a different scale, as in the M/M/1 case.

3 Simulation

Trucks at the processor below the threshold are assigned a cost of 0, as there is meager inteference with city traffic and the pollution level is small. We assume each truck above the threshold k (which we can set) will initiate a cost per truck and per unit time. The costs could include interference with local traffic, noise pollution, fuel polllution, wasted fuel, wasted driver time and frustration. We define the "badness" B for a time interval to be the area above the threshold level and below the graph of the observed number of trucks versus time t, for $0 < t < T$. The definition of B gives an important performance measure of a queueing system with a threshold. The expected badness (or expected excess above the threshold) has already been considered in section 2, for M/M/1 and M/M/c models at the processor. For the simulation section, we can consider more complex models.

On weekdays, there are higher arrival rates compared to weekends, and these rates vary by time of day. Figure 2 shows a plot of actual data, courtesy of the Cross-Border Institute at the University of Windsor. Arrival rates are measured in trucks per minute.

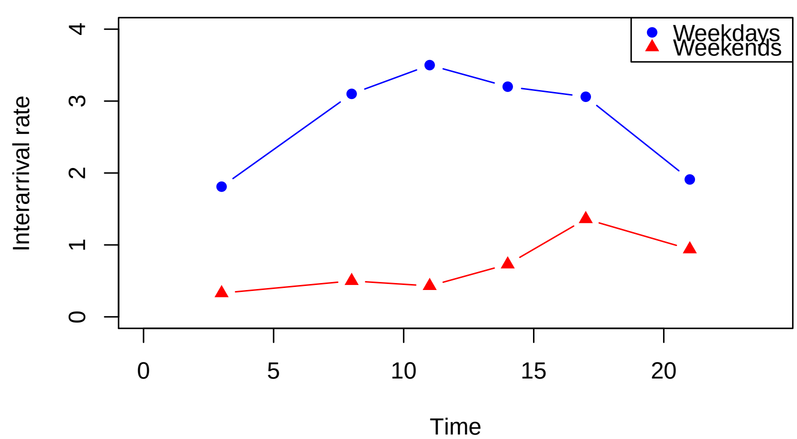

Figure 2: Arrival rate in weekdays and weekends

We restrict our analysis to weekdays. We approximate the arrival rate from midnight to 8 a.m. by $\lambda_1 = 1.8$, from 8 a.m. to 8 p.m. by $\lambda_2 = 3.5$, and from 8 p.m. to midnight by $\lambda_1 = 1.8$. We use these rates for our simulation of truck traffic. We further assume exponential interarrival times. These assumpetions are consistent with trucks behaving independently of each other.

For service times, we assume a mixture, reflecting the policy at border

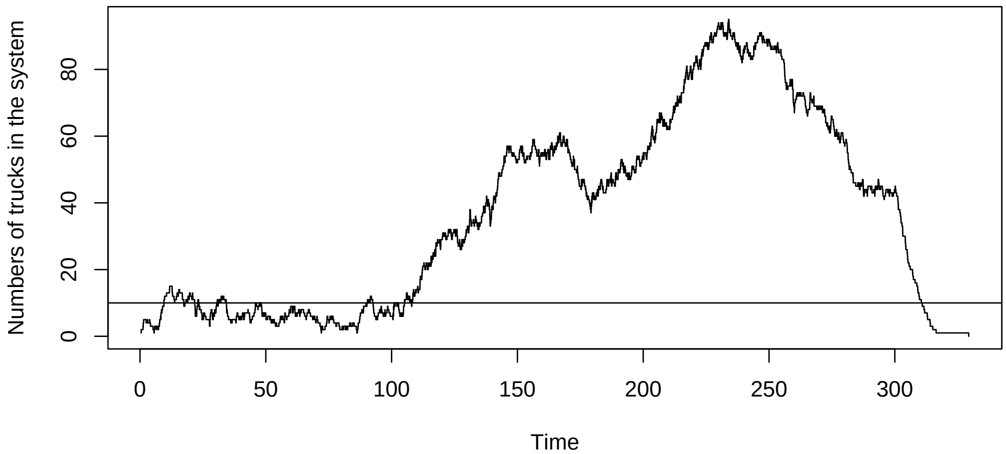

Figure 3: Numbers of trucks in the system without preprocessing facilities

corssings of randomly choosing some trucks for secondary inspection. Service time is taken to be some baseline time plus an exponentially distributed value. We also assume that there is approximately a 1% probability of being inspected extra carefully. In these two different situations, the baseline times are taken to be 0.7 min and 10 min. The threshold level is set at 10. Without preprocessing, we present a typical simulation plot in Figure 3. The area of the curve above 10 (the badness) is large.

In contrast, by adding a preprocessor and thereby reducing the service rate at the processor, we get a very different plot. Assume that there are 5 servers in the processor and 3 servers in the preprocessing station. A typical simulation appears in Figure 4. Note that the vertical scale is very different. Again we set the threshold at 10. Now the area above the thershold (badness) is tiny.

Based on 200 runs for a single day, we find the mean of badness without and with preprocessing facility. Figure 4 shows badness of the two systems. The mean badness per day is 829080 min. and 165 min. There is a more than 99.9% of decrease in total wait time. Since the badness occurs in clusters, this would be very time consuming for drivers and would cause the neighborhood of the border crossing problems in terms of pollution, noise, and interference of local traffic. A box plot shows the comparison in Figure 5.

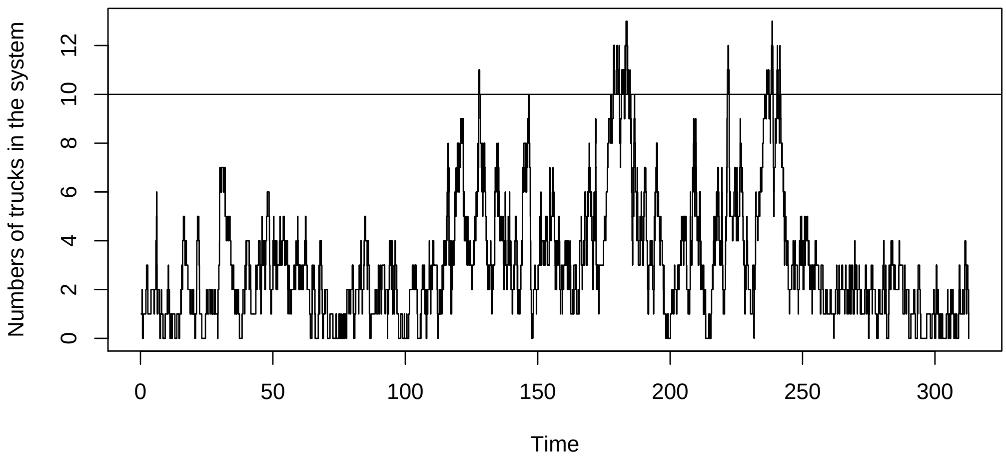

Figure 4: Numbers of trucks in the system with preprocessing facilities

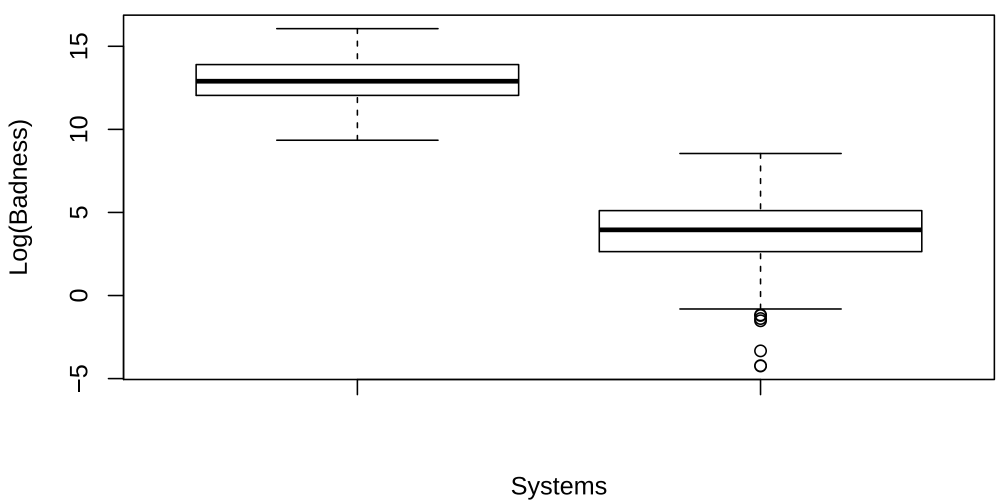

Figure 5: Badness without and with preprocessing facilities

4 Costs

It is clear that a preprocessor to reduce border processing times (or equivalently increase border service rate) has a positive effect in reducing wait times at the border. But there are costs associated with a preprocessor station. These costs include building, maintaining, and staffing the preprocessor. The preprocessor may actually increase the total time (preprocessor plus processor times) for a truck driver, though the border time may be reduced. Extra waiting time for the truck drivers means a financial loss. And if cargo is waiting in transit, this also means a financial loss. If the the cargo is perishable, or is part of a JIT (Just-In-Time) system, it needs rapid delivery. So it is reasonable to include a cost per unit wait time D. We assume that if the number of trucks is greater than a particular threshold k, there will be a cost P per unit time for each truck due to pollution, noise, local traffic congestion. If we build a preprocessing station, there are costs related to the station. The building cost B is paid up front as one lump sum. There will be some maintenance fees and staffing costs S per year.

If we do not take interest rates into consideration, the static payback period (n years) should be the time when all the savings are equal to initial costs. Let T be the total wait time per year of truck drivers without preprocessing facilities, and T_1, T_2 respectively be the wait time in preprocessor and processor. Similarly, t, t_1, t_2 represent the badness (above the threshold K). We calculate the annual savings as $(T - T_1 - T_2) \cdot D + (t - t_2) \cdot P$. So we compute the static payback period n:

$$B + S \cdot n = (T - T_1 - T_2) \cdot D \cdot n + (t - t_2) \cdot P \cdot n.$$

Thus, the static payback period should be:

$$n = \frac{B}{(T-T_1-T_2)\cdot D+(t-t_2)\cdot P-S}.$$

5 Discussion and Conclusion

These are the main results of the paper. This paper has used queueing models and simualtion to highlight the value of preprocessing in cross border traffic. The excess above threshold (or badness) measure has been used to evaluate the performance3. Above the threshold in a queueing system, each excess customer is assigned a cost per unit time. Below the threshold, there is no cost. This type of model could find applications in other areas beside traffic, such as in health care. For example, diabetes could be prevented or delayed by keeping the sugar level below a threshold, through eating smaller more frequent meals. Allergies could be minimized by keeping the certain chemical levels low, where the processing can take place without serious consequence. The effect of chemotherapy could be minimized by keeping the total medication below a threshold. There are many applications of this type of queueing threshold model.

Acknowledgements. We acknowledge funding and support from the Cross-Border Institute (University of Windsor) MITACS Global Internship program, University of Windsor, Nanjing University, and the Chinese Scholarship Council.

References

[1] N.F. Ahmad, D.B. Hoang, M.H. Phung, Robust Preprocessing for Health Care Monitoring Framework, *Proceedings of Healthcom 2009,* (2009), 169 - 174. http://dx.doi.org/10.1109/health.2009.5406196

[2] J. Albert and M. Rizzo, *R by Example,* Springer, New York, 2012. http://dx.doi.org/10.1007/978-1-4614-1365-3

[3] N. Egge, A. Brodsky, I. Griva, An Efficient Preprocessing Algorithm to Speed-Up Multistage Production Decision Optimization Problems, *Proceedings of the 2013 46th Hawaii International Conference on System Sciences,* (2013), 1124 - 1133. http://dx.doi.org/10.1109/hicss.2013.82

[4] D. Gross, J.F. Shortle, J.M. Thompson and C.M. Harris, *Fundamentals of Queueing Theory,* Wiley, Hoboken, 2008. http://dx.doi.org/10.1002/9781118625651

[5] L. Kavraki and J.C. Latombe, Randomized preprocessing of configuration for fast path planning, *1994 IEEE International Conference on Robotics and Automation Proceedings,* (1994), 2138 - 2145. http://dx.doi.org/10.1109/robot.1994.350966

[6] E. Kao, *An Introduction to Stochastic Processes,* Duxbury, Belmont, 1997.

[7] R.D. Knowles and C.W. Matthiessen, Barrier effects of international borders on fixed link traffic generation: the case of Oresundsbron, *Journal of Transport Geography,* **17** (2009), 155 - 165. http://dx.doi.org/10.1016/j.jtrangeo.2008.11.001

[8] S.B. Kotsiantis, D. Kanellopoulos and P. E. Pintelas, Data preprocessing for supervised leaning, *International Journal of Computer Science,* **1** (2006), no. 2, 111 - 117.

[9] J.P. Meltzer, The Internet, Cross-Border Data Flows and International Trade, *Asia and the Pacific Policy Studies,* **2** (2014), no. 1, 90 - 102. http://dx.doi.org/10.1002/app5.60

[10] A.V. Mishra and R.A. Ratti, Home bias and cross border taxation, *Journal of International Money and Finance,* **32** (2013), 169 - 193. http://dx.doi.org/10.1016/j.jimonfin.2012.04.004

[11] N.M. Nawi, W.H. Atomi, and M.Z. Rehman, The Effect of Data Preprocessing on Optimized Training of Artificial Neural Networks, Procedia Technology, **11** (2013), 32 - 39.
http://dx.doi.org/10.1016/j.protcy.2013.12.159

[12] R. Schmieder, R. Edwards, Quality control and preprocessing of metagenomic datasets, *Bioinformatics,* **27** (2011), 863 - 864.
http://dx.doi.org/10.1093/bioinformatics/btr026

[13] T. Zhang and J. Li, Online Task Scheduling for LiDAR Data Preprocessing on Hybrid GPU/CPU Devices: A Reinforcement Learning Approach, *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing,* **8** (2015), 386 - 397.
http://dx.doi.org/10.1109/jstars.2015.2390626

Received: October 12, 2015; Published: January 5, 2016