A Long-Term Solution for Shoreline Evolution

Induced by Wave Action

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Abstract

In this paper new approach of numerical schemes for the shoreline evolution in the long-term scale are presented, and comparisons to analytical solution for some cases are presented for a satisfactory level of suitable numerical scheme. Analytical solutions of shoreline evolution for simple configuration are presented under idealized wave condition. In this study, we also present the shoreline evolution to response the wave height breaking at different value and case. The simulation of shoreline evolution conducts at different of shoreline configuration, which rectangular cut in beach and straight impermeable groin. The simulation result shows that shoreline with straight impermeable groin and finite rectangular beach fill configurations have bigger root mean percentage square error (RMSPE) when the sediment transport rate and time are increased. However the increases of RMSPE is very small which only approximate in the range of 0.1– 0.2 %. In addition, simulations in variation of wave height breaking show that wave height breaking increases the RMSPE increase for both of shoreline configurations. In general the RMSPE shows less than 1% for all cases. These results indicate that this new approach of numerical scheme is very reliable for solving problems of long term shoreline evolution.
Keywords: rectangular cut in beach, root mean square percentage error, shoreline evolution, straight impermeable groin, wave height breaking

1. Introduction

Surf zone processes are created when a longshore current is generated by oblique breaking waves and generate a longshore sediment transport [7]. The transportation of sediment begin with the stirring of sand particles into suspension mode due to the passage of breaking wave and the sediment will be transported by the longshore current. The entrainments of the sand particles into suspension load and bed load transport are depending on the magnitude of bed shear stress which derives from the wave and currents respectively. In practice, it is important to know how much the volume of sand loss and gain along the coast to provide and planning the proper solution. For quantifying the volume of sediment movement, the approximate sediment transport rates must be known by mean of experimental work at field or may be calculated from aerial survey. The aerial survey is carried out over several years to identify the trend for accretion or depletion may be discernible. The beach profile will respond to these changes in manner which tends to restore equilibrium [5].

Many researchers have been conducted the study of longshore sediment transport in the past. Schoones and Theron [11] conducted a comprehensive review of field data for longshore sediment transport. In predicting the shoreline changes, many commercial software are available introducing various models for short and medium term forecasting [6]. However for long term evolution of coastline, there is a lot of afford needed to improve the accuracy and robustness especially on the numerical approach when applying for long term and complex condition. Therefore the main objective of this research is to develop a new approach of numerical scheme to be applied in long term shoreline evolution for various cases. A several tests were used to identify the accuracy of the method.

2. Methodology

The fundamental of one line model concept used in this study relies on basic assumption which described in detail through the paper of Ahmad et al. [1] and not be mentioned here. However the equation involved will be explained to understand the physical processes occurs. The longshore evolution equation is a one dimensional model formulated by conservation of sand volume under the two assumptions stated by Ahmad et al. [1]. The equation as follow:

\[
\frac{\partial y}{\partial t} = \frac{1}{(D_B + D_C)} \left( - \frac{\partial Q}{\partial x} \right)
\]

(1)

where \(x\) is the alongshore coordinate (m); \(y\) is the shoreline positions (m) and perpendicular to \(x\)- axis; \(t\) is time (s); \(Q\) is the long-shore sand transport rate (m\(^3\)/s); \(D_B\) is the average berm height (m) and \(D_C\) is the closure depth (m).
The spatial variations in the long-shore sediment transport rate are considered to lead to change in the shoreline. A general expression for the long-shore sand transport rate was developed by the US Army Corp [4]:

\[ Q = Q_0 \sin(2\alpha_0) \]  

where \( Q_0 \) is the amplitude of the long-shore sand transport rate. The empirical predictive equation for the amplitude of the long-shore sand transport rate is written as [8]:

\[ Q_0 = \frac{\rho}{16} \left( H_b c_g \right) \left( \frac{K}{\rho_s - \rho} \right) (1-n) \]  

where the subscript \( b \) denotes value at the point of breaking, \( c_g \) is the wave group velocity, \( H \) is the wave height, \( \rho_s \) is the density of the sediment (kg/m\(^3\)), \( \rho \) is the density of the sea water, \( n \) is the porosity and \( K \) is the dimensionless coefficient which is a function of particle size.

For the beaches with the mild slope, it can be assumed that breaking wave angle to the shoreline is small. So that the differential equation for shoreline evolution becomes simplify as follow:

\[ \frac{\partial y}{\partial t} = D \frac{\partial^2 y}{\partial x^2} \]  

where \( D = \frac{2Q_0}{D_B + D_C} \).

![Figure 1. Computational molecules BTCS scheme](image)

The shoreline evolution in Equation 4 is discretized based on grid point in the Figure 1 and the backward time centered space (BTCS). The BTCS is not new numerical scheme but considerably as new numerical approach in predicting the coastal shoreline changes especially to solve the long term evolution. The BTCS is the numerical scheme uses finite difference technique and is stepped backward in time using increments of time interval [2, 3]. The information used in forming the finite difference quotient in BTCS comes from bottom of grid point \((i, j)\) that
is, it uses \( y_{i,j-1} \) as well as \( y_{i,j} \). No information for the above of \((i, j)\) is used (Figure 1). BTCS use the finite difference quotient of space comes from both sides of the grid point located at \((i, j)\); that is, it uses \( y_{i,j+1} \) as well as \( y_{i,j-1} \). Grid point \((i, j)\) falls between the two adjacent grid points [3].

Finite difference expression for the BTCS in Equation 4 can be written as [12]:

\[
y_{i,j} - D \frac{\Delta t}{\Delta x^2} \left( y_{i+1,j} - 2y_{i,j} + y_{i-1,j} \right) = y_{i,j-1}
\]

Solve numerically Equation 5 to obtain the solution of long term shoreline evolution.

3. Result and discussion

There are four different cases tested in this study for validating the numerical approach against the analytical solution. It is started with straight impermeable grain and rectangular cut in an infinite beach as case 1 and case 2 respectively with the difference sediment rate. Case 3 and case 4 are similar with configuration of case 1 and case 2 respectively, but the difference is the various wave breaking height are tested while the other parameters are setting as constant. The cases are also tested with variation of wave breaking. More details are described as follows:

**Case 1. Straight Impermeable groin**

Shorelines evolution with straight impermeable groin have the initials of beach is parallel to the x-axis with the same breaking wave angle \( \alpha_0 \) existing everywhere as depicted in Figure 2, thus leading to uniform sand transport rate along the beach. At time \( t = 0 \) a thin groin is instantaneously place at \( x = 0 \) and blocks all transport. Mathematically, this boundary condition can be formulated as follows [12]:

\[
\frac{\partial y}{\partial x} = \tan \alpha_0 \quad \text{at} \quad x = 0
\]

![Figure 2. Initial Shoreline with configuration straight impermeable groin.](image)
The analytical solution describing the accumulation part on up-drift side of the groin is:

$$y(x,t) = \tan \alpha_0 \sqrt{\frac{4Dt}{\pi}} \left\{ e^{-\frac{x^2}{4Dt}} - \frac{x\sqrt{\pi}}{2\sqrt{Dt}} \text{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) \right\}$$

(7)

This equation states that the shoreline at the groin is instant parallel to the wave crests. A groin interrupts the transport of the sand alongshore, cause an accumulation on the up-drift side and erosion on the down-drift side.

In this case, $\alpha_0$ was set to $15^0$, since the boundary condition at the groin which is totally blocking the transport of sand alongshore so that long-shore sediment transport rate taken to be $Q = 0$. The shoreline changes calculated by using the analytical solution in Equation 8 and the results from the numerical solutions by using BTCS scheme are shown in Figure 3.0-5.0.

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**Figure 3.0a:** Comparison numerical result and analytical result in the case shoreline configuration of straight impermeable groin with parameter $D= 300,000$ m$^3$/year; 3.0b: zoom out box 1 in the Figure 3.0a that is simulation calculated 1 year; Figure 3.0c: zoom out box 2 in the Figure 3.0a that is simulation calculated 3 year; 3.0d: zoom out box 3 in the Figure 3.0a that is simulation calculated 5 year.
Figure 4.0a: Comparison numerical result and analytical result in the case shoreline configuration of straight impermeable groin with parameter $D = 500,000 \text{ m}^3/\text{year}$; 4.0b: zoom out box 1 in the Figure 4.0a that is simulation calculated 1 year; 4.0c: zoom out box 2 in the Figure 4.0a that is simulation calculated 3 year; 4.0d: zoom out box 3 in the Figure 4.0a that is simulation calculated 5 year.

Figure 5.0a: Comparison numerical result and analytical result in the case shoreline configuration of straight impermeable groin with parameter $D = 1,000,000 \text{ m}^3/\text{year}$; 5.0b: zoom out box 1 in the Figure 5.0a that is simulation calculated 1 year; 5.0c: zoom out box 2 in the Figure 5.0a that is simulation calculated 3 year; 5.0d: zoom out box 3 in the Figure 5.0a that is simulation calculated 5 year.
The comparison between the analytical and numerical solutions is only investigated on the up-drift side, since the analytical solution on the down-drift side has not been considered in this paper. As shown in the Figures 3.0 – 5.0, in general the evolutions of shorelines are showing a very close between the simulation with the numerical scheme and analytical solution. The presence of the impermeable groin which can contribute the complexity and thus lead the error can be avoided by this scheme. It is clearly indicates that the error due to numerical scheme is not obviously growing when longer time is considered.

**Case 2. A rectangular cut in beach**

This problem represents an excavation or natural employment of rectangular shape as pictured in Figure 6. The initial conditions for rectangular cut in a beach are formulated as [10]:

$$y(x, 0) = \begin{cases} y_0 & |x| \geq a \\ 0 & |x| < a \end{cases}$$  \hspace{1cm} (8)

The analytical solution is

$$y(x, t) = \frac{y_0}{2} \left[ \text{erfc} \left( \frac{a - x}{2\sqrt{Dt}} \right) + \text{erfc} \left( \frac{a + x}{2\sqrt{Dt}} \right) \right]$$  \hspace{1cm} (9)

for \( t > 0 \) and \( -\infty < x < \infty \).

![Figure 6. Initial Shoreline with configuration a rectangular cut in an infinite beach.](image)

In Figure 6, this situation is the inverse problem of the rectangular beach fill [13], so that this situation can be used to evaluate the rate of infilling of certain volumetric percentage of sand. In this problem, \( y_0 \) was set to 30 m and \( a \) was set to 2500 m, since this problem doesn’t block the transport of sand alongshore so
that the boundary condition is specified on the initial and final $x$-axis that depends from Equation 10 as depicted in Figure 6. The shoreline changes calculated by using the analytical solution Equation 10 and numerical solutions by using numerical approach based on BTCS scheme are shown in Figure 7.0-9.0. Figure 7.0-9.0 show that erosion occurs at the grid point 0 to 2500m, while accretion occurs on the grid points 2500m to 1000m in the longshore direction. This erosion and accretion become greater with increasing time and sediment transport. These results show a very close between the simulation with the numerical scheme and analytical solution. It is obviously implies that the error due to numerical scheme is not apparently growing when longer time is considered. It is unknown expectation whether the increment of sediment load can contribute to negative effect or not but in order to see the robustness of the numerical scheme it is ideally to observe the consequences from the increment of sediment load. These results show the increment do not affect the numerical scheme.

Figure 7.0a: comparison numerical result and analytical result in the case shoreline configuration of a rectangular cut in an infinite beach with parameter $D=300,000$ m$^3$/year; Figure 7.0b: zoom out box 1 in the Figure 7.0a that is simulation calculated 1 year; 7.0c: zoom out box 2 in the Figure 7.0a that is simulation calculated 3 year; Figure 7.0d: zoom out box 3 in the Figure 7.0a that is simulation calculated 5 year.
Figure 8.0a: comparison numerical result and analytical result in the case shoreline configuration of a rectangular cut in an infinite beach with parameter D = 500,000 m³/year; Figure 8.0b: zoom out box 1 in the Figure 8.0a that is simulation calculated 1 year; Figure 8.0c: zoom out box 2 in the Figure 8.0a that is simulation calculated 3 year; Figure 8.0d: zoom out box 3 in the Figure 8.0a that is simulation calculated 5 year.

Figure 9.0a: comparison numerical result and analytical result in the case shoreline configuration of a rectangular cut in an infinite beach with parameter D = 1,000,000 m³/year; Figure 9.0b: zoom out box 1 in the Figure 9.0a that is simulation calculated 1 year; 9.0c: zoom out box 2 in the Figure 9.0a that is simulation calculated 3 year; 9.0d: zoom out box 3 in the Figure 9.0a that is simulation calculated 5 year.
**Case 3. Parametric studies of wave height breaking induced shoreline with straight impermeable groin**

To study the effect wave height breaking on shoreline evolution, simulations are conducted using the numerical approach based on BTCS scheme with parameter wave height breaking ranging from 0.2 m to 1.72 m, assuming the gravity constant and density of sea water and sediment in Table 3. For this problem, the shoreline at the groin is instant parallel to the wave crests. A groin interrupts the transport of the sand alongshore, cause an accumulation on the up-drift side and erosion on the down-drift side.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Quantity and Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave group velocity ( C_{gb} )</td>
<td>1.74 m/s</td>
</tr>
<tr>
<td>Dimensionless coefficient which is a function of particle size ( K )</td>
<td>0.39</td>
</tr>
<tr>
<td>Depth Closure ( D_c )</td>
<td>4 m</td>
</tr>
<tr>
<td>Depth Berm ( D_B )</td>
<td>1 m</td>
</tr>
<tr>
<td>Density of Sediment ( \rho_s )</td>
<td>2650 kg/m(^3)</td>
</tr>
<tr>
<td>Density of Sea Water ( \rho )</td>
<td>1025 kg/m(^3)</td>
</tr>
<tr>
<td>Porosity ( n )</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Base in Equation 8 simulated is conducted. In this case, \( \alpha_0 \) was set small angle constant to 15\(^0\) and long-shore sediment transport rate taken to be \( Q = 0 \). The shoreline changes are calculated by using the analytical solution equation and numerical solutions by using numerical approach based on BTCS scheme. The comparison between the analytical and numerical simulations as pictured in Figure 10 (a-d), since the analytical solution on the down-drift side has not been considered in this paper.

Specific numerical results are enlarged in Figure 4.10b, Figure 4.10c and Figure 4.10d which corresponding with variation wave height breaking. These results indicate that numerical result and analytical solution are very close. It is clearly implies that the error due to numerical scheme is not obviously growing when variation of wave height breaking is considered.
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Figure 10.0a: comparison numerical result and analytical result in the case shoreline configuration of straight impermeable groin with different wave height breaking; Figure 10.0b: zoom out box 1 in the Figure 10.0a that is simulation calculated at \( H_b = 0.2 \text{m} \); Figure 10.0c: zoom out box 2 in the Figure 10.0a that is simulation calculated at \( H_b = 1 \text{m} \); Figure 10.0d: zoom out box 3 in the Figure 10.0a that is simulation calculated at \( H_b = 1.72 \text{m} \).

Case 4. Parametric studies of wave height breaking induced shoreline with pattern rectangular cut in beach

In case 4, the parametric study of wave height breaking at shoreline evolution with groin configuration has been conducted at different wave height breaking. To investigate the effectiveness from the numerical approach based on BTCS scheme, similar parametric study of wave height breaking have been conducted at another configuration of shoreline. In this section, the problem represents an excavation or natural employment of rectangular shape. The initial conditions for rectangular cut in a beach are formulated as similar case 2. The comparison between the analytical and numerical simulations from this problem is pictured in Figure 11(a-d). Simulations are conducted at shorelines evolution with configuration a rectangular cut at ranging wave height breaking from \( H_b = 0.2 \text{ m} \) to \( H_b = 1.72 \text{ m} \). As shown in the Figures 11.0, in general the evolution of shorelines is showing a very close between the simulation result and analytical solution. The variation of wave height breaking which can contribute the wider wave condition and thus lead the error can be avoided by this scheme. It is clearly indicates that the error due to numerical scheme is not apparently when variation
of wave height is considered. It is unknown expectation whether the variation of wave height can contribute to negative effect or not but in order to see the robustness of the numerical scheme it is ideally to observe the consequences from the variation of wave height. These results show the variation of wave height does not higher affect the numerical scheme.

Figure 11.0a: comparison numerical result and analytical result in the case shorelines evolution with configuration a rectangular cut in beach with different wave height breaking; Figure 11.0b: zoom out box 1 in the Figure 11.0a that is simulation calculated at \( H_b = 0.2m \); 11.0c: zoom out box 2 in the Figure 11.0a that is simulation calculated at \( H_b = 1m \); Figure 11.0d: zoom out box 3 in the Figure 11.0a that is simulation calculated at \( H_b = 1.72m \).

4. Root Mean Square Percentage Error Analysis

In this research the accuracy of the result was analysed by using Root Mean Square Percentage Error (RMSPE) method. The method is given by Hyndman et al. [9] and written as:

\[
RMSPE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( 100 \cdot \frac{(y_i) - (y_0)}{(y_0)} \right)^2}
\]

where \( y_i \) is shoreline simulation; \( y_0 \) is shoreline observation and \( N \) is number of cells.
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In general the root mean square percentage error shows less than 1% for all cases. The scheme of BTCS shows an efficient numerical method which can be applied for determining the shoreline changes. The results as shown in Table 1.0 and 2.0 for case 1 and case 2 respectively, the RMPSE increases when the sediment transport rate and time are increased. However the increases of RMPSE is very small which only approximate in the range of 0.1 – 0.2 %. Simulation in case 3 and case 4 show the RMSPE increases when the wave breaking increase too but in small scale as represented in Table 3. These results show that increasing from wave height breaking give alteration accuracy from the simulation is not significant from this case. It is indicated that the new approach of numerical scheme in this paper is appropriate for solving of the long term shoreline evolution as well as for practical engineering applications in coastal areas.

**Case 1 - Straight Impermeable groin**

Table 1.0 Root Mean Square Percentage Error for case: straight impermeable groin

<table>
<thead>
<tr>
<th>Time (year)</th>
<th>D1 ( (m^3/\text{year}) )</th>
<th>D2 ( (m^3/\text{year}) )</th>
<th>D3 ( (m^3/\text{year}) )</th>
<th>RMSPE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2734</td>
<td>0.3337</td>
<td>0.3557</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.2974</td>
<td>0.3456</td>
<td>0.5267</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.3058</td>
<td>0.3928</td>
<td>0.6591</td>
<td></td>
</tr>
</tbody>
</table>

Note: D1 = 300,000; D2 = 500,000; D3 = 1,000,000

**Case 2 - A rectangular cut in an infinite beach**

Table 2.0: Root Mean Square Percentage Error for case: shoreline with pattern rectangular cut in beach

<table>
<thead>
<tr>
<th>Time (year)</th>
<th>D1 ( (m^3/\text{year}) )</th>
<th>D2 ( (m^3/\text{year}) )</th>
<th>D3 ( (m^3/\text{year}) )</th>
<th>RMSPE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1673</td>
<td>0.1913</td>
<td>0.2496</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.1997</td>
<td>0.2276</td>
<td>0.4206</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.2475</td>
<td>0.2867</td>
<td>0.5530</td>
<td></td>
</tr>
</tbody>
</table>

Note: D1 = 300,000; D2 = 500,000; D3 = 1,000,000

**Other cases - Case 3 and case 4**

Table 3.0. Root Mean Square Percentage Error for Parametric studies of wave height breaking

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
<th>Wave Height Breaking ( (H_b) ) (m)</th>
<th>RMSPE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Shoreline with straight impermeable groin</td>
<td>0.2</td>
<td>0.3117</td>
</tr>
<tr>
<td>4</td>
<td>Shoreline with pattern rectangular cut in beach</td>
<td>0.5729</td>
<td>0.6961</td>
</tr>
</tbody>
</table>
5. Conclusion

A new approach of numerical scheme is developed based on BTCS scheme. This approach is suitable for solving the long term shoreline evolution. Numerical scheme in this case study is deal with shoreline position. When input wave data attacked the shoreline, shoreline will move back and forth depending on the input wave data and initial shoreline pattern (include coastline structure) in the numerical domain. Numerical simulation in this case calculates changes back and forth of the shoreline position. This simulation is calculated over time. When the time spent on running the simulation getting longer, the resulting error is also greater. In this case actually can be solved by simple numerical scheme i.e FTCS. In this numerical scheme the changes back and forth of the shoreline position in next time is calculated directly by known value of the shoreline. It causes the calculation of the shoreline getting unstable when the duration of the simulation was a long time. In order to overcome this problem, in this study numerical approach based on BTCS scheme is applied in this case. In this numerical scheme, calculation of the changes back and forth of the shoreline position in next time is not calculated directly by known value of the shoreline. But this calculation come from shoreline position before the time and calculate the next shoreline position together with calculation shoreline the before the time. In other word, this calculation is simultaneously between shoreline positions that known and shoreline position unknown. This numerical result from this scheme show good agreement with analytical solution with indication average low RMSPE in the all of the case in this study like shown in Table 1.0, Table 2.0 and Table 3.0. Based on these tables, case 1 and 2 shows that RMSPE will increase when time scale increase. In case 1 RMSPE will become more error in $D = 500.000 \text{ m}^3/\text{years}$ during 1 year than others constant $D$. This is also even in time scale 1 year and 5 years. While case 2 shows RMSPE increase when constant $D$ increases for 1 year until 5 years. Parametric studies have been conducted to investigate the shoreline evolution at three configurations of shoreline. The influences of various wave heights breaking on the simulation shoreline evolution are systematically investigated. In case 3 and 4 show that RMSPE increase when wave height breaking increase, these results show that decreasing from wave height breaking accuracy from the simulation is become significant accurate. In general the RMSPE shows less than 1% for all cases. These results indicate that this new approach of numerical scheme is very reliable for solving problems of long term shoreline evolution.

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