The Method of Determining the Locations of Reinforcing Elements in a Composite Orthotropic Plate Undergoing Dynamic Impact.

Part 2. Calculation Algorithm

Alexey A. Loktev
Moscow State University of Civil Engineering (MGSU)
26, Yaroslavskoe shosse, Moscow, 129337, Russian Federation

Ekaterina A. Gridasova
Far Eastern Federal University (FEFU)
8, Sukhanova str., Vladivostok, 690001, Russian Federation

Vladimir V. Kramchaninov
Moscow State University of Railway Engineering (MSURE)
22/2, Chasovaya str., Moscow, 125993, Russia Federation

Roman N. Stepanov
Moscow State University of Civil Engineering (MGSU)
26, Yaroslavskoe shosse, Moscow, 129337, Russian Federation

Abstract
In this investigation, we realize the idea to link reinforcement positions of plane constructions with points of origin of the largest kinematic and dynamic characteristics after impact on the target. It is assumed that the largest dynamic characteristics, e.g. stresses, arise in the intersection points of wave surfaces, which
have experienced multiple reflections from the boundaries of the target. The origin and propagation of wave surfaces with finite velocities in the proposed model of the deformation of a flat target become possible due to the use of hyperbolic equations of Uflyand-Mindlin-Reissner type describing dynamic displacements of the points of the plate with consideration of the transverse shear deformation and rotational inertia of cross-sections. In this paper, we propose an algorithm taking into account different rheological properties of the interacting bodies, which is based on the analytical method of representation of unknown quantities in the form of expansions in a spatial coordinate and time, initial and boundary conditions, compatibility conditions. The final integro-differential equations are solved using numerical computational schemes. Moreover, the creation of algorithms for calculating impact interactions with a given wave portrait in colliding bodies, as well as their implementation in a software application with a simple and intuitive interface are desirable. In this paper, we developed a computational algorithm and a software system and tested them on the example of a flat target composed of materials with various mechanical characteristics. Furthermore, we provide specific recommendations for locations of reinforcing filaments or rods in the composite material.

**Keywords:** Dynamic impact, Uflyand-Mindlin-Reissner plate, Elastic waves, Orthotropic properties, Reflected waves, Boundary conditions, Spherical functions, Hyperbolic equations, Contact force

1 Introduction

In this paper, to determine the reinforcement positions of the flat element along its thickness, we propose to construction picture of propagation of the fronts of elastic waves [1-4] and separately to find points where the reflected longitudinal and transverse waves meet [3,5]. In these points the largest compressive or tensile stresses will occur [6-8] and exactly in their vicinity it is proposed to place reinforcing elements in the form of fibers, filaments or rods. Since we consider the material with orthotropic properties, the reinforcement can be done in three main directions of anisotropy. By varying mechanical characteristics of the reinforcement elements one can achieve significant changes in the properties of the final composite and choose them in such a way that the certain dynamic, kinematic or geometric characteristics of the behavior of the structure or its elements under external dynamic load can be minimized.

2 Contact problem

The peculiarity of this investigation is the development of the method for calculation of reinforcement positions of a flat element under dynamic impact load, which is often transmitted through the contact with a different body. The contact force is related with displacements of the points of the plane element in the point of interaction by a following functional equation [4, 8]
Method of determining the locations of reinforcing elements

\[ y(t) = V_0 t - \frac{1}{m_0} \int_0^t P(t)(t-t_1)dt_1, \]  

(1)

where \( y(t) = \alpha(t) + w(t) \) – the full displacement of the contact body, by means of which a load is passed on the flat target in the direction of the impact, \( m \) – the impactor’s mass, \( t \) – time elapsed from the moment of contact of the impactor and the target, \( t_1 \) – the integration variable.

Time dependencies \( \alpha(t) \) and \( P(t) \) are determined by solving the contact problem [7], the results of which are combined with the results of solution of the wave problem. From numerous existing solutions of the impact interaction between two bodies we can use as a basis the classical Hertz ratio, for which there exist a significant amount of theoretical and experimental data that will allow us to compare the results obtained here

\[ \alpha(t) = b P(t)^{2/3}, \]  

(2)

where \( b = \left( 9\pi^2 (k_1 + k^2) / 16R \right)^{1/3} \), \( k_1 = \left( 1 - \sigma_1 \right)^2 / E_1 \), \( k = \left( 1 - \sigma, \sigma_0 \right) / E \), \( \sigma, \sigma_1, E_1 \) – the Poisson’s ratio and the modulus of elasticity of the impactor, respectively.

3 Computational scheme

The solution of equation (1) is found numerically based on the assumption that on each sufficiently small interval \((n-1)\tau \leq t \leq n\tau\) unknown magnitudes change linearly

\[ \dot{P}(n\tau) = (P_n - P_{n-1}) / \tau. \]  

(3)

After the substitution of the expressions for the target sag [8] at a given point, i.e., at fixed values of coordinates \( r, \theta \), and local deformation (2), in equation (1) we obtain a nonlinear integro-differential equation for the force of interaction, which can be solved using iterative computational scheme [2,7,8]. This procedure is mostly often used while representing unknown quantities in the form of expansions in a series of special functions [9]. This is due to the fact that the equation obtained from (1) after substitution of dependencies for displacements of the target’s points in the form of expansions in a series of special functions is a quite complicated integro-differential equation with a summation of the two indexes.

In this paper, for a final solution of the described problem and for acquiring graphical or tabular time dependencies of dynamic and kinematic characteristics of the target’s deformation we use the following relations obtained by assuming a linear dependence of an unknown quantity on time, if the time interval is negligible [7,8]:

\[ \tau_i = \tau i, \quad s_i = s_{i-1} + V_{i-1}\tau + y_{i-1} \frac{\tau^2}{2}, \]  

(4)
The numerical procedure described above can be implemented using both existing mathematical packages and standard features of programming languages. The described procedure can be used to determine the points of meeting of the direct and reflected from the boundaries of the target elastic waves. In this investigation, the software package [10] that implements a mathematical model of wave propagation in an elastic media was improved. This software package provides a better computing capacity and at the same time allows the user to trace the impact of each of the parameters on the final values at each stage of the simulation and calculation [11], which distinguishes it from existing commercial applications.

4 Numerical investigation

For the software implementation of the mathematical model we developed a computer system that contains three main subsystems responsible for the simulation of wave processes, calculation of dynamic characteristics, processing and storage of data [11]. For a normal operation of the software application, one must consider the magnitudes \( E_r, E_\theta, \sigma_r, \sigma_\theta, \rho, h \), the studied time period \( t \) and the parameter of wave’s damping upon reflection (0\( < l < 1 \)). Figure 1 shows the points of intersection of direct and reflected waves (the initial data for a normal operation of the software system take the following values: \( E_r = 250 \) GPa, \( E_\theta = 150 \) GPa, \( \sigma_r = 0.3, \sigma_\theta = 0.3, \rho = 6000 \) kg/m\(^3\), \( h = 0.1 \) m, \( t = 25 \) ms, \( l = 0.5 \)). It can be clearly seen that direct and reflected waves interact in different points of the plate from 0 to \( h \). Further analysis of the data shows that the greatest number of "growing" interactions is located exactly in the middle of the plate (Fig. 1).
Method of determining the locations of reinforcing elements

Fig. 1 Points of wave interactions (interactions in one direction are indicated by the triangles pointing up (red) and interactions in mixed directions – down (green)).

The developed computational algorithms are also suitable for determining the dynamic characteristics [12] of the behavior of the plate’s points under impact load. Figure 2 shows the dependence of the maximum contact force on the dimensionless thickness of the plate with different values of the initial impact velocity.

Fig. 2 The dependence of the maximum force of interaction on the dimensionless plate thickness for different initial impact velocities (curve 1 – 5 m/s, 2 – 10 m/s, 3 – 15 m/s).

The maximum contact force increases with increasing thickness of the plate. Other parameters take the following values: \( \bar{E} = 2.4 \times 10^{-6} \), h = 1 m, m = 25 kg. From Figure 2 it is evident that the value of the contact force increases faster for large initial impact velocities. Figure 3 presents graphical dependencies of the contact force on time, the impact velocity and the ratio of plate thickness to the radius of the buffer at different values of the ratio \( c_4/c_5 \) (\( c_4 \) and \( c_5 \) are velocities of transverse waves in two planes of the transverse shear), i.e., at different values of the shear stiffness in the direction of main coordinate axes.
5 Conclusions

The conducted numerical experiments allow us to conclude about the necessity to take preventive measures to protect objects against destructive effects of wave processes. In situations where after the impact after a certain period of time there are points where multidirectional waves meet, it is necessary to strengthen the object from the delamination of the material, and if there are points at which the waves are moving collinear, i.e., they increase the stress, it is necessary to strengthen the object from deformation of compression. In both cases, additional reinforcement can serve as a remedy for the protection of the object. In addition to the substantial increase of stresses in the middle of the object, the local maxima at a distance of \( \frac{1}{4} \) and \( \frac{3}{4} \) of the height of the object were observed, that is why the step of reinforcement should be not less than 25\% of the height of the plate.

References


http://dx.doi.org/10.1007/bf01593905


**Received:** March 29, 2015; **Published:** April 27, 2015