A Uniform Bound on Negative Binomial Approximation with \(w\)-Functions

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Abstract

This paper uses Stein’s method and \(w\)-functions to determine a uniform bound for the Kolmogorov distance between the cumulative distribution function of a non-negative integer-valued random variable \(X\) and the negative binomial cumulative distribution function with parameters \(r > 0\) and \(p = 1 - q \in (0,1)\), where \(\frac{rq}{p} = E(X)\) and \(E(X)\) is the mean of \(X\). Two examples are provided to illustrate applications of the result obtained.

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1 Introduction

The topics related to the negative binomial approximation have yielded useful results in applied probability and statistics. The first study of negative binomial approximation has concerned an approximation of the distribution of a sum of dependent Bernoulli random variables, which was presented by Brown and Phillips [1]. They used Stein’s method to give a uniform bound on the negative binomial approximation to the distribution of a sum of dependent Bernoulli random variables, and they also applied the result to approximate the Pólya distribution. For independent geometric summands, Vellaisamy and Upadhye [9]...
used Kerstan’s method to give a uniform bound for approximating the distribution of a sum of independent geometric random variables by a negative binomial distribution. However, the results in [1] and [9] are inappropriate for a non-negative integer-valued random variable. Later, Teerapabolarn and Boondirek [8] adapted and applied Stein’s method and \( w \)-functions to approximate the distribution of a non-negative integer-valued random variable by an appropriate negative binomial distribution as follows. Let \( NB \) be the negative random variable with parameter \( r > 0 \) and \( p = 1 - q \in (0,1) \) that has probabilities

\[
p_{NB}(k) = \frac{\Gamma(r+k)}{k! \Gamma(r)} p^r q^k, \quad k \in \mathbb{N} \cup \{0\},
\]

and has mean \( E(NB) = \frac{rq}{p} \) and variance \( Var(NB) = \frac{rq}{p^2} \). For \( r = 1 \), the negative random variable is the geometric random variable with parameter \( p \), denoted by \( G \). Let \( X \) be a non-negative integer-valued random variable with probability mass function \( p_X(x) > 0 \) for every \( x \) in the support of \( X \), denoted by \( S(X) \), have mean \( \mu = E(X) \) and variance \( \sigma^2 = Var(X) \). For \( A \subseteq \mathbb{N} \cup \{0\} \), a uniform bound for the total variation distance between the distributions of \( X \) and \( NB \), presented in Teerapabolarn and Boondirek [8], is of the form

\[
d_{TV}(X, NB) \leq \frac{p(1-p^r)}{rq} E \left[ \frac{(r+X)q}{p} - w(X) \right] + \frac{rq - \mu}{p}, \quad (1.2)
\]

where \( d_{TV}(X, NB) = \sup_A \left| P(X \in A) - P(NB \in A) \right| \) and \( w(x) = \frac{\sum (\mu-k)p_X(k)}{\sigma^2 p_X(x)} \), \( x \in S(X) \), is the \( w \)-function associated with \( X \). In addition, if \( \frac{rq}{p} = \mu \) then the result in (1.2) becomes

\[
d_{TV}(X, NB) \leq \frac{p(1-p^r)}{rq} E \left[ \frac{(r+X)q}{p} - w(X) \right]. \quad (1.3)
\]

For \( A = \{x_0\}, \ x_0 \in S(X) \), Malingam and Teerapabolarn [4] and Teerapabolarn [7] used the same tools in [8] to give a non-uniform bound for the point metric between the distributions of \( X \) and \( NB \) as follows:

\[
|p_X(0) - p_{NB}(0)| \leq \frac{(1-p^r)p}{rq} \left( E \left[ \frac{(r+X)q}{p} - \sigma^2 w(X) \right] + (1-p_X(0)) \right) \frac{rq - \mu}{p}. \quad (1.4)
\]

and if \( \frac{rq}{p} = \mu \), then [7] improved the bound (1.4) to be the form
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\[ |p_X(0) - p_{NB}(0)| \leq \frac{rq - 1 - p^r}{r(r + 1)q^2} \mathbb{E}(r + X)q - p\sigma^2 w(X). \]  

(1.5)

When \( x_0 \in S(X) \setminus \{0\} \), for \( r \in (0,1) \), [7] showed that

\[ |p_X(x_0) - p_{NB}(x_0)| \leq \min \left\{ \frac{1}{x_0}, \frac{1 - p^r}{rq} \right\} p\mathbb{E} \left[ \frac{(r + X)q}{p} - \sigma^2 w(X) \right] \]

\[ + (1 - p_X(0)) \left\{ \frac{rq}{p} - \mu \right\} \]  

(1.6)

and if \( \frac{rq}{p} = \mu \), then

\[ |p_X(x_0) - p_{NB}(x_0)| \leq \min \left\{ \frac{1}{x_0}, \frac{1 - p^r}{rq} \right\} p\left[ \mathbb{E} \left[ \frac{(r + X)q}{p} - \sigma^2 w(X) \right] \right] \]

\[ + (1 - p_X(0)) \left\{ \frac{rq}{p} - \mu \right\} \]  

(1.7)

For \( r \geq 1 \), [4] showed that

\[ |p_X(x_0) - p_{NB}(x_0)| \leq \min \left\{ \frac{1}{x_0}, \frac{1 - p^r}{(r + x_0 - 1)q} \right\} p\mathbb{E} \left[ \frac{(r + X)q}{p} - \sigma^2 w(X) \right] \]

\[ + (1 - p_X(0)) \left\{ \frac{rq}{p} - \mu \right\} \]  

(1.8)

and if \( \frac{rq}{p} = \mu \), then

\[ |p_X(x_0) - p_{NB}(x_0)| \leq \min \left\{ \frac{1}{x_0}, \frac{1 - p^r}{(r + x_0 - 1)q} \right\} p\mathbb{E} \left[ \frac{(r + X)q}{p} - \sigma^2 w(X) \right]. \]  

(1.9)

Consider the bounds in (1.2)–(1.9), it is seen that if the mean \( E(NB) = \frac{rq}{p} = \mu \), then the bounds in (1.3), (1.5), (1.7) and (1.9) are sharper than those reported in (1.2), (1.4), (1.6) and (1.8), respectively. For \( x_0 \in S(X) \), if \( A = \{0,\ldots,x_0\} \), then the result in (1.3) can be expressed in the following form

\[ d_K(X, NB) \leq \frac{p(1 - p^r)}{rq} \mathbb{E} \left[ \frac{(r + X)q}{p} - w(X) \right], \]  

(1.10)

where \( d_K(X, NB) = \sup_{x_0 \geq 0} |P(X \leq x_0) - P(NB \leq x_0)| \) is the Kolmogorov distance between the cumulative distribution functions of \( X \) and \( NB \). In this paper, we are
interested to improve the bound in (1.10) to be more appropriate for measuring the accuracy of this approximation when
\[ E(NB) = \frac{r\mu}{p} = \mu. \]

The tools for giving our main result are consist of Stein’s method and \( w \)-functions, which are utilized to provide the desired result as mentioned in Section 2. In Section 3, we use Stein’s method and \( w \)-functions to yield a new result of the approximation. In Section 4, we give two examples to illustrate applications of the result. Conclusion of this study is presented in the last section.

2 Method

The methodology in this study consists of Stein’s method and \( w \)-functions. For \( w \)-functions, Cacoullos and Papathanasiou [2] defined a function \( w \) associated with the non-negative integer-valued random variable \( X \) in the relation

\[ w(x)p_X(x) = \frac{1}{\sigma^2} \sum_{i=0}^{x} (\mu - i) p_X(i), \quad x \in S(X) \]  

and Majsnerowska [3] expressed the relation (11) as the form

\[ w(0) = \frac{\mu}{\sigma^2}, \quad w(x) = \frac{1}{\sigma^2} \left\{ \mu + \frac{\sigma^2 w(x-1)p_X(x-1)}{p_X(x)} - x \right\}, \quad x \in S(X) \backslash \{0\}, \]  

where \( w(x) \geq 0 \) and \( p_X(x) > 0 \) for every \( x \in S(X) \). The next relation is an important property for obtaining the main result, which was started by [2].

If a non-negative integer-valued random variable \( X \) is defined as in Section 1, then

\[ E[(X - \mu)f(X)] = \sigma^2 E[w(X)\Delta f(X)], \]  

for any function \( f : \mathbb{N} \cup \{0\} \to \mathbb{R} \) for which \( E[w(X)\Delta f(X)] < \infty \), where \( \Delta f(x) = f(x+1) - f(x) \) and \( E[w(X)] = 1. \)

For Stein’s method, Stein [5] introduced a powerful and general method for bounding the error in the normal approximation. This method was developed and applied in the setting of the negative binomial approximation by Brown and Phillips [1]. Stein’s equation for the negative binomial distribution with parameters \( r > 0 \) and \( p = 1 - q \in (0,1) \) is, for given \( h \), of the form

\[ h(x) - \mathbb{N}_{r,p}(h) = (1 - p)(r + x)f(x+1) - xf(x), \]  

where \( \mathbb{N}_{r,p}(h) = \sum_{k=0}^{\infty} h(k) \frac{\Gamma(r+k)}{k!\Gamma(r)} p^r q^k \) and \( f \) and \( h \) are bounded real-valued.
functions defined on \( \mathbb{N} \cup \{0\} \). For \( A \subseteq \mathbb{N} \cup \{0\} \), let \( h_A : \mathbb{N} \cup \{0\} \to \mathbb{R} \) be defined by
\[
h_A(x) = \begin{cases} 
1 & \text{if } x \in A \\
0 & \text{if } x \not\in A.
\end{cases}
\] (2.5)

Following [1] and [8] and writing \( C_x = \{0, \ldots, x\} \), the solution \( f_A \) of (14) can be written as
\[
f_A(x) = \begin{cases} 
\frac{x!\Gamma(r)}{\Gamma(r + x)xp'q^x} \left[ \mathbb{N}_{r,p}(h_A - C_{x+1}) - \mathbb{N}_{r,p}(h_A) \mathbb{N}_{r,p}(h_{C_{x+1}}) \right] & \text{if } x \geq 1, \\
0 & \text{if } x = 0.
\end{cases}
\] (2.6)

Note that, it follows from [1] that
\[
f_A = -f_{A^c},
\] (2.7)
where \( A^c \) is the complement of \( A \). For \( x_0 \in \mathbb{N} \cup \{0\} \) and \( h_{x_0} = h_{\{x_0\}} \), by following (2.6) the solution \( f_{x_0} = f_{\{x_0\}} \) is of the form
\[
f_{x_0}(x) = \begin{cases} 
\frac{x!\Gamma(r)}{\Gamma(r + x)xp'q^x} \mathbb{N}_{r,p}(h_{x_0}) \mathbb{N}_{r,p}(h_{C_{x+1}}) & \text{if } x \leq x_0, \\
\frac{x!\Gamma(r)}{\Gamma(r + x)xp'q^x} \mathbb{N}_{r,p}(h_{x_0}) \mathbb{N}_{r,p}(1 - h_{C_{x+1}}) & \text{if } x > x_0, \\
0 & \text{if } x = 0.
\end{cases}
\] (2.8)

Moreover, it follows from [1] that
\[
\Delta f_{x_0}(x) = f_{x_0}(x+1) - f_{x_0}(x) \begin{cases} < 0 & \text{if } x_0 > x, \\
> 0 & \text{if } x_0 = x.
\end{cases}
\] (2.9)

For \( A = C_{x_0} \), \( x_0 \in \mathbb{N} \cup \{0\} \), the solution \( f_{C_{x_0}} \) of (2.4) can be written as
\[
f_{C_{x_0}}(x) = \begin{cases} 
\frac{x!\Gamma(r)}{\Gamma(r + x)xp'q^x} \mathbb{N}_{r,p}(h_{C_{x+1}}) \mathbb{N}_{r,p}(1 - h_{C_{x_0}}) & \text{if } x \leq x_0, \\
\frac{x!\Gamma(r)}{\Gamma(r + x)xp'q^x} \mathbb{N}_{r,p}(h_{C_{x_0}}) \mathbb{N}_{r,p}(1 - h_{C_{x-1}}) & \text{if } x > x_0, \\
0 & \text{if } x = 0.
\end{cases}
\] (2.10)

The following lemma gives a uniform bound for \( \Delta f_x \), which is also need for proving the desired result.
Lemma 2.1. Let \( x \in \mathbb{N} \), then the following inequality holds:

\[
\Delta f_x(x) \leq \frac{1 - p^{r+1}}{(r+1)q}.
\] (2.11)

Proof. [1] showed that \( \Delta f_x \) to be a decreasing function in \( x \in \mathbb{N} \). From which, it follows that \( \Delta f_x(x) - \Delta f_x(x+1) > 0 \) for every \( x \in \mathbb{N} \). Therefore, we obtain

\[
\Delta f_x(x) \leq \Delta f_x(1) = \sum_{k=2}^{\infty} \frac{\Gamma(r+k) p^k q^k}{k! \Gamma(r)} + \sum_{k=2}^{\infty} \frac{1 - p^r - r p^r q + (r+1) p^r q}{(r+1)q}
\]

which implies that (2.11) holds.

Lemma 2.2. For \( x_0 \in \mathbb{N} \cup \{0\} \) and \( x \in \mathbb{N} \), then we have the following:

1. \( \sup_{x_0,x} \left| \Delta f_{c_{x_0} x} \right| \leq \frac{1 - p^{r+1}}{(r+1)q} \) \hspace{1cm} (2.12)

2. \( \sup_{x_0,x} \left| \Delta f_{c_{x_0} x} \right| \leq \frac{1}{2} \) for \( r = 1 \). \hspace{1cm} (2.13)

Proof. 1. With Lemma 2.1, (2.7) and (2.9), we have that

\[
\frac{1 - p^{r+1}}{(r+1)q} \geq \sum_{k \in c_{x_0}} \Delta f_k(x) = \Delta f_{c_{x_0} x} \geq \Delta f_{c_{x_0} x}^c(x) = -\Delta f_x(x) \geq \frac{1 - p^{r+1}}{(r+1)q}.
\]

Hence, the inequality (2.12) is obtained.

2. It is directly obtained from [6].

3 Main result

The following theorem presents a new uniform bound for the Kolmogorov distance between the cumulative distribution function of the non-negative integer-valued random variable \( X \) and an appropriate negative binomial cumulative distribution function with parameters \( r \) and \( p \).

Theorem 3.1. With the above definitions, let \( \frac{r}{p} = \mu \), then we have the following inequality.

\[
d_K(X,NB) \leq \frac{p(1 - p^{r+1})}{(r+1)q} E \left[ \frac{(r+X)q}{p} - \sigma^2 w(X) \right]. \] (3.1)
Proof. For $x_0 \in S(X)$, replacing $h$ by $h_{c_{x_0}}$ and $x$ by $X$ and taking the expectation in (2.4), yields
\[
P(X \leq x_0) - P(NB \leq x_0) = E[q(r + X)f(X + 1) - Xf(X)],
\] (3.2)
where $f = f_{c_{x_0}}$ is defined in (2.10). Let $\delta(X) = E[q(r + X)f(X + 1) - Xf(X)]$ then we obtain
\[
\delta(X) = E[qrf(X + 1) + qXf(X + 1) - Xf(X)]
\]
\[
= rqEf(X + 1) + qE[X\Delta f(X)] - pE[Xf(X)]
\]
\[
= rqE[f(X + 1)] + qE[X\Delta f(X)] - pE[(X - \mu)f(X)] + \mu E[f(X)]
\]
\[
= rqE[\Delta f(X)] + qE[X\Delta f(X)] + (rq - p\mu)E[f(X)] - pE[(X - \mu)f(X)]
\]
\[
= rqE[\Delta f(X)] + qE[X\Delta f(X)] - pE[(X - \mu)f(X)].
\] (3.3)
Since, $E[w(X)] = 1$ and $E[w(X)\Delta f(X)] = E[w(X)] \Delta f(X) < \infty$. Then by (2.3), (2.4) and (3.3), we have that
\[
d_K(X, NB) = \left|rqE[\Delta f(X)] + qE[X\Delta f(X)] - p\sigma^2 E[w(X)\Delta f(X)]\right|
\]
\[
\leq E\left|[(r + X)q - p\sigma^2 w(X)]\Delta f(X)\right|
\]
\[
\leq \sup_{x_0, x}|\Delta f(x)|E\left|(r + X)q - p\sigma^2 w(X)\right|
\]
\[
\leq \frac{p(1 - p^{r+1})}{(r + 1)q} E\left|(r + X)q - p\sigma^2 w(X)\right| \quad \text{(by (2.12)).}
\]
Hence, we have the inequality (3.1). \qed

Corollary 3.1. If \( \frac{(r+X)q}{p} - \sigma^2 w(X) \geq / < 0 \) for every $x \in S(X)$ and $\frac{rq}{p} = \mu$, then
\[
d_K(X, NB) \leq \frac{p(1 - p^{r+1})}{(r + 1)q} \left|(r + \mu)q - p\sigma^2 w(X)\right|.
\] (3.4)

In the case of $r = 1$, a new uniform bound on the geometric approximation with $w$-functions by using Lemma 2.2 (2) is as follows.

Corollary 3.2. For geometric approximation, if $\frac{q}{p} = \mu$, then we have the following:
\[
d_K(X, G) \leq \frac{1}{2} E\left|(r + X)q - p^2 w(X)\right|
\] (3.5)
and if \( \frac{(X+1)d}{p} - \sigma^2 w(X) \geq l < 0 \) for every \( x \in S(X) \),
\[
d_K(X,G) \leq \frac{1}{2} |(r+\mu)q - p\sigma^2|.
\] (3.6)

**Corollary 3.3.** If \( p = 1-q \in (0,1) \) and \( r > 0 \), then
\[
\frac{1 - p^{r+1}}{(r+1)q} \leq \frac{1 - p^r}{rq}.
\] (3.7)

**Proof.** Let \( \lambda = rq \), then we have that \( e^\lambda = 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots \). Therefore
\[
(1+\lambda)e^{-\lambda} < 1 \iff e^{-\lambda} + \lambda e^{-\lambda} < 1 \iff e^{-\lambda} < 1 - \lambda e^{-\lambda}.
\] (3.8)

Because \( e^{-q} = 1 + (-q) + \frac{(-q)^2}{2!} + \frac{(-q)^3}{3!} + \cdots = 1 - (1 - p) + \frac{(-q)^2}{2!} + \frac{(-q)^3}{3!} + \cdots \),
\[
p < e^{-q} \iff p^r < e^{-rq} = e^{-\lambda}.
\] (3.9)

From (3.8) and (3.9), it follows that
\[
p^r < 1 - \lambda e^{-\lambda} \iff \lambda e^{-\lambda} < 1 - p^r \iff p^r < \frac{1 - p^r}{rq} \iff rqp^r < 1 - p^r
\]
\[
\iff rqp^r (1-p) < q (1-p^r) \iff rqp^r - rqp^{r+1} < q - qp^r
\]
\[
\iff -rqp^{r+1} < q - rqp^r - qp^r \iff rq - rqp^{r+1} < rq + q - rqp^r - qp^r
\]
\[
\iff rq(1 - p^{r+1}) < (r+1)q(1-p^r) \iff \frac{1 - p^{r+1}}{(r+1)q} < \frac{1 - p^r}{rq},
\]
this yields the desired result. \( \square \)

**Remark.** With Corollary 3.3, it is observed that the bound in the Theorem 3.1 is sharper than that reported in (1.10).

### 4 Examples

This section gives applications for the negative binomial approximation to the beta binomial and beta negative binomial distributions.

**Example 4.1.** An application to the beta binomial distribution

Let us consider the binomial distribution with parameters \( n \) and \( p \), if \( p \) is a random variable that has a beta distribution with shape parameters \( \alpha > 0 \) and \( \beta > 0 \), then the distribution is called the beta binomial distribution with parameters \( n \), \( \alpha \) and \( \beta \). Let the beta binomial random variable \( X \) have probabilities.
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\[ p_X(x) = \binom{n}{x} \frac{\Gamma(\alpha + x)\Gamma(\beta + n - x)\Gamma(\alpha + \beta)}{\Gamma(\alpha + \beta + n)\Gamma(\beta)\Gamma(\alpha)}, \quad x = 0,1,\ldots,n, \]

and have mean \( \mu = \frac{na}{\alpha + \beta} \) and variance \( \sigma^2 = \frac{na\beta(\alpha + \beta + n)}{(\alpha + \beta)^2(\alpha + \beta + 1)} \). Following the relation in (2.2), we have \( w(x) = \frac{(\alpha + x)(\alpha - x)}{(\alpha + \beta)^2} \). Putting \( p = \frac{\alpha + \beta}{\alpha + \beta + n} \) and \( r = \alpha \) to the result in Theorem 3.1, then \( \frac{(\alpha + x)q}{p} - \sigma^2 w(x) = \frac{(\alpha + x)x}{\alpha + \beta} \geq 0 \) for all \( 0 \leq x \leq n \). Applying the Corollaries 3.1 and 3.2, the following corollary is as follows.

**Corollary 4.1.** If \( r = \alpha \) and \( p = \frac{\alpha + \beta}{\alpha + \beta + n} \), then we have the following:

\[ d_K(X, NB) \leq \frac{\alpha(1 - p^{\alpha + 1})}{(\alpha + \beta + 1)p}, \]

and for \( r = 1 \),

\[ d_K(X, G) \leq \frac{n}{(\beta + 1)(\beta + 2)}. \]

The result gives a good approximation when \( \beta \) is large and \( n \) and \( \alpha \) are small. Therefore, the negative binomial distribution with parameters \( \alpha \) and \( \frac{\alpha + \beta}{\alpha + \beta + n} \) can be used as an approximation of the beta binomial distribution with parameters \( n, \alpha \) and \( \beta \) when \( \beta \) is large and \( n \) and \( \alpha \) are both small.

**Example 4.2.** An application to the beta negative binomial distribution

It is well-known that the beta-negative binomial distribution is a negative binomial distribution whose probability of success parameter \( p \) follows a beta distribution with shape parameters \( \alpha > 0 \) and \( \beta > 0 \). Let \( X \) be the beta negative binomial random variable with probability mass function

\[ p_X(x) = \frac{\Gamma(r + \alpha)\Gamma(x + \beta)\Gamma(r + x)\Gamma(\alpha + \beta)}{\Gamma(r + x + \alpha + \beta)\Gamma(r)\Gamma(x + 1)\Gamma(\alpha)\Gamma(\beta)}, \quad x = 0,1,\ldots, \]

where \( \alpha, \beta, r \in \mathbb{R}^+ \). The mean and variance of \( X \) are \( \mu = \frac{r\beta}{\alpha - 1} \) and \( \sigma^2 = \frac{r\beta(r + \alpha - 1)(\alpha + \beta - 1)}{(\alpha - 2)(\alpha - 1)^2} \), respectively. Following the relation in (2.2), we have

\[ w(x) = \frac{(\alpha + x)(\beta + x)}{(\alpha - 1)^2}. \]

Setting \( p = \frac{\alpha + 1}{\alpha + \beta + 1} \) to the result in Theorem 3.1, then

\[ \frac{\alpha + 1}{\alpha - 1} - \sigma^2 w(x) = -\frac{(\alpha + x)x}{\alpha - 1} \leq 0 \quad \text{for all } x \geq 0. \]

Using the Corollaries 3.1 and 3.2, the following corollary is as follows.
Corollary 4.2. If \( p = \frac{a^{-1}}{a + \beta^{-1}} \), then we have the following:

\[
d_k (X, NB) \leq \frac{r(1 - p^{r+1})}{(\alpha - 2)p},
\]

and for \( r = 1 \),

\[
d_k (X, G) \leq \frac{\beta}{(\alpha - 1)(\alpha - 2)}.
\]

The result gives a good approximation when \( \alpha \) is large and \( r \) and \( \beta \) are small. Therefore, the negative binomial distribution with parameters \( r \) and \( p \) can be used as an approximation of the beta negative binomial distribution with parameters \( \alpha, \beta \) and \( r \) when \( \alpha \) is large and \( r \) and \( \beta \) are both small.

5 Conclusion

In this study, a uniform bound for the Kolmogorov distance between the cumulative distribution function of a non-negative integer-valued random variable and an appropriate negative binomial cumulative distribution function with parameters \( r \) and \( p \) was obtained by using Stein’s method and \( w \)-functions, where \( \frac{rq}{p} = \mu \). The main result gives a good negative binomial approximation when the uniform bound is small. Furthermore, by theoretical comparison, the uniform bound in the present study is sharper than that presented in (1.10).

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