Fish Consumption Impact on Coronary Heart Disease Mortality in Morocco: A Mathematical Model

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Abstract

In this work a Mathematical model is proposed to study the relationship between fish consumption and coronary heart disease (CHD) mortality in Morocco.

The aim of this study is to investigate CHD mortality depending on various fish consumption levels by proposing different scenarios.

The mathematical model is based on ordinary differential equations system. The dynamics of a population at risk of CHD, the dynamics of fish population living along the Moroccan coasts, the relationship between CHD mortality and the quantities consumed from the total caught in Morocco are studied. Furthermore, simulation is carried out with different scenarios depending on the total fish harvested and total fish consumed.

This model highlights the negative correlation between CHD mortality and fish consumption.

Mathematics Subject Classification: 92B05

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1 Introduction

Changes in lifestyle and eating habits lead to the fast propagation of chronic diseases in the world. Indeed, chronic diseases related to diet are considered as the leading cause of mortality worldwide. It is estimated that one third of deaths in the world is due to diseases affecting the heart and blood vessels (World Health Organization 2014).

Obviously, the human body needs fats. However, some fats increase the risk of developing cardiovascular diseases, especially fats found in dairy products, meat and hydrogenated oils. According to the World Health Organization (WHO) (Amine et al. 2002), other fats, such as those found in oils of sunflower and soybean, may reduce this risk. Fish oil (found in fatty fish) is also beneficial.

To study the relationship between cardiac diseases and fish consumption, several studies have been carried out. Kris-Etherton et al. (2003) have studied the effects on cardiovascular diseases (CVD) and concluded that \( \text{Omega} - 3(n - 3) \) acid supplements which are highly contained in fatty fish can reduce cardiac events. Virtanen et al. (2008) pointed out that consuming fish can reduce risk of major chronic diseases (cardiovascular disease, cancer...). Raatz et al. (2013) performed a statistical study on an USA population and found that CVD risk is decreased while eating fish frequently. Hooper et al. (2006) investigated the effects of fish consumption, through the relative risk measure, not on the CVD in general, but on a particular type of cardiovascular disease which is Coronary hearth disease (CHD). In 2004, a meta-analysis of 13 cohort studies was conducted to examine the association between fish intake and CHD mortality. Significant inverse associations were reported between fish consumption and fatal CHD (He et al. 2004). Another meta-analysis of observational studies on fish intake and CHD pooled data of 19 studies and indicated that fish consumption was associated with significantly lower risk of fatal and total CHD, suggesting that fish consumption may be an important component of lifestyle modification for the prevention of CHD (Whelton et al. 2004). In 2010 an updated meta-analysis of seven cohort studies concluded that fish consumption has a significant protective effect on fatal CHD (Zheng et al. 2012).

Eco-epidemiological models have been proposed to study and analyse the effect of diseases in ecological systems (Chattopadhyay et al. 2001; Flake et al. 2003). To the best of our knowledge, no mathematical model has been proposed to deal with the impact of fish consumption on cardiovascular disease mortality. The motivation of this work is that Morocco has an important stock of fish living along the 3500 Kms of Mediterranean and Atlantic coasts, with sardine as the most abundant species (Serghini 2008; Serghini et al. 2007). This lat-
ter specie is characterized by its richness in Omega3(n − 3). However, the individual Moroccan consumption of fish is low (8kg/person/year) and this country ranks as one of those who have a high rate of CHD mortality (Mendis et al. 2011). In this paper, we propose a mathematical model dealing with the relationship between fish consumption and CHD mortality by studying the dynamics of a population at risk of CHD and that of fish population living along the Moroccan coasts. Stability analysis is carried out and simulation of different scenarios is considered.

In Morocco, fish is an important source of animal protein, employment and income. Nevertheless, the future availability of this food source will depend on the sustainable use of marine fish stocks (Boutayeb et al. 2013).

2 Model description

For establishing the model, we consider a population of size $N$ constituted of persons with and without CHD risk. Let $S$ and $R$ denote, respectively, the number of persons without CHD risk and with CHD risk. These populations evolve continuously in the time interval $[0,T]$, at any time $t \in [0,T]$, we have

$$P_t := N(t) = S(t) + R(t).$$

The following scheme (figure 1) illustrates the dynamics of the two populations ($S$ and $R$) into two compartments with related settings and probabilities.

![Figure 1: The dynamics of populations with and without CHD risk.](image)

Where, $\Lambda$ represents the recruitment of persons without CHD risk, $\mu$ the natural mortality rate, $\delta$ the mortality rate due to CHD and $\alpha$ the probability to have CHD. Finally, $\beta$ is the rate of patients with CHD who are cured, $RR$ is the relative risk depending on the fish consumption.

The quasi totality of observational studies considered in the meta-analyses cited earlier, standardized and categorized the level of fish consumption into five classes:

- Lowest class: no fish consumption or less than one serving per month.
- Second class: one to three servings per month.
• Third class: one serving a week.
• Fourth class: two to four servings per week.
• Highest class: more than five servings per week.

The relative risk \( RR \) indicates the impact of fish consumption (classes 2 to 5) compared to little or no consumption (Lowest class with \( RR=1 \)). Following this remark and based on pooled data given by He et al. (2004), we propose a RR following a logistic equation of the form:

\[
RR = \frac{a_1}{a_2 + e^{a_3 \gamma X}},
\]

with

\[
\gamma = \frac{qE}{P_i} aT_f,
\]

where, \( a_1, a_2, a_3 \) are coefficients of non-linear regression applied to data of the TABLE 2 from He et al. (2004), \( a \) is the rate of fish consumed from the total harvested and \( T_f \) is a coefficient that transforms the consumption of fish per capita and per year to a frequency per month.

The relative risk is represented by the figure 2 below.

Suppose that at any time \( t \), \( X(t) \) is the biomass of the fish population in Moroccan coasts. The growth of fish population is assumed to follow the logistic equation with carrying capacity \( K \) and biotic potential \( r \). A total harvesting effort is denoted \( E \) with catchability coefficient \( q \).

With the descriptions above, the proposed model can be written as follows:

\[
\begin{align*}
\frac{dS}{dt} &= \Lambda - (\mu + \alpha)S + \beta R \\
\frac{dR}{dt} &= -(\mu + \delta RR + \beta)R + \alpha S \\
\frac{dX}{dt} &= r (1 - \frac{X}{K}) X - qEX
\end{align*}
\]

\[
(1)
\]

3 Equilibrium analysis

For the model above, equilibrium points are defined such that there is no variations in \( R, S \) and \( X \) with respect to \( t \), namely:

\[
\frac{dS}{dt} = \frac{dR}{dt} = \frac{dX}{dt} = 0.
\]

Thus, we can easily check that our model has two equilibrium points given by:
Figure 2: Non linear regression ($a_1 = -0.73, a_2 = -1.71, a_3 = -0.0616$).

- First point: $P_0 = (S_0, R_0, X_0 = 0)$

\[
\begin{align*}
S_0 &= \frac{\Lambda(\beta + \mu + \delta A_1)}{\beta\mu + \mu^2 + \mu\alpha + \delta A_1(\mu + \alpha)} \\
R_0 &= \frac{\alpha \Lambda}{\beta\mu + \mu^2 + \mu\alpha + \delta A_1(\mu + \alpha)}
\end{align*}
\]

where,

\[
A_1 = \frac{a_1}{a_2 + 1}.
\]

This point corresponds to the case when there is no biomass of fish population. Consequently, the rate of mortality due to CHD is not affected by fish consumption.

- Second point: $P^* = (S^*, R^*, X^*)$
\[
S^* = \frac{\Lambda(\beta + \mu + \delta A_2)}{\beta \mu + \mu^2 + \mu \alpha + \delta A_2(\mu + \alpha)}
\]
\[
R^* = \frac{\alpha \Lambda}{\beta \mu + \mu^2 + \mu \alpha + \delta A_2(\mu + \alpha)}
\]
\[
X^* = \frac{K}{r} (r - qE)
\]

where,
\[
A_2 = \frac{a_1}{a_2 + e^{a_3 \gamma K} (r - qE)}.
\]

The second equilibrium point exists if \( E < \frac{r}{q} \). We can note that the equilibrium population at risk of CHD is in response to the fish consumption.

In the rest of the paper, we assume that the fishing effort \((E)\) should be less than the Biotechnical Productivity \( \left( \frac{r}{q} \right) \),

\[
E < \frac{r}{q}. \tag{2}
\]

Otherwise, a problem of over-fishing will take place, which will contribute to the extinction of the fish population.

4 Stability analysis

The local stability analysis based on variational principle is used. The Jacobian of the system (1) at any point \( P(S, R, X) \) is written as:

\[
J(P) = \begin{pmatrix}
-\mu - \alpha & \beta & 0 \\
\alpha & -\beta - \mu - \frac{\delta a_1}{a_2 + e^{a_3 \gamma X}} & \frac{\delta a_1 a_3 \gamma e^{a_3 \gamma X} R}{(a_2 + e^{a_3 \gamma X})^2} \\
0 & 0 & r - 2r \frac{X}{K} - qE
\end{pmatrix}
\]

- Consequently, the Jacobian at the point \( P_0 \) is given by:

\[
J(P_0) = \begin{pmatrix}
-\mu - \alpha & \beta & 0 \\
\alpha & -\beta - \mu - \delta A_1 & \frac{\delta A_2^2 a_3 \gamma R}{a_1} \\
0 & 0 & r - qE
\end{pmatrix}
\]
This matrix has an obvious eigenvalue
\[
\lambda_0^3 = r - qE,
\]
which is positive according to (2). Thus, \( P_0 \) is an unstable node.

- At the second equilibrium point \( P^* \) one can write the Jacobian as:
\[
J(P^*) = \begin{pmatrix}
-\mu - \alpha & \beta & 0 \\
\alpha & -\beta - \mu - \delta A_2 & \frac{\delta A_2^2 a_3 \gamma e^{a \gamma x^* R^*}}{a_1} \\
0 & 0 & -r + qE
\end{pmatrix}
\]
The characteristic equation for this Jacobian is:
\[
(-r + qE - \lambda)(\lambda^2 + \lambda B_1 + B_2) = 0,
\]
with:
\[
B_1 = 2\mu + \alpha + \beta + \delta A_2,
\]
and
\[
B_2 = \mu^2 + \mu(\alpha + \beta) + \delta A_2(\alpha + \mu).
\]
Therefore, one of the eigenvalues is:
\[
\lambda_1^3 = -r + qE,
\]
which is negative according to (2).
The other two eigenvalues are the roots of the quadratic:
\[
\lambda^2 + \lambda B_1 + B_2 = 0,
\]
using Routh-Hurwitz stability criterion, it can be concluded that \( P^* \) is stable, since \( B_1 \) and \( B_2 \) are positive.

5 Simulations
Simulations are performed by taking the following parameters:

\[
\Lambda = 500000, \quad \mu = 0.014, \quad \alpha = 0.06, \quad \beta = 0.005, \quad \delta = 0.006, \quad r = 1,
\]
\[
K = 1.1 \times 3.75 \times 10^9, \quad q = 0.04, \quad E = 10, \quad P_t = 32 \times 10^6,
\]
\[
S_0 = P_t \times 0.75, \quad R_0 = P_t \times 0.25, \quad X_0 = 225 \times 10^7.
\]
We consider different scenarios depending on the parameter \( a \) which is the rate of fish consumed from the total harvested (figure 3).
Figure 3: Mortality due to CHD in ten years

- **a=0**

No fish is consumed at all: we find that the total mortality due to CHD in ten years is 834620.

- **a=0.30** (present situation)

30% of the total harvested fish is consumed: we find that the total mortality due to CHD in ten years is 549470.

- **a=0.60** (suggested strategy)

60% of the total harvested fish is consumed: we find that the total mortality due to CHD in ten years is 451970.

- **a=1**

The total harvested fish is completely consumed: we find that the total mortality due to CHD in ten years is 396690.

These simulations show that when the consumption of harvested fish goes from 0% to 60%, CHD mortality is reduced by 54%.
6 Conclusion

In this paper, we proposed a mathematical model based on system of ordinary differential equations. The relative risk was used in order to compare the risk of CHD mortality within a population distributed in five classes according to their consumption levels.

Our model showed that in absence of biomass of fish population \( (X^* = 0) \), the equilibrium point is unstable. Otherwise, in presence of the biomass of fish population \( X^* = \frac{K}{r}(r - qE) \), and under the condition that the fishing effort \( (E) \) is less than the Biotechnical Productivity \( \left( \frac{r}{q} \right) \), the equilibrium point is stable. Furthermore, simulation was carried out with different values of \( a \) (the rate of fish consumed from the total harvested) validating the negative correlation between the level of fish consumption and the risk of CHD mortality.

Note that the WHO recommended that fish consumption should reach 14 kg/year/capita while the Moroccan individual consumes only 8 kg/year/capita since this country dedicates only 30% of the total harvested fish to the local consumption. Therefore, this work encourages the national health authorities to respond with a strategy, for the medium or long term, so that the quantities of fish harvested are consumed locally rather than being exported.

References


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