A Finite Capacity Multiserver Interdependent Retrial Queueing Model with Controllable Arrival Rates

B. Antline Nisha and M. Thiagarajan

Department of Mathematics
St.Joseph’s College (Autonomous), Tiruchirappalli
Tamil Nadu, India-620 002

Abstract

We consider a finite capacity interdependent retrial queueing model with controllable arrival rates, when \( c = 1, 2 \). The steady state solution and system characteristics are derived for this model. The analytical results are numerically illustrated.

Mathematics Subject Classification: 60K25; 68M20; 90B22

Keywords: multiserver; finite capacity; retrial queue; interdependent; primary arrival and service processes

1 Introduction

Retrial queues have been widely used to model many problems in telephone switching systems, telecommunication networks, computers competing to get service from a central processor. For recent bibliographies on retrial queues see Artalejo J.R., [2, 3], Falin G.I. [5, 6], Medhi J. [7]. In general it is assumed that the arrival stream of primary calls, the service times and retrial times are mutually independent. But the primary arrival and service processes are interdependent in practical situations. Much work has been reported in the literature regarding interdependent standard queueing model with controllable arrival rates by Aftab Begum M.I. and Maheswari D. [1], Srinivasan A. and Thiagarajan M. [8, 9].
Recently Antline Nisha B., and Thiagarajan M. [11, 12] have studied interdependent retrial queueing model with controllable arrival rates. An attempt is made in this paper to obtain the relevant results of the M/M/c/K interdependent retrial queueing model with controllable arrival rates is considered, when \( c = 1,2 \).

2 Description of the Model

Consider a multiserver finite capacity retrial queueing system in which primary customers arrive according to the Poisson flow of rate \( \lambda_0 \) and \( \lambda_1 \). If an arriving primary call finds some server free it immediately occupies a server and leaves the system after service. Otherwise, if the servers are engaged, it produces a source of repeated calls. It is assumed that periods between successive retrials are exponentially distributed with parameter \( \theta \) and service times are exponentially distributed with parameter \( \mu \). Without loss of generality it is assumed that \( \mu = 1 \).

It is also assumed that the primary arrival process \( x_1(t) \) and the service process \( x_2(t) \) of the systems are correlated and follow a bivariate Poisson process given by

\[
P(X_1(t) = x_1, X_2(t) = x_2) = e^{-(\lambda_i + \mu_n - \epsilon)t} \sum_{j=0}^{\min(x_1,x_2)} \frac{(\lambda_i-\epsilon)^{x_1-j} (\mu_n-\epsilon)^{x_2-j}}{j! (x_1-j)! (x_2-j)!}
\]

\( x_1, x_2 = 0,1,2, \ldots \), \( \lambda_i \), \( \mu_n < 0 \), \( i = 0,1 \);

with parameters \( \lambda_0 \), \( \lambda_1 \), \( \mu_n \) and \( \epsilon \) as mean faster rate of primary arrivals, mean slower rate of primary arrivals, mean service rate and mean dependence rate (covariance between the primary arrival and service processes) respectively.

At time \( t \), let \( N(t) \) be the number of sources of repeated calls and \( C(t) \) be the number of busy servers. The system state at time \( t \) can be described by means of a bivariate process \( \{C(t),N(t)\}, t \geq 0 \), where \( C(t) = 1,2 \) or \( 0 \) according as the server is busy or idle, the process will be called CN process. For the M/M/c/K interdependent retrial queue, stationary distribution exits, if \( \frac{\lambda_0 - \epsilon}{\mu - \epsilon} < 1 \) and \( \frac{\lambda_1 - \epsilon}{\mu - \epsilon} < 1 \), where \( (\mu - \epsilon) = 1 \). In this paper, when \( c=1 \), the results are obvious from the earlier work "The M/M/1/K interdependent retrial queueing model with controllable arrival rates".

3 Steady State Equations

Let \( P_{0,n,0}, P_{1,n,0} \) and \( P_{2,n,0} \) denote the steady state probability that there are \( n \) customers in the queue when the system is faster rate of primary arrivals. Let \( P_{0,n,1}, P_{1,n,1} \) and \( P_{2,n,1} \) denote the steady state probability that there are \( n \) customers in the queue when the system is slower rate of primary arrivals. The
servers are idle and busy with one server and two servers. We observe that only $P_{0,n,0}$, $P_{1,n,0}$ exists when $n = 0, 1, 2, \ldots, r-1, r; P_{2,n,0}$ exists when $n = 0, 1, 2, \ldots, r-2, r-1; P_{0,n,0}, P_{1,n,0}, P_{n,0,1}$ and $P_{1,n,1}$ exists when $n = r+1, r+2, \ldots, R-2, R-1; P_{2,n,0}$, $P_{2,n,1}$ exists when $n= r$, $r+1$, $r+2, \ldots, R-2; P_{0,n,1}$ and $P_{1,n,1}$ exists when $n = R, R+1, \ldots, K; P_{2,n,1}$ exists when $n = R-1, R, \ldots, K-1$.

Further $P_{j,n,0} = P_{j,n,1} = 0$ if $n > K$, $j=0,1,2$

The steady state equations are

$$-[(\lambda_0 - \epsilon) + n\theta]p_{0,n,0} + p_{1,n,0} = 0, \quad (0 \leq n \leq R - 1) \ldots \ldots (3.1)$$

$$-[(\lambda_0 - \epsilon) + 1 + n\theta]p_{1,n,0} + (\lambda_0 - \epsilon)p_{0,n,0} + (n+1)\theta \ p_{0,n+1,0}$$

$$+ 2p_{2,n,0} = 0, \quad (0 \leq n \leq R - 2) \ldots \ldots \ldots \ldots \ldots \ldots (3.2)$$

$$-[(\lambda_0 - \epsilon) + 2]p_{2,n,0} + (\lambda_0 - \epsilon)p_{1,n,0} + (n+1)\theta \ p_{1,n+1,0}$$

$$+ (\lambda_0 - \epsilon)p_{2,n-1,0} = 0, \quad (0 \leq n \leq r - 1) \ldots \ldots \ldots (3.3)$$

$$-[(\lambda_0 - \epsilon) + 2]p_{2,r,0} + (\lambda_0 - \epsilon)p_{1,r,0} + (r+1)\theta \ p_{1,r+1,0}$$

$$+ (r+1)\theta \ p_{1,r+1,1} + (\lambda_0 - \epsilon)p_{2,r-1,0} = 0 \ldots \ldots \ldots (3.4)$$

$$-[(\lambda_0 - \epsilon) + 2]p_{2,n,0} + (\lambda_0 - \epsilon)p_{1,n,0} + (n+1)\theta \ p_{1,n+1,0}$$

$$+ (\lambda_0 - \epsilon)p_{2,n-1,0} = 0,\quad (r + 1 \leq n \leq R - 2) \ldots \ldots \ldots (3.5)$$

$$-[(\lambda_0 - \epsilon) + 2]p_{2,R-1,0} + (\lambda_0 - \epsilon)p_{1,R-1,0}$$

$$+ (\lambda_0 - \epsilon)p_{2,R-2,0} = 0, \ldots \ldots \ldots \ldots \ldots \ldots (3.6)$$

$$-[(\lambda_1 - \epsilon) + n\theta]p_{0,n,1} + p_{1,n,1} = 0, \quad (r + 1 \leq n \leq K) \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.7)$$

$$-[(\lambda_1 - \epsilon) + 1 + n\theta]p_{1,n,1} + (\lambda_1 - \epsilon)p_{0,n,1} + (n+1)\theta \ p_{0,n+1,1}$$

$$+ 2p_{2,n,1} = 0,\quad (r + 1 \leq n \leq K - 1) \ldots \ldots \ldots (3.8)$$

$$-[(\lambda_1 - \epsilon) + 2]p_{2,r+1,1} + (\lambda_1 - \epsilon)p_{1,r+1,1} + (r+2)\theta \ p_{1,r+2,1}$$

$$- \{[(\lambda_0 - \epsilon)(r + 1)]\theta/2\} \ p_{0,r+1,1} = 0 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.9)$$

$$-[(\lambda_1 - \epsilon) + 2]p_{2,n,1} + (\lambda_1 - \epsilon)p_{1,n,1} + (n+1)\theta \ p_{1,n+1,1}$$

$$+ (\lambda_1 - \epsilon)p_{2,n-1,1} = 0, \quad (r + 2 \leq n \leq R - 1) \ldots \ldots \ldots \ldots \ldots (3.10)$$

$$-[(\lambda_1 - \epsilon) + 2]p_{2,R,1} + (\lambda_1 - \epsilon)p_{1,R,1} + (R+1)\theta \ p_{1,R+1,1}$$

$$+ (\lambda_1 - \epsilon)p_{2,R-1,1} + (\lambda_0 - \epsilon)p_{2,R-2,0} = 0 \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.11)$$

$$-[(\lambda_1 - \epsilon) + 2]p_{2,n,1} + (\lambda_1 - \epsilon)p_{1,n,1} + (n+1)\theta \ p_{1,n+1,1}$$
\[ + (\lambda_1 - \varepsilon)p_{2,n-1,1} = 0, \quad (R + 1 \leq n \leq K - 1) \quad \ldots \ldots \ldots (3.12) \]

\[-[2p_{2,k,1} + (\lambda_1 - \varepsilon)p_{1,k,1} + (\lambda_1 - \varepsilon)p_{2,k-1,1} = 0 \quad \ldots \ldots \ldots \ldots \ldots (3.13)\]

From (3.1) – (3.3), we get,

\[P_{0,n,0} = \frac{(-\varepsilon)^n}{n!\theta^n} \prod_{i=1}^{n} \frac{(-\varepsilon + i\theta)^2}{2 + 3(-\varepsilon + 2i\theta)} P_{0,0,0}\]

\[P_{1,n,0} = A_1 P_{0,n,0}, \quad (0 \leq n \leq r) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.14)\]

\[P_{2,n,0} = A_2 \frac{(-\varepsilon)^n}{n!\theta^n} \prod_{i=1}^{n} \frac{(-\varepsilon + i\theta)^2}{2 + 3(-\varepsilon + 2i\theta)} P_{0,0,0}, \quad (0 \leq n \leq r - 1) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.15)\]

Where \[A_1 = (\lambda_0 - \varepsilon + n\theta), \quad A_2 = (1 + \lambda_0 - \varepsilon) + (n + 1)\theta\]

From (3.1), (3.2) and (3.5) we recursively derive

\[P_{0,n,0} = \frac{(-\varepsilon)^n}{n!\theta^n} \prod_{i=1}^{n} \frac{(-\varepsilon + i\theta)^2 + i\theta}{2 + 3(-\varepsilon + 2i\theta)} P_{0,0,0}\]

\[-\left[ \frac{2A_3(r+1)!}{n!\theta^{n-1-r}(2 + 3(-\varepsilon + 2i\theta))} \left( \sum_{m=1}^{n} m\theta^{m-r} (-\varepsilon)^{n-m} \right) \right] P_{0,n+1,1}\]

\[P_{1,n,0} = A_1 P_{0,n,0}, \quad (r + 1 \leq n \leq R - 1) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.16)\]

\[P_{2,n,0} = A_2 \frac{(-\varepsilon)^n}{n!\theta^n} \prod_{i=1}^{n} \frac{(-\varepsilon + i\theta)^2 + i\theta}{2 + 3(-\varepsilon + 2i\theta)} P_{0,0,0}\]

\[+ \left[ \frac{A_3(r+1)!}{n!\theta^{n-1-r}(2 + 3(-\varepsilon + 2i\theta))} \left( \sum_{m=1}^{n} m\theta^{m-r} (-\varepsilon)^{n-m} \right) \right] P_{0,n+1,1}, \quad (r \leq n \leq R - 2) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.17)\]

Where \[A_3 = [(\lambda_1 - \varepsilon)(r + 1)\theta]\]

Using (3.16), (3.17) in (3.1), (3.2) and (3.6), we get,

\[P_{0,r+1,1} = \frac{A_4}{A_5} P_{0,0,0}\]

Where \[A_4 = \frac{(-\varepsilon)^R \prod_{i=1}^{R-1} (-\varepsilon + i\theta)^2 + i\theta}{\theta^{R-1}(R-1)! \prod_{i=0}^{R-2} (2 + 3(-\varepsilon + 2i\theta))}\]

\[A_5 = 2[(\lambda_1 - \varepsilon) + (r + 1)\theta] \left( \frac{(r+1)!}{(R-1)!\theta^{R-2-r}} \sum_{m=1}^{R-2} m\theta^{R-2-r} (-\varepsilon)^{n-m} \right)\]
A finite capacity multiserver interdependent retrial queueing model

\[ \prod_{i=m+1}^{R-1} \frac{(\lambda_0 - \varepsilon + i\theta)^2 + i\theta}{2 + 3(\lambda_0 - \varepsilon) + 2i\theta} + (r + 1)\theta \] \quad \text{...(3.18)}

From (3.7) – (3.9), we recursively derive,

\[ P_{0,n,1} = \left[ \frac{A_6(r+1)!}{m!n^{n-1-r}(2 + 3(\lambda_1 - \varepsilon) + 2n\theta)} \left( \sum_{m=r}^{\infty} m!\theta^{m-r} \frac{(-r)(\lambda_0 - \varepsilon)^{n-1-m}}{r!} \right) \prod_{i=m+1}^{n} \frac{(\lambda_0 - \varepsilon + i\theta)^2 + i\theta}{2 + 3(\lambda_0 - \varepsilon) + 2i\theta} + \frac{(n-1)!\theta^{n-1-r}}{r!} \right] P_{0,r+1,1} \]

\[ P_{1,n,1} = A_7 P_{0,n,1}, \quad (r + 1 \leq n \leq R) \quad \text{...(3.19)} \]

Where \( A_6 = [2 + 3(\lambda_1 - \varepsilon) + 2(r + 1)\theta], \quad A_7 = [(\lambda_1 - \varepsilon) + 2n\theta] \)

\[ P_{2,n,1} = \left[ \frac{A_8(r+1)!}{m!n^{n-1-r}(2 + 3(\lambda_1 - \varepsilon) + 2n\theta)} \left( \sum_{m=r}^{\infty} m!\theta^{m-r} \frac{(-r)(\lambda_1 - \varepsilon)^{n-1-m}A_6}{r!} \right) \prod_{i=m+1}^{n} \frac{(\lambda_1 - \varepsilon + i\theta)^2 + i\theta}{2 + 3(\lambda_1 - \varepsilon) + 2i\theta} + \frac{m!\theta^{n-r}}{2r!} \right] P_{0,r+1,1}, \quad (r + 1 \leq n \leq R - 1) \]

Where \( A_8 = (1 + (\lambda_1 - \varepsilon) + (n + 1)\theta) \).

From (3.7), (3.8), (3.12) and (3.13), we recursively derive,

\[ P_{0,n,1} = \left[ \frac{A_6(r+1)!}{m!n^{n-1-r}(2 + 3(\lambda_1 - \varepsilon) + 2n\theta)} \left( \sum_{m=r}^{\infty} m!\theta^{m-r} \frac{(-r)(\lambda_1 - \varepsilon)^{n-1-m}}{r!} \right) \prod_{i=m+1}^{n} \frac{(\lambda_1 - \varepsilon + i\theta)^2 + i\theta}{2 + 3(\lambda_1 - \varepsilon) + 2i\theta} \right] P_{0,r+1,1} \]

\[ P_{1,n,1} = A_7 P_{0,n,1}, \quad (R + 1 \leq n \leq K) \quad \text{...(3.21)} \]

\[ P_{2,n,1} = \left[ \frac{A_8(r+1)!}{m!n^{n-1-r}(2 + 3(\lambda_1 - \varepsilon) + 2n\theta)} \left( \sum_{m=r}^{\infty} m!\theta^{m-r} \frac{(-r)(\lambda_1 - \varepsilon)^{n-1-m}A_6}{r!} \right) \prod_{i=m+1}^{n} \frac{(\lambda_1 - \varepsilon + i\theta)^2 + i\theta}{2 + 3(\lambda_1 - \varepsilon) + 2i\theta} \right] P_{0,r+1,1}, \quad (R \leq n \leq K - 1) \quad \text{...(3.22)} \]

4 Characteristics of the model

The probability that the system is in faster rate of primary arrivals is

\[ P(0) = \left[ \sum_{n=0}^{R-1} P_{0,n,0} + \sum_{n=r+1}^{R-1} P_{0,n,0} \right] + \left[ \sum_{n=0}^{R-1} P_{1,n,0} + \sum_{n=r+1}^{R-1} P_{1,n,0} \right] \]

\[ + \left[ \sum_{n=0}^{R-1} P_{2,n,0} + \sum_{n=r+1}^{R-1} P_{2,n,0} \right] \]

\[ \text{...(4.1)} \]

The probability that the system is in slower rate of primary arrivals is,

\[ P(1) = \left[ \sum_{n=r+1}^{R} P_{0,n,1} + \sum_{n=R+1}^{K} P_{0,n,1} \right] + \left[ \sum_{n=r+1}^{R} P_{1,n,1} + \sum_{n=R+1}^{K} P_{1,n,1} \right] \]

\[ + \left[ \sum_{n=r+1}^{R} P_{2,n,1} + \sum_{n=R}^{K} P_{2,n,1} \right] \]

\[ \text{...(4.2)} \]
The probability $P_{0,0,0}$ that the system is empty can be calculated from the normalizing condition $P(0) + P(1) = 1. P_{0,0,0}$ is calculated from (4.1) and (4.2). Let $L_q$ denote the average number of customers in the orbit, then we have

$$L_q = L_{q_0} + L_{q_1} \quad (4.3)$$

$$L_{q_0} = \left[ \sum_{n=0}^\infty n P_{0,n,0} + \sum_{n=r+1}^{n=\infty} n P_{0,n,0} \right] + \left[ \sum_{n=0}^\infty n P_{1,n,0} + \sum_{n=r+1}^{n=\infty} n P_{1,n,0} \right]$$

$$L_{q_1} = \left[ \sum_{n=0}^\infty n P_{0,n,1} + \sum_{n=r+1}^{n=\infty} n P_{0,n,1} \right] + \left[ \sum_{n=0}^\infty n P_{1,n,1} + \sum_{n=r+1}^{n=\infty} n P_{1,n,1} \right]$$

From (3.14) - (3.22) and (4.3), we can calculate the value of $L_q$. The expected waiting time of the customers in the orbit is calculated as $\bar{W}_q = \frac{L_q}{\lambda}$. Where $\lambda = \lambda_0 P(0) + \lambda_1 P(1)$. $W_q$ is calculated from (4.1) - (4.3). Let $U_{ser}$ denote the utilization of the server then we have,

$$U_{ser} = U_{0ser} + U_{1ser} \quad (4.4)$$

$$U_{0ser} = \left[ \sum_{n=0}^\infty 2P_{1,n,0} + \sum_{n=r+1}^{n=\infty} 2P_{1,n,0} \right] + \left[ \sum_{n=0}^\infty 2P_{2,n,0} + \sum_{n=r+1}^{n=\infty} 2P_{2,n,0} \right]$$

$$U_{1ser} = \left[ \sum_{n=r+1}^\infty 2P_{2,n,1} + \sum_{n=r+1}^{n=\infty} 2P_{2,n,1} \right] + \left[ \sum_{n=0}^\infty 2P_{2,n,1} + \sum_{n=0}^{n=\infty} 2P_{2,n,1} \right]$$

The expected number of customers in the system is calculated from (4.3) and (4.4)

$$L_s = L_q + U_{ser} \quad (4.5)$$

This model includes the particular case that taking $\varepsilon = 0$ and $\lambda_0$ tends to $\lambda_1$, this model reduces to M/M/c/K retrial queueing model. When $\lambda_0$ tends to $\lambda_1$, $\varepsilon = 0$ and $\theta \to \infty$ this model reduces to standard M/M/c/K queueing model.

5 Numerical Illustration

For various values of $r, R, K, \lambda_0, \lambda_1, \varepsilon, \theta$ the values of $P_{0,0,0}$, $P(0)$, $P(1),$ $W_q$, $U_{ser}$, $L_s$ are computed and tabulated

<table>
<thead>
<tr>
<th>$r$</th>
<th>$R$</th>
<th>$K$</th>
<th>$\lambda_0$</th>
<th>$\lambda_1$</th>
<th>$\theta$</th>
<th>$\varepsilon$</th>
<th>$P_{0,0,0}$</th>
<th>$P(0)$</th>
<th>$P(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>9</td>
<td>12</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>0.5</td>
<td>1.49315368x10^{-6}</td>
<td>0.01449112</td>
<td>0.98550887</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>12</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>0.5</td>
<td>1.975763067x10^{-7}</td>
<td>0.01066697</td>
<td>0.98933303</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>12</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>0.5</td>
<td>1.069813221x10^{-7}</td>
<td>0.00577583</td>
<td>0.99422417</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>12</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>0.5</td>
<td>2.047784914x10^{-7}</td>
<td>0.00720759</td>
<td>0.99279241</td>
</tr>
</tbody>
</table>
A finite capacity multiserver interdependent retrial queueing model

Table 5.1 (Continued):

<table>
<thead>
<tr>
<th></th>
<th>5</th>
<th>9</th>
<th>12</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>1</th>
<th>7.646466238x10^{-7}</th>
<th>0.01241134</th>
<th>0.98758866</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>9</td>
<td>12</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>0.5</td>
<td>5.206538816x10^{-8}</td>
<td>0.00375284</td>
<td>0.99624716</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>9</td>
<td>15</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>0.5</td>
<td>2.110491802x10^{-8}</td>
<td>0.00074283</td>
<td>0.99925717</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>9</td>
<td>15</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>0.5</td>
<td>1.097503083x10^{-8}</td>
<td>0.00084134</td>
<td>0.99915866</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>10</td>
<td>15</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>0.5</td>
<td>1.14396084x10^{-8}</td>
<td>0.00118444</td>
<td>0.99881556</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>9</td>
<td>12</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>0.5</td>
<td>1.449153325x10^{-6}</td>
<td>0.01019383</td>
<td>0.98980617</td>
</tr>
</tbody>
</table>

Table 5.2

<table>
<thead>
<tr>
<th>$L_q$</th>
<th>$W_q$</th>
<th>$U_{ser}$</th>
<th>$L_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.03167451</td>
<td>2.498865784</td>
<td>1.930838074</td>
<td>11.96251258</td>
</tr>
<tr>
<td>10.05095486</td>
<td>2.499408149</td>
<td>1.931182423</td>
<td>11.98213728</td>
</tr>
<tr>
<td>12.53241003</td>
<td>2.503589942</td>
<td>1.922444678</td>
<td>14.44388846</td>
</tr>
<tr>
<td>10.33100937</td>
<td>2.063227693</td>
<td>1.897629568</td>
<td>12.2283894</td>
</tr>
<tr>
<td>10.22142551</td>
<td>2.039223201</td>
<td>1.916252656</td>
<td>12.13767816</td>
</tr>
<tr>
<td>10.49979454</td>
<td>2.098383928</td>
<td>1.895831048</td>
<td>12.39562559</td>
</tr>
<tr>
<td>13.15041884</td>
<td>2.629693088</td>
<td>1.890567851</td>
<td>15.04098669</td>
</tr>
<tr>
<td>13.43155166</td>
<td>2.685858387</td>
<td>1.826486993</td>
<td>15.25803865</td>
</tr>
<tr>
<td>13.23914096</td>
<td>2.647201101</td>
<td>0.016867957</td>
<td>13.25600892</td>
</tr>
<tr>
<td>10.31518408</td>
<td>2.06152016</td>
<td>1.897644246</td>
<td>12.21282833</td>
</tr>
</tbody>
</table>

6 Conclusion

It is observed from the tables 5.1 and 5.2 that when the value of $\lambda_0$, $\lambda_1$, $\eta$ and $K$ increases keeping the other parameters fixed, $P_{0,0,0}$ and $P(0)$ decrease but $P(0)$, $L_q$, $W_q$ and $L_s$ increase. When the value of $\theta$, $\epsilon$ and $R$ increases keeping the other parameters fixed, $P_{0,0,0}$ and $P(0)$ increase but $P(1)$, $L_q$, $W_q$ and $L_s$ decrease.

References


Received: March 3, 2015; Published: April 3, 2015