Understanding and controlling the transport at which particles and heat escape from the reactor chamber is critical to the successful design and operation of a magnetic fusion device.

The goal of this paper is to determine the transport coefficients in dusty plasma particularly electrical and thermal conductivities by Extended Irreversible Thermodynamics (EIT).

Transport coefficients are determined for different types of particles electrons ions and impurities.

**Keywords:** Plasma, Perpendicular Transport coefficients, Extended Irreversible Thermodynamics, electrical conductivity, thermal conductivity

**1. Introduction**

Physics knowledge in plasma confinement and transport relevant to design of a reactor-scale tokamak is reviewed and methodologies for projecting confinement properties to ITER are provided [1-2].

Theoretical approaches to describing a turbulent plasma transport in a tokamak are outlined and phenomenology of major energy confinement regimes observed in tokamak turbulence and transport in plasma [3].
There exist general equations describing these different phenomena, and the special effects are characterized by coefficients generally called transport coefficients [4].

The transport phenomena in plasma strongly limit the particles and energy confinement and represent a crucial obstacle to controlled thermonuclear fusion. The scientific questions to solve the confinement of plasma inside of the reactor this means the limitation of anomalous transport of particles and energy as well as pollution of plasma by impurities coming from internal walls [5].

Impurities influence the plasma performance through a number of processes. In the core the impurities dilute the plasma fuel thus diminishing the efficiency of the fusion process. They can also influence through dilution the instabilities which drive transport and thus reduce core confinement [6].

Recent descriptions of heat and particles transport in plasma have opened a promising field of application for Extended Irreversible Thermodynamics. Extended irreversible thermodynamics is a branch of non-equilibrium thermodynamics that goes beyond the local equilibrium hypothesis of classical irreversible thermodynamics. The space of state variables is enlarged by including the fluxes of mass, momentum and energy and eventually higher order fluxes. The formalism is well-suited for describing high-frequency processes and small-length scales materials. [7-8]

A new formulation of nonequilibrium thermodynamics, known as extended irreversible thermodynamics, has fuelled increasing attention. The basic features of this formalism and several applications are reviewed. Extended irreversible thermodynamics includes dissipative fluxes (heat flux, viscous pressure tensor, electric current) in the set of basic independent variables of the entropy [9].

Starting from this hypothesis, and by using methods similar to classical irreversible thermodynamics, evolution equations for these fluxes are obtained. These equations reduce to the classical constitutive laws in the limit of slow phenomena, but may also be applied to fast phenomena, such as second sound in solids, ultrasound propagation or generalized hydrodynamics [10-11].

This is achieved in (EIT) by enlarging the space of fundamental independent variables, such as the variables and certain additional variables (heat flux, particle flux, etc) [12-13].

Recently D. Jou and J. Casas-Vázquez adopt the extended irreversible thermodynamic theory in order to derive transport equations in semi conductors [14].

The purpose of this paper is to determine the transport coefficients in plasma by (EIT). In the second section we established the fundamental hypotheses of (EIT) and the corresponding evolution equations for the fluxes. In the third section, we develop the transport equations of particle and heat fluxes and we determine the transport coefficients for multispecies of plasma (electrons, ions and impurities).

In the last section we comment the result obtained from graphical presentation of perpendicular transport coefficients as function as magnetic field.
2. Generalized Gibbs equation

As in classical irreversible thermodynamics (CIT), the entropy and the Gibbs equation play a central role in extended irreversible thermodynamics (EIT). Here, it is assumed that the entropy will not only depend on the classical variable, namely the specific internal energy $u$, but in addition on the dissipative flux, so the generalized Gibbs equation takes the form: [15-17]

$$dS^a = \frac{du^a}{T_a} - \frac{\mu^a_{e}}{T_a} de^a - \frac{1}{\rho_a T_a} [(\alpha_{11}^a q^a + \alpha_{12}^a j^a) \cdot dq^a + (\alpha_{21}^a q^a + \alpha_{22}^a j^a) \cdot dj^a]$$ \hspace{1cm} (1)

Where $U$ is the internal energy, $\mu^a_{e}$ the chemical potential, $\alpha_{ij}^a$ phenomenological coefficient, $e^a$ total electric charge contributed by particle $a$, $j^a$ particle flux, $q^a$ heat flux and $T_a$ is the absolute temperature of particle $a$.

The balance equations of total electric charge contributed by particle $a$ and internal energy are given by:

$$\rho^a \partial_t e^a = -\nabla \cdot j^a$$ \hspace{1cm} (2)

$$\rho^a \partial_t U^a = -\nabla \cdot q^a + j^a \cdot E$$ \hspace{1cm} (3)

Here $E$ and $j^a \cdot E$ are respectively the electric and the joule heating term.

In virtue of the balance equations (2) and (3) for $U$ and $e^a$, one obtains for the entropy balance

$$\rho^a \partial_t S^a + \nabla \cdot \left( \frac{1}{T_a} q^a - \frac{\mu^a_{e}}{T_a} j^a \right) = q^a \cdot \left( \nabla T_a^{-1} - \frac{\alpha_{11}^a dq^a}{T_a \cdot dt} - \frac{\alpha_{21}^a dj^a}{T_a \cdot dt} \right) + j^a \cdot \left( \frac{E^a}{T_a} - \nabla \frac{\mu^a_{e}}{T_a} \right) - \frac{\alpha_{12}^a dq^a}{T_a \cdot dt} - \frac{\alpha_{22}^a dj^a}{T_a \cdot dt}$$ \hspace{1cm} (4)

This equation can be cast in the general form of a balance equation

$$\rho \dot{S} = -\nabla \cdot J^s + \sigma^s$$ \hspace{1cm} (5)

Where the quantity $\sigma^s$ is the entropy production ($\sigma^s > 0$) and $J^s$ is the entropy flux.

The entropy production obeys

$$\sigma^s = \mu_{11} q^a \cdot q^a + \mu_{12} j^a \cdot q^a + \mu_{21} q^a \cdot j^a + \mu_{22} j^a \cdot j^a$$ \hspace{1cm} (6)

The coefficients $\mu_{ij} > 0$ as a consequence of the entropy production with $\sigma^s$ is positive ($\sigma^s > 0$)

Considering equations (4) and (5), we obtain the simplest evolution equations for $q^a$ and $j^a$ compatible with a definite positive entropy production, one assumes linear
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relations between the thermodynamic forces and the fluxes $q^a$ and $j^a$. This result in

$$
\frac{E^a}{T_a} - \nabla \frac{\mu^a}{T_a} - \frac{\alpha_{12}^a dq^a}{T_a} dt - \frac{\alpha_{22}^a dj^a}{T_a} dt = \mu_{21}^a q^a + \mu_{22}^a j^a
$$

(7)

$$
\nabla T_a^{-1} - \frac{\alpha_{11}^a dq^a}{T_a} dt - \frac{\alpha_{12}^a dj^a}{T_a} dt = \mu_{11}^a q^a + \mu_{12}^a j^a
$$

(8)

Let us assume that $\nabla T_a^{-1}$ and $\frac{E^a}{T_a} - \nabla \frac{\mu^a}{T_a}$ vanish in system of equation (7) and (8), so that they refer to fluctuations near an equilibrium state. The equations (7) and (8), became:

$$
- \frac{\alpha_{12}^a dq^a}{T_a} dt - \frac{\alpha_{22}^a dj^a}{T_a} dt = \mu_{21}^a q^a + \mu_{22}^a j^a
$$

(9)

$$
- \frac{\alpha_{11}^a dq^a}{T_a} dt - \frac{\alpha_{12}^a dj^a}{T_a} dt = \mu_{11}^a q^a + \mu_{12}^a j^a
$$

(10)

After resolution of equations (9) and (10) we have

$$
\left( \frac{\partial q^a}{\partial t^a} \right) = -T((q^a)^T)^{-1}. \mu^a. (q^a)
$$

(11)

This is equivalent to

$$
\partial_t j^a = -T \left( \frac{\alpha_{11}^a \mu_{21}^a - \alpha_{12}^a \mu_{11}^a}{\alpha_{11}^a \alpha_{22}^a - \alpha_{12}^a \alpha_{21}^a} \right) . q^a - T \left( \frac{\alpha_{12}^a \mu_{22}^a - \alpha_{11}^a \mu_{12}^a}{\alpha_{11}^a \alpha_{22}^a - \alpha_{12}^a \alpha_{21}^a} \right) . j^a
$$

(12)

$$
\partial_t q^a = -T \left( \frac{\alpha_{12}^a \mu_{12}^a - \alpha_{11}^a \mu_{22}^a}{\alpha_{12}^a \alpha_{22}^a - \alpha_{12}^a \alpha_{21}^a} \right) . q^a - T \left( \frac{\alpha_{12}^a \mu_{12}^a - \alpha_{11}^a \mu_{22}^a}{\alpha_{12}^a \alpha_{22}^a - \alpha_{12}^a \alpha_{21}^a} \right) . j^a
$$

(13)

3. Parallel transport coefficients of Plasma with impurities

We consider plasma consisting of multispecies (electron ($e$), ions ($i$), impurities ($I$)), the generalized Gibbs equation (1) became:

$$
dS = \sum_{a=e,i,l} \frac{dU^a}{T_a} - \sum_{a=e,i,l} \frac{\mu^a}{T_a} dt - \sum_{a=e,i,l} \frac{1}{\rho_a T_a} \left[ \sum_{b=e,i,l} \left( \frac{(\alpha_{11}^{ab} q^a + \alpha_{12}^{ab} j^a)}{\alpha_{21}^{ab} q^a + \alpha_{22}^{ab} j^a} \right) dq^b + (\alpha_{21}^{ab} q^a + \alpha_{22}^{ab} j^a) . dj^b \right]
$$

(14)

And the evolutions equations of the fluxes can be written

$$
\partial_t j^a = \sum_{b=e,i,l} (k_1^{ab} . q^b + k_2^{ab} . j^b)
$$

(15)

$$
\partial_t q^a = \sum_{b=e,i,l} (k_3^{ab} . q^b + k_4^{ab} . j^b)
$$

(16)
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Considering equation (12) and (13) the coefficients $k_1^{ab}, k_2^{ab}, k_3^{ab}, k_4^{ab}$ are given

$$k_1^{ab} = -T_a \left( \frac{a_{11}^{ab} b_{21}^{ab} - a_{12}^{ab} b_{11}^{ab}}{a_{11}^{ab} b_{22}^{ab} - a_{12}^{ab} b_{12}^{ab}} \right), \quad k_2^{ab} = -T_a \left( \frac{a_{11}^{ab} b_{22}^{ab} - a_{12}^{ab} b_{12}^{ab}}{a_{11}^{ab} b_{21}^{ab} - a_{12}^{ab} b_{11}^{ab}} \right)$$

$$k_3^{ab} = -T_a \left( \frac{a_{21}^{ab} b_{12}^{ab} - a_{22}^{ab} b_{11}^{ab}}{a_{21}^{ab} b_{22}^{ab} - a_{22}^{ab} b_{21}^{ab}} \right), \quad k_4^{ab} = -T_a \left( \frac{a_{21}^{ab} b_{22}^{ab} - a_{22}^{ab} b_{21}^{ab}}{a_{21}^{ab} b_{12}^{ab} - a_{22}^{ab} b_{11}^{ab}} \right)$$

The coefficients $\mu_i^b$ are given by

$$\mu_{11}^b = \frac{1}{\lambda^b T_b^2}, \quad \mu_{12}^b = \frac{-\left(\Pi^b + \mu_1^b\right)}{\lambda^b T_b^2},$$

$$\mu_{21}^b = \frac{\varepsilon \tau^b - \mu_1^b}{\lambda^b T_b^2}, \quad \mu_{22}^b = \frac{\left(\Pi^b + \mu_1^b\right)\left(\varepsilon_1^b - e_b^b\right)}{\lambda^b T_b^2} + \frac{1}{\sigma^b T_b}$$

With

$$\lambda^b = \frac{5n_b k_B T_b}{2m_b} \tau^b, \quad \sigma^b = n_b \bar{\varepsilon}_b^2 \tau^b \frac{m_b}{m_b}, \quad \Pi^b = \frac{\pi^2 k_B T_b}{2e_b^2}, \quad \varepsilon^b = \frac{\pi^2 T_b k_B^2}{2e_b^2}, \quad \mu^b = k_B T_b \log \left( \frac{1}{n_b} \left( \frac{2\pi k_B T_b}{m_b} \right)^\frac{3}{2} \right)$$

Where $\tau^b$, $n_b$, and $k_B$ are respectively the relaxation time of particle b, the particle b density and the Boltzmann constant.

Determination of the coefficients $\alpha_{ij}^{ab}$ [18]

$$\alpha_{11}^{ab} = \frac{k_B}{\langle \delta q^a \delta q^b \rangle}; \quad \alpha_{12}^{ab} = \frac{k_B}{\langle \delta q^a \delta j^b \rangle}; \quad \alpha_{21}^{ab} = \frac{k_B}{\langle \delta j^a \delta q^b \rangle}; \quad \alpha_{22}^{ab} = \frac{k_B}{\langle \delta j^a \delta j^b \rangle}$$

(17)

With $\delta q^a$ is the fluctuation of the heat flux, $\delta j^a$ is the fluctuation of the particle flux.

Determination of $\alpha_{ij}^{ab}$:

The fluctuations of the heat flux are given

$$\delta q^a = \int \left( \frac{1}{2} m C^2 - \frac{5}{2} k_B T \right) \delta f^a$$

(18)

Using this expression, frequently called the subtracted heat flux, to compute the second moments of the fluctuations, one finds that:

$$\langle \delta q^a \delta q^b \rangle = \int \Delta F \int \Delta F' \left( \frac{1}{2} m_a C^2 - \frac{5}{2} k_B T_a \right) C' \left( \frac{1}{2} m_b C'^2 - \frac{5}{2} k_B T_b \right) C' \delta f^a(C) \delta f^b(C')$$

(19)

$$\langle \delta j^a \delta q^b \rangle = e_a \int \Delta F \int \Delta F' \left( \frac{1}{2} m_a C'^2 - \frac{5}{2} k_B T_a \right) C' \delta f^a(C) \delta f^b(C')$$

(20)
\[ \langle \delta q^a \delta j^b \rangle = \int dc \int dc' \left( \frac{1}{2} C \left( \frac{1}{2} m_a C'^2 \frac{k_B}{2} T_a \right) CC' \langle \delta f^a(C) \delta f^b(C') \rangle \right) \tag{21} \]

\[ \langle \delta j^a \delta j^b \rangle = \int dc \int dc' CC' \langle \delta f^a(C) \delta f^b(C') \rangle \tag{22} \]

Where \( C = c - \nu \) the particle velocity relative to the mean motion, with \( c \) is the velocity of a particle, \( \nu \) is the mean velocity.

\[ \langle \delta f^a(C) \delta f^b(C') \rangle = \frac{1}{V} f_{eq}(C) \delta (C - C') \tag{23} \]

Where \( V \) is the volume of the system.

The local equilibrium Maxwell-Boltzmann distribution function \( f_{eq} \)

\[ f_{eq} = n \left( \frac{m}{2\pi k_B T} \right)^{3/2} \exp \left( -\frac{m C^2}{2 k_B T} \right) \]

After calculations, we found that

\[ \alpha_{11}^{ab} = \frac{(48 V \pi^{3/2} k_B^{1/2})/(5 n_a n_b T_a T_b \sqrt{\pi})}{\left[ -\frac{3 \beta_b^5/\beta_a^5}{(1 + (\beta_b/\beta_a)^2)^{5/2}} - \frac{3 \beta_a^5/\beta_b^5}{(1 + (\beta_a/\beta_b)^2)^{5/2}} - \frac{5 \beta_b^5}{(1 + (\beta_b/\beta_a)^2)^{5/2}} + \frac{5 \beta_a^5}{(1 + (\beta_a/\beta_b)^2)^{5/2}} \right]} \]

\[ \alpha_{12}^{ab} = \frac{24 V \pi^{3/2} k_B^{3/2}}{(e_b n_a n_b T_a \sqrt{\pi} \beta_b^5) \left[ \frac{3}{(1 + (\beta_b/\beta_a)^2)^{5/2}} - \frac{5}{(1 + (\beta_a/\beta_b)^2)^{5/2}} \right]} \]

\[ \alpha_{21}^{ab} = \frac{24 V \pi^{3/2} k_B^{3/2}}{(e_a n_a n_b T_b \sqrt{\pi} \beta_a^5) \left[ \frac{3}{(1 + (\beta_a/\beta_b)^2)^{5/2}} - \frac{5}{(1 + (\beta_b/\beta_a)^2)^{5/2}} \right]} \]

\[ \alpha_{22}^{ab} = \frac{12 V k_B}{(e_a e_b n_a n_b \sqrt{\pi}) \left( \frac{2 \pi^2 T_b}{m_b} \right)^{3/2} (1 + (\beta_a/\beta_b)^3)^{3/2}} \]

With: \( \beta_a = \left( \frac{m_a}{2 T_a} \right)^{1/2} \) and \( \beta_b = \left( \frac{m_b}{2 T_b} \right)^{1/2} \)

Here \( m_a, n_a, V \) and \( e_a \), are respectively the mass, the density of particle \( a \), volume of the system and the charge of particle \( a \).

The state of the plasma after a short transition time remains close to the local plasma equilibrium. For this reason, the local plasma equilibrium will be a reference
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state. The distribution function can the conveniently be written in the form: [17]

\[ f^a = f_0^a + f_1^a \]  \hspace{1cm} (24)

With \( f_1^a \) is a deviation of distribution function, and then can be expanded in a series of irreducible Hermite polynomial as

\[ f_1^a = \sum_{n=0}^{\infty} h_r^{a(2n+1)} H_r^{(2n+1)}(v) f_0^a(v) \]  \hspace{1cm} (25)

With \( h_r^{a(2n+1)} \) the hermitien moment and \( H_r^{(2n+1)} \) the irreducible Hermite polynomials

We limited in the 13 Moment approximation \((n=1)\) so

\[ f_1^a = (h_r^{a(1)} H_r^{a(1)} + h_r^{a(3)} H_r^{a(3)}) f_0^a \]  \hspace{1cm} (26)

With \( H_r^{a(1)} = \sqrt{2} \beta_a C \) And \( H_r^{a(3)} = \frac{1}{\sqrt{5}} \beta_a C [2(\beta_a C)^2 - 5] \)

We derive a relation between the heat flux and the Hermitian moments in order to determine the dimensionless equations of fluxes

\[ h_r^{a(1)} = \frac{1}{n_a} \left( \frac{m_a}{T_a} \right)^{\frac{1}{2}} j_r^a \]  \hspace{1cm} (27)

\[ h_r^{a(3)} = \frac{1}{\sqrt{5}} \frac{1}{n_a T_a} \left( \frac{m_a}{T_a} \right)^{\frac{1}{2}} q_r^a \]  \hspace{1cm} (28)

Where \( h_r^{a(1)} \) and \( h_r^{a(3)} \) are respectively the dimensionless particle flux and the dimensionless heat flux of particle \( a \) \((a = e, i, l)\).

The evolution equations of the fluxes \( j \) and \( q \) have the forms

\[ \partial_t j^a = \partial_t (e_a \int C f^a dC) \]  \hspace{1cm} (29)

\[ \partial_t q^a = \partial_t (\int \left( \frac{1}{2} m_a C^2 - \frac{5}{2} k_B T \right) C f^a dC) \]  \hspace{1cm} (30)

By considering the equations (26) and (27), the equations (28) and (29) can be written

\[ \tau^a \partial_t h_r^{a(1)} + h_r^{a(1)} = G_r^{a(1)} + \frac{e_a x^a}{m_a c} \epsilon_{rmn} h_r^{a(1)} B \]  \hspace{1cm} (31)

\[ \tau^a \partial_t h_r^{a(3)} + h_r^{a(3)} = G_r^{a(3)} + \frac{e_a x^a}{m_a c} \epsilon_{rmn} h_r^{a(3)} B \]  \hspace{1cm} (32)

Where \( \tau^a \) is the relaxation time of particle \( a \), and \( B \) is the magnetic field.
With $G_r^{(1)}$ and $G_r^{(3)}$ are the source term related to the thermodynamic forces by relation

$$G_r^{(1)} = \tau_a \frac{1}{n_a} \left( \frac{m_a}{T_a} \right)^{1/2} \left( \frac{1}{m_a} \nabla_r (n_a T_a) + \frac{e a m_a}{m_a} E_r \right)$$  \hspace{1cm} (33)$$

$$G_r^{(3)} = -\tau_a \sqrt{\frac{5}{2}} \left( \frac{m_a}{T_a} \right)^{1/2} \frac{1}{m_a} \nabla_r (T_a)$$  \hspace{1cm} (34)$$

Considering equations (12) and (13) the evolution equations of particle and heat flux has the form

$$\partial_t j^a = \sum_{b=1,2} (k_{1}^{ab} \cdot q^b + k_{2}^{ab} \cdot j^b)$$  \hspace{1cm} (35)$$

$$\partial_t q^a = \sum_{b=1,2} (k_{3}^{ab} \cdot q^b + k_{4}^{ab} \cdot j^b)$$  \hspace{1cm} (36)$$

So the dimensionless evolution equations of particle and heat fluxes in parallel direction become:

$$\partial_t h^{a(1)} = \sum_{b=e,i} (k_1^{ab} h^{b(3)} + k_2^{ab} h^{b(1)})$$  \hspace{1cm} (37)$$

$$\partial_t h^{a(3)} = \sum_{b=e,i} (k_3^{ab} h^{b(3)} + k_4^{ab} h^{b(1)})$$  \hspace{1cm} (38)$$

With:

$$k_1^{ab} = \sqrt{\frac{5}{2}} \frac{n_b}{n_a} T_b \left( \frac{m_a T_b}{m_b T_a} \right)^{1/2} k_1^{ab}, \quad k_2^{ab} = \frac{n_b}{n_a} \left( \frac{m_a T_b}{m_b T_a} \right)^{1/2} k_2^{ab}$$

$$k_3^{ab} = \sqrt{\frac{5}{2}} \frac{n_b}{n_a} T_b \left( \frac{m_a T_b}{m_b T_a} \right)^{1/2} k_3^{ab}, \quad k_4^{ab} = \frac{n_b}{n_a} \left( \frac{m_a T_b}{m_b T_a} \right)^{1/2} k_4^{ab}$$

The dimensionless evolution equation (31) of particle flux in parallel direction became

$$\tau^a \partial_t h^{a(1)} + h^{a(1)} = G_r^{a(1)}$$  \hspace{1cm} (39)$$

Where the term of magnetic field vanish

For electrons (e), the equation (39) become

$$\tau^e \partial_t h^{e(1)} + h^{e(1)} = G_r^{e(1)}$$  \hspace{1cm} (40)$$

The expression of electron relaxation time $\tau_e$ is [19]

$$\tau^e = \frac{3}{4\sqrt{2\pi}} \frac{m_e^{1/2} T_e^{3/2}}{Z^2 e^4 n_i \ln \Lambda}$$  \hspace{1cm} (41)$$

Where $\ln \Lambda$ is the coulomb logarithm $(\ln \Lambda = \ln \left( \frac{(3/2) (T_e + T_i) \lambda_D}{Z e^2} \right))$ and $\lambda_D$ is the Debye length.
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We replace \( \tau^e \partial_t h_{r}^{e(1)} \) in equation (40) by its expression from equation (37), we find:

\[
\tau^e \sum_{b=e,i,l} (k_1^{ab} h_{ij}^{b(3)} + k_2^{ab} h_{ij}^{b(1)}) + h_{ij}^{e(1)} = G_{ij}^{e(1)}
\]

(42)

This is equivalent to:

\[
\tau^e k_1^{ee} h_{ij}^{e(3)} + \tau^e k_2^{ee} h_{ij}^{e(1)} + \tau^e k_1^{ei} h_{ij}^{i(3)} + \tau^e k_2^{ei} h_{ij}^{i(1)} + \tau^e k_1^{li} h_{ij}^{l(3)} + \tau^e k_2^{li} h_{ij}^{l(1)} + h_{ij}^{e(1)} = G_{ij}^{e(1)}
\]

(43)

Considering Fourier transformed

\[
\partial_t h_{ij}^{b(1)} = i \omega h_{ij}^{b(1)}
\]

\[
\partial_t h_{ij}^{b(3)} = i \omega h_{ij}^{b(3)}
\]

We place in the asymptotic limit where we neglect \( \partial_t h_r \), with we take \( \omega = 0 \), the evolutions equations (43) reduces to

\[
h_{ij}^{e(1)} = L_{11}^{ee} G_{ij}^{e(1)} + L_{13}^{ee} G_{ij}^{e(3)} + L_{11}^{ei} G_{ij}^{i(1)} + L_{13}^{ei} G_{ij}^{i(3)} + L_{11}^{li} G_{ij}^{l(1)} + L_{13}^{li} G_{ij}^{l(3)}
\]

(44)

Similar for ions (i) and impurities (I) we determine \( h_{ij}^{i(1)} \) and \( h_{ij}^{l(1)} \) so

\[
h_{ij}^{i(1)} = L_{11}^{ee} G_{ij}^{e(1)} + L_{13}^{ee} G_{ij}^{e(3)} + L_{11}^{ii} G_{ij}^{i(1)} + L_{13}^{ii} G_{ij}^{i(3)} + L_{11}^{ii} G_{ij}^{i(1)} + L_{13}^{ii} G_{ij}^{i(3)}
\]

(45)

\[
h_{ij}^{l(1)} = L_{11}^{ee} G_{ij}^{e(1)} + L_{13}^{ee} G_{ij}^{e(3)} + L_{11}^{li} G_{ij}^{l(1)} + L_{13}^{li} G_{ij}^{l(3)} + L_{11}^{li} G_{ij}^{l(1)} + L_{13}^{li} G_{ij}^{l(3)}
\]

(46)

The \( L_{ij}^{ab} \) coefficients with \( a = e,i,1 \) and \( i = 1,3, j = 1,3 \) are the pure transport coefficients and \( L_{ij}^{ab} \) \((a, b = e, i, I \text{ with } a \neq b)\) the mixed transport coefficients.

Similar of particle flux we develop now the dimensionless evolution equation of heat flux of multispecies of plasma so we have

\[
h_{ij}^{e(3)} = L_{31}^{ee} G_{ij}^{e(1)} + L_{33}^{ee} G_{ij}^{e(3)} + L_{31}^{ei} G_{ij}^{i(1)} + L_{33}^{ei} G_{ij}^{i(3)} + L_{31}^{li} G_{ij}^{l(1)} + L_{33}^{li} G_{ij}^{l(3)}
\]

(47)

\[
h_{ij}^{i(3)} = L_{31}^{ee} G_{ij}^{e(1)} + L_{33}^{ee} G_{ij}^{e(3)} + L_{31}^{ii} G_{ij}^{i(1)} + L_{33}^{ii} G_{ij}^{i(3)} + L_{31}^{ii} G_{ij}^{i(1)} + L_{33}^{ii} G_{ij}^{i(3)}
\]

(48)

\[
h_{ij}^{l(3)} = L_{31}^{ee} G_{ij}^{e(1)} + L_{33}^{ee} G_{ij}^{e(3)} + L_{31}^{li} G_{ij}^{l(1)} + L_{33}^{li} G_{ij}^{l(3)} + L_{31}^{li} G_{ij}^{l(1)} + L_{33}^{li} G_{ij}^{l(3)}
\]

(49)
Identification of the transport coefficients

The transport matrix $L_{ij}^{ab}$ (whose coefficients are the transport coefficients) has the characteristic Onsager symmetry, which reduces here to the simple matrix symmetry $L_{ij}^{ab} = L_{ji}^{ab}$ so the pure transport coefficients have the form

$$L_{11}^{ee} = \frac{1}{1 + \tau_{e}^{ee} k_{2}}, \quad L_{13}^{ee} = -\frac{\tau_{e}^{ee} k_{3}^{ee}}{1 + \tau_{e}^{ee} k_{2}}, \quad L_{33}^{ee} = \frac{1}{1 + \tau_{e}^{ee} k_{3}}$$

$$L_{11}^{ii} = \frac{1}{1 + \tau_{i}^{ii} k_{2}^{ii}}, \quad L_{13}^{ii} = -\frac{\tau_{i}^{ii} k_{3}^{ii}}{1 + \tau_{i}^{ii} k_{2}^{ii}}, \quad L_{33}^{ii} = \frac{1}{1 + \tau_{i}^{ii} k_{3}^{ii}}$$

With the expression of ion relaxation time is

$$\tau_{i} = \frac{3}{4\sqrt{2} Z_{i}^{\frac{1}{2}} e^{4} n_{i}^{\frac{3}{2}}}$$

Where $m_{i}, Z_{i}$ and $T_{i}$ are respectively the mass of ion, the charge and the ion temperature.

And the expression of relaxation time of impurities is

$$\tau_{i} = \frac{3}{4\sqrt{2} Z_{i}^{\frac{1}{2}} e^{4} n_{i}^{\frac{3}{2}}}$$

Where $m_{i}, Z_{i}$ and $T_{i}$ are respectively the mass of impurities, the charge and the impurities temperature.

The mixed transport coefficients

$$L_{11}^{ei} = -\frac{\tau_{e}^{ee} k_{2}^{ei}}{1 + \tau_{e}^{ee} k_{2}}, \quad L_{31}^{ei} = -\frac{\tau_{e}^{ee} k_{3}^{ei}}{1 + \tau_{e}^{ee} k_{2}}, \quad L_{11}^{eI} = -\frac{\tau_{e}^{ee} k_{2}^{eI}}{1 + \tau_{e}^{ee} k_{2}}, \quad L_{31}^{eI} = -\frac{\tau_{e}^{ee} k_{3}^{eI}}{1 + \tau_{e}^{ee} k_{2}}$$

4. The perpendicular transport coefficients of Plasma

In this paragraph we study the transport phenomena in presence of a constant magnetic field (B) where the particles (electrons ($e$), ions ($i$) and impurities (I), move perpendicular to the magnetic fields.

B is parallel to the Z axis so we project the dimensionless evolution equations in the x and y direction.
For particle fluxes

We determine now the dimensionless evolution equation of particle flux of multispecies of plasma so we have

\[
    h_x^{e(1)} = L_{1}^{e} G_x^{e(1)} + L_{13}^{e} G_x^{e(3)} + L_{11}^{e} G_x^{i(1)} + L_{11}^{e} G_x^{i(3)} + L_{13}^{e} G_x^{i(1)} + L_{13}^{e} G_x^{i(3)} + \left(-\frac{eB\tau}{mc}\right) h_y^{e(1)}
\]

(52)

\[
    h_y^{e(1)} = L_{1}^{e} G_x^{e(1)} + L_{13}^{e} G_x^{e(3)} + L_{11}^{e} G_x^{i(1)} + L_{11}^{e} G_x^{i(3)} + L_{13}^{e} G_x^{i(1)} + L_{13}^{e} G_x^{i(3)} - \left(-\frac{eB\tau}{mc}\right) h_x^{e(1)}
\]

(53)

We suppose

\[
    x_a = \frac{e_a B \tau_a}{mc}, \quad x_a = \Omega_a \tau_a \quad \text{With} \quad \Omega_a = \frac{e_a B}{mc} \quad \text{is the Larmor frequency of species} \ a, \ a = (e, i) \text{and} \ B \text{is the magnetic field}
\]

After introduce (53) in (52) we use

For electrons

\[
    h_x^{e(1)} = L_{1}^{e} G_x^{e(1)} + L_{13}^{e} G_x^{e(3)} + L_{11}^{e} G_x^{i(1)} + L_{11}^{e} G_x^{i(3)} + L_{13}^{e} G_x^{i(1)} + L_{13}^{e} G_x^{i(3)} + \left(-\frac{eB\tau}{mc}\right) h_y^{e(1)}
\]

(54)

\[
    h_y^{e(1)} = L_{1}^{e} G_y^{e(1)} + L_{13}^{e} G_y^{e(3)} + L_{11}^{e} G_y^{i(1)} + L_{11}^{e} G_y^{i(3)} + L_{13}^{e} G_y^{i(1)} + L_{13}^{e} G_y^{i(3)} - \left(-\frac{eB\tau}{mc}\right) h_x^{e(1)}
\]

(55)

For ions

\[
    h_x^{i(1)} = L_{1}^{i} G_x^{i(1)} + L_{13}^{i} G_x^{i(3)} + L_{11}^{i} G_x^{i(3)} + L_{13}^{i} G_x^{i(1)} + \left(-\frac{eB\tau}{mc}\right) h_y^{i(1)}
\]

(56)
\[ h_y^{(1)} = \frac{L_{1e}^{i}}{1+x_i^2} G_x^{e(1)} + \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} + \frac{L_{1i}^{i}}{1+x_i^2} G_y^{i(1)} + \frac{L_{1i}^{i}}{1+x_i^2} G_y^{i(1)} + \frac{L_{1i}^{i}}{1+x_i^2} G_y^{i(1)} \]
\[ \frac{L_{1i}^{i}}{1+x_i^2} G_y^{i(1)} = x_i \frac{L_{1i}^{i}}{1+x_i^2} G_x^{e(1)} - x_i \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} - x_i \frac{L_{1i}^{i}}{1+x_i^2} G_y^{i(1)} - x_i \frac{L_{1i}^{i}}{1+x_i^2} G_y^{i(1)} \]
\[ x_i \frac{L_{1i}^{i}}{1+x_i^2} G_y^{i(1)} = x_i \frac{L_{1i}^{i}}{1+x_i^2} G_y^{i(1)} \]  
\[ (57) \]

For impurities

\[ h_x^{(1)} = \frac{L_{1e}^{i}}{1+x_i^2} G_x^{e(1)} + \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} + \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} + \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} + \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} + \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} + \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} \]
\[ \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} = x_i \frac{L_{1i}^{i}}{1+x_i^2} G_x^{e(1)} + x_i \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} + x_i \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} + x_i \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} + x_i \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} + x_i \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} + x_i \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} \]
\[ x_i \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} = x_i \frac{L_{1i}^{i}}{1+x_i^2} G_x^{i(1)} \]  
\[ (58) \]

For heat fluxes

Similar of particle flux we develop now the dimensionless evolution equation of heat flux of multispecies of plasma so we have

\[ h_y^{(3)} = L_{31}^{i} G_x^{e(3)} + L_{31}^{i} G_x^{e(3)} + L_{31}^{i} G_x^{i(3)} + L_{31}^{i} G_x^{i(3)} + L_{31}^{i} G_x^{i(3)} + \left( \frac{eB \tau_a}{m_e c} \right) h_y^{(3)} \]  
\[ (60) \]

\[ h_x^{(3)} = L_{31}^{i} G_y^{e(3)} + L_{31}^{i} G_y^{e(3)} + L_{31}^{i} G_y^{i(3)} + L_{31}^{i} G_y^{i(3)} + L_{31}^{i} G_y^{i(3)} + \left( \frac{eB \tau_a}{m_e c} \right) h_x^{(3)} \]  
\[ (61) \]

\[ x_a = \frac{eB \tau_a}{m_e c} \quad (a = e, i, I) \]

After introduce (61) in (60) we use
For electrons

\[ h_e^{(3)} = \frac{L_{e}^{e}}{1+x_e^2} G_e^{(1)} + \frac{L_{e}^{e}}{1+x_e^2} G_e^{(3)} + \frac{L_{e}^{i}}{1+x_e^2} G_i^{(1)} + \frac{L_{e}^{i}}{1+x_e^2} G_i^{(3)} \]

\[ + \frac{L_{e}^{i}}{1+x_e^2} G_i^{(1)} + \frac{L_{e}^{i}}{1+x_e^2} G_i^{(3)} + \frac{L_{e}^{i}}{1+x_e^2} G_i^{(1)} + \frac{L_{e}^{i}}{1+x_e^2} G_i^{(3)} \]

\[ x_e \frac{L_{e}^{i}}{1+x_e^2} G_i^{(1)} + x_e \frac{L_{e}^{i}}{1+x_e^2} G_i^{(3)} \]

\[ (62) \]

For ions

\[ h_i^{(3)} = \frac{L_{i}^{e}}{1+x_i^2} G_e^{(1)} + \frac{L_{i}^{e}}{1+x_i^2} G_e^{(3)} + \frac{L_{i}^{i}}{1+x_i^2} G_i^{(1)} + \frac{L_{i}^{i}}{1+x_i^2} G_i^{(3)} \]

\[ + \frac{L_{i}^{i}}{1+x_i^2} G_i^{(1)} + \frac{L_{i}^{i}}{1+x_i^2} G_i^{(3)} + \frac{L_{i}^{i}}{1+x_i^2} G_i^{(1)} + \frac{L_{i}^{i}}{1+x_i^2} G_i^{(3)} \]

\[ x_i \frac{L_{i}^{i}}{1+x_i^2} G_i^{(1)} + x_i \frac{L_{i}^{i}}{1+x_i^2} G_i^{(3)} \]

\[ (64) \]

For impurities

\[ h_e^{(3)} = \frac{L_{e}^{e}}{1+x_e^2} G_e^{(1)} + \frac{L_{e}^{e}}{1+x_e^2} G_e^{(3)} + \frac{L_{e}^{i}}{1+x_e^2} G_i^{(1)} + \frac{L_{e}^{i}}{1+x_e^2} G_i^{(3)} \]

\[ + \frac{L_{e}^{i}}{1+x_e^2} G_i^{(1)} + \frac{L_{e}^{i}}{1+x_e^2} G_i^{(3)} + \frac{L_{e}^{i}}{1+x_e^2} G_i^{(1)} + \frac{L_{e}^{i}}{1+x_e^2} G_i^{(3)} \]

\[ x_e \frac{L_{e}^{i}}{1+x_e^2} G_i^{(1)} + x_e \frac{L_{e}^{i}}{1+x_e^2} G_i^{(3)} \]

\[ (66) \]

\[ h_y^{(3)} = \frac{L_{y}^{e}}{1+x_y^2} G_e^{(1)} + \frac{L_{y}^{e}}{1+x_y^2} G_e^{(3)} + \frac{L_{y}^{i}}{1+x_y^2} G_i^{(1)} + \frac{L_{y}^{i}}{1+x_y^2} G_i^{(3)} \]

\[ + \frac{L_{y}^{i}}{1+x_y^2} G_i^{(1)} + \frac{L_{y}^{i}}{1+x_y^2} G_i^{(3)} + \frac{L_{y}^{i}}{1+x_y^2} G_i^{(1)} + \frac{L_{y}^{i}}{1+x_y^2} G_i^{(3)} \]

\[ x_i \frac{L_{y}^{i}}{1+x_y^2} G_i^{(1)} + x_i \frac{L_{y}^{i}}{1+x_y^2} G_i^{(3)} \]

\[ (67) \]
5. Result and discussion

In Figs 1-5 we plotted the perpendicular electrical conductivity, the thermoelectric conductivity, electron ion and impurity thermal conductivities as function of $X_\alpha$ ($\alpha = e, i, I$).

**Fig 1:** Perpendicular ions thermal conductivity as function of $X_i$.

In this curve we plotted the perpendicular ions thermal conductivity as function $X_i$ of. This plot shows that the perpendicular ions thermal conductivity decreases with increasing of $X_i$.

**Fig 2:** Perpendicular electrical conductivity as function of $X_e$

This plot show that the electrical conductivity decreases with increasing of $X_e$

**Fig 3:** Perpendicular electron thermal conductivity as function of $X_e$. 

We notice that the perpendicular electron thermal conductivity decreases with increasing of $X_e$.

**Fig 4:** Perpendicular impurity thermal conductivity as function of $X_I$.

We notice that the perpendicular ion thermal conductivity decreases with increasing of $X_I$.

**Fig 5:** Perpendicular thermoelectric conductivity as function of $X_e$.

We notice that the perpendicular thermoelectric conductivity increases with increasing of $X_e$.

The perpendicular transport coefficients are monotonously decreasing function of $X_\alpha$. So for a very strong magnetic field, the particles would stick to the field lines, and there would be no transport in any direction perpendicular to (B). This situation is opposed by the collisions the latter make the particles jump from one field line to another, thus making a perpendicular transport possible.

In the perpendicular direction the collisions favor the transport thus for large values of the parameter $X_\alpha = \Omega_\alpha \tau_\alpha$, i.e. for large magnetic field the coefficients are decreasing functions of $X_\alpha$. This situation is clearly illustrated in Figs (1-5).

### 6. Conclusion

In this paper we have been interested to calculate by Extended Irreversible Thermodynamics the transport coefficients like electrical and thermal conducti-
vities after developing transport equations of dissipative fluxes like particles and heat fluxes of multispecies plasma (electron, ions, and impurities).

In absence of magnetic field all the transport coefficient are proportional to the relaxation time this important characteristic can be easily understood. It implies that as the collision frequency increases the transport coefficients decrease in order words, the collisions tend to oppose the transport of matter and energy; they act as an obstacle to the free flow of these quantities. This result is in perfect agreement with kinetic theory result [20-21]

The asymptotic perpendicular transport coefficients are proportional to the collision frequency this property vividly illustrates that the collisions oppose the parallel transport but favor the perpendicular one. It may be said that in plasma in presence of a constant magnetic field, when the collision frequency is increased And the equality such coefficients, which has been obtained here from purely EIT, is supported by kinetic theory.

References


Application of extended irreversible thermodynamics


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