Casson Fluid Flow and Heat Transfer over an Unsteady Porous Stretching Surface

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Abstract

The unsteady two-dimensional flow of a non-Newtonian fluid over a stretching porous surface having a prescribed surface temperature is investigated. Similarity transformations are employed to transform the governing partial differential equations into ordinary differential equations. The velocity and the temperature have been analyzed for different variations of the governing parameters Casson parameter, heat source parameter, Hartmann number, and Prandtl number are analyzed and discussed in detail. It was found that heat source and magnetic field modify the flow patterns and increase the temperature of the fluid.

Keywords: Parallel Porous Plate, Boundary Layer, Heat Source, Magnetic Field, Casson fluid

1 Introduction

Convective heat transfer plays a vital role during the handling and processing of non-Newtonian fluid flows. Mechanics of non-Newtonian fluid flows present a special challenge to engineers, physicists, and mathematicians. Because of the complexity of these fluids, there is not a single constitutive equation which exhibits all properties of such non-Newtonian fluids. Many papers have been published on the boundary layer flow and heat transfer problems where the stretching force and surface temperature are varying with time. Several authors [3, 5] studied the problem for unsteady flow of viscous fluid between parallel porous plates. In recent times, A Ishak et al. [1], Mukhopadhyay [6, 7], A J Chamkha et al. [2] and C K Kirubhashankar et al. [4] obtained similarity
solutions for unsteady flow and heat transfer over a stretching sheet under different conditions. Motivated by the previously mentioned investigations on non-Newtonian fluids due to a stretching sheet and its vast applications in many industries, in the present investigation, a mathematical model for the unsteady fluid flow through parallel plate channel with heat source is presented. The main objective of the present work is to obtain analytical expressions for axial velocity and temperature fields of the fluid. The effects of magnetic field ($M$), Casson parameter ($\gamma$), heat source parameter ($S$) and Prandtl number ($Pr$) on the axial velocity and temperature distribution are investigated and analyzed with the help of their graphical representations.

2 Mathematical analysis of the flow

Let us consider the unsteady two-dimensional flow and heat transfer of an incompressible Casson fluid over an exponentially shrinking/stretching porous sheet at $y = 0$, with the flow being confined in $y > 0$. The fluid is electrically conducting in the presence of a uniform magnetic field applied normal to the sheet, and the induced magnetic field is neglected under the approximation of small Reynolds number. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid as follows:

$$\tau_{ij} = \begin{cases} 
\left( \mu_B + \frac{p_y}{\sqrt{2\pi}} \right) 2\varepsilon_{ij}, & \pi > \pi_c, \\
\left( \mu_B + \frac{p_y}{\sqrt{2\pi}c} \right) 2\varepsilon_{ij}, & \pi < \pi_c,
\end{cases}$$

where $\tau_{ij}$ is the $(i,j)$-th component of the stress tensor, $\pi = e_{ij}e_{ij}$ and $e_{ij}$ are the $(i,j)$-th component of the deformation rate, $\pi$ is the product of the component of deformation rate with itself, $\pi_c$ is a critical value of this product based on the non-Newtonian model, $\mu_B$ is plastic dynamic viscosity of the non-Newtonian fluid, and $p_y$ is the yield stress of the fluid.

Considering $u$ and $v$ as velocity components in the directions of $x$ and $y$ respectively at time $t$ in the flow field, we may write the two dimensional boundary layer equations in presence of transverse magnetic field as

$$\frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = \nu \left( 1 + \frac{1}{\gamma} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K} u - \frac{\sigma B^2 u}{\rho} + g\beta(T - T_0) \quad (1)$$

$$\frac{\partial T}{\partial t} = \frac{K_T}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p} (T - T_0) \quad (2)$$

where $\nu$ is the kinematic fluid viscosity, $\rho$ is the fluid density, $\gamma = \frac{\mu_B}{\sqrt{2\pi c}} p_y$ is the Casson parameter, $\sigma$ is the electrical conductivity of the fluid, and $H_0$ is the strength of magnetic field applied in the $y$ direction. $\rho$ and $\mu$ is the density and
viscosity of the blood. $K_T$ is thermal conductivity; $C_p$ is the specific heat at constant pressure. $Q$ is the quantity of heat, $T$ is the temperature and $\beta$ is the volumetric expansion parameter while $\theta$ is the temperature distribution.

The boundary conditions are taken as:

\[
\theta = e^{-\lambda_1 t}, \quad u = e^{-\lambda_1 t} \quad \text{at} \quad y = -1
\]

\[
\theta = 0, \quad u = 0 \quad \text{at} \quad y = 1
\]

Let us introduce the non-dimensional variables,

\[
x^* = \frac{x}{h}, \quad y^* = \frac{y}{h}, \quad u^* = \frac{u}{m/2 \rho h}, \quad v^* = \frac{v}{m/2 \rho h}, \quad t^* = \frac{t}{\rho h^2 / \mu},
\]

\[
p^*(x,t) = \frac{dp}{dx}, \quad \theta^* = \frac{\theta}{\mu m / 2 \rho^2 h^3}, \quad K^* = \frac{K}{h^2}
\]

Substituting equation (3) into equations (1) and (2), we get

\[
\frac{\partial u}{\partial t} + p = \left(1 + \frac{1}{\gamma}\right) \frac{\partial^2 u}{\partial y^2} - \left(Ha^2 + \frac{1}{K}\right)u + g \beta \theta
\]

\[
\frac{\partial \theta}{\partial t} = \frac{1}{\nu Pr} \frac{\partial^2 \theta}{\partial t^2} + \frac{S}{\nu Pr} \theta
\]

where the heat source parameter, $S = \frac{Q h^2}{K_T}$, Prandtl number, $Pr = \frac{\rho C_p}{K_T}$.

### 3 Analytical Solution of the Problem

With the above discussions in the previous section, let us choose the solutions of the equations (4) and (5) respectively as

\[
u(y,t) = F(y)e^{-\lambda_1 t}
\]

\[
\theta(y,t) = H(y)e^{-\lambda_1 t}
\]

The boundary conditions are transformed to

\[
H = 1, \quad F = 1, \quad \text{at} \quad y = -1
\]

\[
H = 0, \quad F = 0, \quad \text{at} \quad y = 1
\]

By virtue of (6) – (7), we obtain the equations (4) and (5) respectively as

\[
F^*(y) + \frac{\gamma}{1 + \gamma} \left(\lambda^2 - Ha^2 - \frac{1}{K}\right)F(y) = \frac{\gamma}{1 + \gamma} (p - g \beta H(y))
\]

\[
H^*(y) + \left(S + \lambda^2 P, u\right)H(y) = 0
\]

Solution of equation (10) using the boundary condition (8) is as follows

\[
H(y) = \frac{1}{2 \cos \delta} \cos(\delta y) - \frac{1}{2 \sin \delta} \sin(\delta y)
\]
where $\delta = \sqrt{S + \lambda^2 P_r\nu}$

From (7) and (11) the temperature distribution is given by

$$\theta(y, t) = \left(\frac{1}{2\cos \delta} \cos(\delta y) - \frac{1}{2\sin \delta} \sin(\delta y)\right)e^{-\lambda^2 t} \quad (12)$$

Using the equation (12) into equation (9) we get

$$F^*(y) + \alpha^2 F(y) = \frac{1}{1 + \gamma} \left(p - g\beta\left(\frac{1}{2\cos \delta} \cos(\delta y) - \frac{1}{2\sin \delta} \sin(\delta y)\right)\right) \quad (13)$$

where $\alpha^2 = \frac{1}{1 + \gamma} \left(\lambda^2 - Ha^2 - \frac{1}{K}\right)$

From equation (6) and (13) the velocity of the flow of the fluid parallel to the direction of the channel is obtained as,

$$u(y, t) = \left(c_1 + c_2 \cos \delta y + c_3 \sin \delta y + c_4 \cos \alpha y + c_5 \sin \alpha y\right)e^{-\lambda^2 t} \quad (14)$$

where $p_1 = \frac{p^*}{e^{-\lambda^2 t}}$, $\alpha = \frac{1}{\sqrt{1 + \gamma} \left(\lambda^2 - Ha^2 - \frac{1}{K}\right)}$, $c_1 = \frac{p_1}{\alpha^2}$,

$$c_2 = -\frac{g\beta}{2(\alpha^2 - \delta^2)\cos \delta}, \quad c_3 = \frac{g\beta}{2(\alpha^2 - \delta^2)\sin \delta}, \quad c_4 = \frac{1 - 2c_1 + 2c_2 \cos \delta}{2\cos \alpha}, \quad c_5 = -\frac{1 + 2c_2 \sin \delta}{2\sin \alpha}$$

Equations (12) and (14) show the axial velocity and temperature distribution.

**4 Results and Discussions**

In this section, we discuss the different physical parameters. The obtained computational results are presented graphically and the variations in velocity and temperature are discussed.

From Figures 1 – 2, the temperature field decreases with increasing the heat source parameter ($S$) and Prandtl number ($P_r$). And temperature field increases with increasing the heat source parameter ($S$) for $y \geq 0.5$ and $y \geq 0.16$ respectively. From figures 3 – 6, for $y \leq -0.3$, the axial velocity increases with increasing the heat source parameter ($S$) and the effect reverses in $-0.3 \leq y \leq 1$. The axial velocity decreases with increasing the magnetic field ($Ha^2$), Prandtl number ($P_r$) and Casson parameter ($\gamma$). And axial velocity increases with increasing the magnetic field ($Ha^2$) and Casson parameter ($\gamma$) for $y \geq -0.5$ and $y \geq -0.2$ respectively. The effect of increasing values of the Casson parameter is to suppress the velocity field. Prandtl number can be used to increase the rate of
cooling in conducting flows. The results may be helpful for possible technological applications in liquid-based systems involving stretchable materials.

5 Conclusion

The present study provides the solution for the unsteady Casson fluid flow through the parallel plate channel with heat source and external transverse magnetic field is presented. The present work is the effect of magnetic field, heat source and Casson parameter seems to be significant.

Figure 1 Temperature Field for different values of Heat Source Parameter (S) at \( \lambda = 0.5, P_r = 1, \ \nu = 0.5, \text{ and } t = 1. \)

Figure 2 Temperature Field for different values of Prandtl Number \( (P_r) \) at \( S = 1, \ \lambda = 0.5, \ \nu = 0.2, \text{ and } t = 1. \)

Figure 3 Axial velocity for different values of Heat Source Parameter (S) at \( \lambda = 1.5, P_r = 1, \ \nu = 0.2, \ t = 1, \ \gamma = 0.2, \ g = 9.8, \ \beta = 0.5, \ \rho = 0.5 \text{ and } Ha = 1. \)

Figure 4 Axial velocity for different values of values of Magnetic Field Parameter \( (Ha) \) at \( S = 1, \ \lambda = 0.5, \ P_r = 1, \ \nu = 0.2, \ t = 1, \ \gamma = 0.2, \ g = 9.8, \ \beta = 0.5 \text{ and } \rho = 0.5. \)
Figure 5 Axial velocity for different values of Prandtl Number ($P_r$) at $S = 1$, $\lambda = 0.5$, $P_r = 1$, $\nu = 0.2$, $t = 1$, $\gamma = 1$, $g = 9.8$, $\beta = 0.5$, $p = 0.5$ and $Ha = 1$.

Figure 6 Axial velocity for different values of Casson Parameter ($\gamma$) at $S = 1$, $\lambda = 0.5$, $P_r = 1$, $\nu = 0.2$, $t = 1$, $\gamma = 0.2$, $g = 9.8$, $\beta = 0.5$ and $p = 0.5$.

References


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