MHD Flow and Heat Generation Near
Stagnation Point towards a Stretching Sheet in
Porous Medium

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Abstract

The steady two-dimensional laminar flow towards a stretching sheet with heat generation in electrically conducting incompressible viscous fluid is studied for porous medium in the presence of uniform transverse magnetic field. The governing nonlinear momentum and energy differential equations are reduced to ordinary differential equations by using suitable similarity transformation and then by a perturbation technique to obtain the approximate numerical results. The local skin-friction coefficient and the local Nusselt number at the sheet are computed corresponding to various parameters. The effects of various physical parameters, such as porosity parameter, magnetic parameter, stretching parameter, Prandtl number and heat generation coefficient for velocity and temperature distributions have discussed through graphs and their numerical values for various values of physical parameters are presented through tables.

Keywords: MHD Flow; Heat Generation; Stagnation Point; Stretching Sheet; Porous Medium
1. Introduction

Flow and heat transfer of a viscous incompressible fluid over a stretching surface is playing a pivotal role in different realms of technological, engineering and industrial manufacturing processes. Its numerous applications are vital for advancement in the field of metallurgy, chemical engineering and biological systems such as paper production, hot rolling, wire drawing, glass fiber, cooling of metal, drawing plastic films, design of various chemical processing equipment and many others. The desired quality and parameters of final product, depends on the rate of heat transfer at the stretching surface. Crane [7] was the first who considered flow of an incompressible viscous fluid over a linearly stretching plate and produced an exact analytical solution for the steady two-dimensional boundary layer flow problems. Gupta and Gupta [10], Carragher and Crane [4], Dutta et al. [8], Ariel [1] and Jat and Chaudhary [14] discussed the effects of heat transfer on a stretching surface taking into account different aspects of the problem.

In fluid dynamics, the study of stagnation point flow is of great technical importance due to its large numbers of applications in technology and engineering, for example, cooling of nuclear reactors, cooling of electronic devices by fans, drag reduction, thermal oil recovery and many hydodynamics processes; thus the analysis of stagnation point fluid flow is very important in both theory and practice of above applications. The pioneering work in this area was carried out by Hiemenz [12] who presented the two-dimensional stagnation point flow on an infinite wall that the Navier-Stokes equations governing the flow reduced to nonlinear ordinary differential equations using similarity transformation and derived an exact solution to the Navier-Stokes equations. Later the stagnation point flow problem was extended by Massoudi and Ramezan [19], Ariel [2], Lok et al. [17]. The combination of both stagnation point flow and stretching surface problem was studied by Chiam [5]. Further, the behavior of stagnation point flow over stretching surface under various physical situations were made by Mahapatra and Gupta [18], Paullet and Weidman [21], Jat and Chaudhary [13], Reza and Gupta [22], Gorder et al. [9].

Flow and heat transfer phenomenon in porous media has received considerable attention due to its wide applications in geophysics, engineering and thermal sciences, such as petroleum recovery, packed bed reactors, underground disposal of nuclear and non nuclear waste, food processing and many others. Cortell [6] investigated fluid flow and heat transfer in a porous medium over a stretching surface and an analytical solution obtained for this flow problem. Wu et al. [23], Merrill et al. [20], Harris et al. [11] extended this work. Recently Attia [3], Layek et al. [16] and Kazem et al. [15] obtained either numerical or analytical solution to a stagnation point flow over stretching sheet in porous medium.

Looking into importance of increasing applications of MHD effects in engineering and technology, the object of present paper is to study the stagnation point flow and heat transfer over stretching sheet for an electrically conduction fluid in the presence of a magnetic field. A uniform magnetic field is applied
normal to the sheet. The governing momentum and energy equations are solved numerically by perturbation technique. The results of velocity distribution, temperature distribution, local skin-friction coefficient and surface heat transfer are discussed for different physical parameters.

2. Mathematical Formulations

Consider the steady two-dimensional stagnation point flow of viscous incompressible and electrically conducting fluid over stretching sheet placed in the plane \( y = 0 \), the flow being in a region \( y > 0 \) and the space above the plane sheet is filled with the porous medium in the presence of an externally applied magnetic field of constant strength \( B_0 \) normal to the sheet. The stretching sheet has a linear velocity \( u_0(x) \) and uniform temperature \( T_v \), while the velocity of the flow external to the boundary layer is \( u_e(x) \) and temperature \( T_e \). Two equal and opposing forces are applied along the x-axis; therefore the sheet is stretched, keeping the origin fixed. The velocity distribution in the frictionless flow near stagnation point is given by

\[
U(x) = a x, \quad V(x) = -a y
\]

Whereas \((U, V)\) are the velocity components for the potential flow at any point \((x, y)\) and the constant \( a (>0) \) is proportional to the free stream velocity far away from the stretching sheet. The system of boundary layer equations (which model the Figure 1) are given by:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = U \frac{du}{dx} + \frac{\partial^2 u}{\partial y^2} + \frac{\nu}{K} [U(x)-u] - \frac{\sigma B_0^2 u}{\rho} \tag{2}
\]

\[
\rho C_p \left( \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} \right) = \kappa \frac{\partial^2 T}{\partial y^2} + Q(T-T_v) \tag{3}
\]

![Figure 1. Physical model and coordinate system](image-url)
Where \( u \) and \( v \) are the velocity components for the potential flow along \( x \)-axis and \( y \)-axis respectively, \( \nu = \frac{u}{\rho} \) is the coefficient of kinematic viscosity, \( \mu \) is the coefficient of viscosity, \( \rho \) is the fluid density, \( K \) is the Darcy permeability, \( \sigma \) is the electrical conductivity, \( C_p \) is the specific heat at constant pressure, \( T \) is the temperature, \( \kappa \) is the thermal conductivity and \( Q \) is the volumetric rate of heat generation or absorption. The other symbols have their usual meanings.

The boundary conditions are:

\[
\begin{align*}
  y = 0: & \quad u = u_c(x) = c x, \quad v = 0; \quad T = T_u \\
  y = \infty: & \quad u = u_c(x) = a x; \quad T = T_u
\end{align*}
\]  

(4)

Where \( c \) is positive constant, proportional to the velocity of the stretching sheet.

3. Mathematical Analysis

The continuity equation (1) is identically satisfied by stream function \( \psi(x, y) \), defined by

\[
\psi(x, y) = \sqrt{u/v} \times f(\eta), \quad \eta = \frac{c}{\sqrt{D}} y, \quad T = T_u + (T_v - T_u)\theta(\eta)
\]

(6)

For the solution of momentum and energy equations (2) and (3), introducing the following dimensionless variables:

\[
\psi(x, y) = \sqrt{u/v} \times f(\eta), \eta = \frac{c}{\sqrt{D}} y, \quad T = T_u + (T_v - T_u)\theta(\eta)
\]

(6)

Using equations (5) and (6), equations (2) and (3) are reduced to

\[
f'' + f f'' - f'^2 - (\lambda + M)f' + C(C + \lambda) = 0
\]

(7)

\[
\theta' + Pr f \theta' + Pr B \theta = 0
\]

(8)

The corresponding boundary conditions are:

\[
\begin{align*}
  \eta = 0: & \quad f = 0, \quad f' = 1; \quad \theta = 1 \\
  \eta = \infty: & \quad f' = C, \quad \theta = 0
\end{align*}
\]

(9)

Where prime \((\cdot)\) denote differentiation with respect to \( \eta \), \( \lambda = \frac{\nu}{c K} \) is the Porosity parameter, \( M = \frac{B \sigma}{\rho c} \) is the Magnetic parameter, \( C = \frac{a}{c} \) is the Stretching parameter, \( Pr = \frac{\mu C_p}{\kappa} \) is the Prandtl number and \( B = \frac{Q}{c \rho C_p} \) is the Heat generation or absorption coefficient.
4. Numerical Method for Solution

For numerical solution of the equations (7) and (8), we apply a perturbation technique as:

\[ f(\eta) = \sum_{i=0}^{\infty} M^i f_i(\eta), \quad \theta(\eta) = \sum_{i=0}^{\infty} M^i \theta_i(\eta) \] (10)

Substituting the values of \( f(\eta) \) and \( \theta(\eta) \) by equation (10) and its derivatives in equations (7) and (8), and then equating the coefficients of like powers of \( M \), we get the following set of equations:

\[ f_0'' + f_0 f''_0 - \lambda f'_0 = -C(C + \lambda) \] (11)

\[ \theta_0'' + \Pr f_0 \theta'_0 + \Pr B \theta_0 = 0 \] (12)

\[ f'_0 + f_0 f'_0 - (2 f'_0 + \lambda) f''_0 + f_0^2 f_i = f_i' \] (13)

\[ \theta'_0 + \Pr f_0 \theta'_0 + \Pr B \theta_i = -\Pr f_i \theta'_0 \] (14)

\[ f_0' + f_0 f'_0 - (2 f'_0 + \lambda) f''_0 + f_0^2 f_i = -f_i f'_0 + f_i f'_0 + f_i' \] (15)

\[ \theta'_0 + \Pr f_0 \theta'_0 + \Pr B \theta_2 = -\Pr f_i \theta'_0 - \Pr f_2 \theta_0 \] (16)

With the boundary conditions:

\[ \eta = 0: \quad f_i = 0, \quad f'_0 = 1, \quad f'_0 = 0; \quad \theta_0 = 1, \quad \theta_j = 0; \]
\[ \eta = \infty: \quad f'_0 = C, \quad f'_0 = 0; \quad \theta_i = 0; \quad i \geq 0, \quad j > 0 \] (17)

The equation (11) is that obtained by Attia [3] for the non-magnetic case and the remaining equations from (12) to (16) are ordinary linear differential equations and have been solved numerically by Runge-Kutta method of fourth order under the boundary condition (17). The velocity and the temperature distributions for different values of parameters are shown in Figures 2-4 respectively.

5. Local Skin-friction Coefficient and Local Nusselt Number

The interesting physical quantities are the local skin-friction coefficient \( C_f \) and the local Nusselt number \( Nu \), which are defined as:

\[ C_f = \frac{\tau_u}{\rho u_c^2/2} = \frac{\mu \left( \frac{\partial u}{\partial y} \right)_{y=0}}{\rho u_c^2/2} \quad \text{and} \quad Nu = -\frac{\left( \frac{\partial T}{\partial y} \right)_{y=0} x}{T_u - T_w} \] (18)

In the present case which can be expressed as:

\[ C_f = \frac{2}{\sqrt{Re_x}} f''(0) = \frac{2}{\sqrt{Re_x}} \left[ \sum_{i=0}^{\infty} M^i f_i''(0) \right], \quad Nu = -\sqrt{Re_x} \theta'(0) = -\sqrt{Re_x} \left[ \sum_{i=0}^{\infty} M^i \theta_i'(0) \right] \] (19)
Where $\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}$ is the surface shear stress and $Re_x = \frac{u_x x}{\nu}$ is the local Reynolds number. The numerical values of functions $f'(0)$ and $\theta'(0)$, which are proportional to the local skin-friction coefficient $c_f$ and the local Nusselt number $Nu$ at the sheet respectively, for various values of the physical parameters are presented in Tables 1-2.

6. Results and Discussion

Figure 2 shows the variation of velocity distribution $f'(\eta)$ against $\eta$ for various values of the porosity parameter $\lambda$, the magnetic parameter $M$ and the stretching parameter $C$. It may be observed that the velocity decreases as the porosity parameter $\lambda$ increases, for the stretching parameter $C < 1$; while it increases as the porosity parameter $\lambda$ increases, for the stretching parameter $C > 1$. It is further observed that the velocity decreases as the magnetic parameter $M$ increases, whereas it increases as the stretching parameter $C$ increases, for a fixed $\eta$.

Figures 3 and 4 illustrate various characteristics of the temperature distribution against $\eta$ for various values of the parameters such as the porosity parameter $\lambda$, the magnetic parameter $M$, the stretching parameter $C$, the Prandtl number $Pr$ and the heat generation $B$. From these figures it may be observed that the temperature distribution increases with the increasing value of the magnetic parameter $M$ and reverse phenomena occurs for the stretching parameter $C$. It is further observed that in Figure 3 the temperature distribution decreases with the increasing value of the Prandtl number $Pr$. In Figure 4, temperature distribution increases with the increasing value of the porosity parameter $\lambda$, for the stretching parameter $C < 1$, whereas it decreases with the increasing value of the porosity parameter $\lambda$, for the stretching parameter $C > 1$.

![Figure 2. Velocity distribution against $\eta$ for various values of $\lambda, M$ and $C$.](image-url)
Figure 3. Temperature distribution against $\eta$ for various values of $M$, $C$ and $Pr$ with $\lambda=3$ and $B=0.1$.

Tables 1 and 2 illustrate the effects of different flow variables such as the porosity parameter $\lambda$, the magnetic parameter $M$, the stretching parameter $C$, the Prandtl number $Pr$ and the heat generation $B$ on the dimensionless surface shear stress $\varphi'(0)$ and the local heat transfer rate $\theta'(0)$. It is noteworthy that the local skin-friction coefficient $C_f$ and the local Nusselt number $Nu$ decreases with increasing value of the porosity parameter $\lambda$, for the stretching parameter $C<1$, whereas these increases with increasing value of the porosity parameter $\lambda$, for the stretching parameter $C>1$. It is further observed that these decreases with increasing value of the magnetic parameter $M$, and increases with increasing value of the stretching parameter $C$. Moreover Table 2 shows that the local Nusselt number $Nu$ increases with increasing value of the Prandtl number $Pr$.

Figure 4. Temperature distribution against $\eta$ for various values of $\lambda$, $M$ and $C$ with $Pr=1$ and $B=0.1$. 
Table 1. Values of $f''(0)$ for various values of $\lambda, M$ and $C$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$M = 0.00$</th>
<th>$M = 0.25$</th>
<th>$M = 0.00$</th>
<th>$M = 0.25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.66730</td>
<td>-0.82530</td>
<td>0.90953</td>
<td>0.70149</td>
</tr>
<tr>
<td>3</td>
<td>-1.09102</td>
<td>-1.18009</td>
<td>1.25330</td>
<td>1.11525</td>
</tr>
</tbody>
</table>

Table 2. Values of $\theta(0)$ for various values of $\lambda, M, C$ and Pr with $B = 0.1$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Pr</th>
<th>$C = 0.5$</th>
<th>$C = 1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M = 0.00$</td>
<td>$M = 0.25$</td>
<td>$M = 0.00$</td>
</tr>
<tr>
<td>0</td>
<td>0.1</td>
<td>-0.21477</td>
<td>-0.20759</td>
</tr>
<tr>
<td>1.0</td>
<td>-0.62200</td>
<td>-0.58974</td>
<td>-0.84370</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.21025</td>
<td>-0.20756</td>
<td>-0.29206</td>
</tr>
<tr>
<td>1.0</td>
<td>-0.58879</td>
<td>-0.57562</td>
<td>-0.85660</td>
</tr>
</tbody>
</table>

7. Conclusions

The steady two-dimensional stagnation point flow and heat generation of a viscous incompressible electrically conducting fluid over a stretching sheet in porous medium in the presence of a magnetic field has been analyzed. The similarity equations are obtained and solved numerically by a perturbation technique. The effects of the porosity parameter, the magnetic parameter, the stretching parameter and the Prandtl number are studied in detail. It is found that for the stretching parameter $C<1$, the velocity boundary layer thickness, the local skin-friction coefficient and the local Nusselt number decreases and the thermal boundary layer thickness increases with increasing value of the porosity parameter whereas, for the stretching parameter $C>1$, reverse phenomenon take place in the same case. It is further observed that the velocity boundary layer thickness, the local skin-friction coefficient and the local Nusselt number decreases and the thermal boundary layer thickness increases with increasing value of the magnetic parameter while the reverse phenomenon is observed with increasing value of the stretching parameter. Further it can be seen that the local Nusselt number increases and the thermal boundary layer thickness decreases with increasing value of the Prandtl number.

References

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