Finite Element Magnetic Field Response of an Exponential Conductivity Ground Profile

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Abstract

In this paper, mathematical model of finite element method for the magnetic field of an exponential conductivity ground profile is presented and computed to find the magnetic field at various locations by assuming that the Earth structure having exponential conductivity profile. There is a source providing a DC voltage and a receiver on the ground surface. Numerical technique the finite element method (FEM) is introduced by using the Galerkin’s method of Weighted Residuals to find approximate solutions of partial differential equation. Matlab programing is conducted to calculate and plot graphs of magnetic field at various locations. The results perform very well to the intensity of magnetic field for cross sections of ground structure.

Mathematics Subject Classification: 86A25

Keywords: Finite element, Magnetic, Galerkin

1 Introduction

Currently, human studied the Earth structure widely in order to utilize the natural resources embedded beneath the Earth for developing the agricultural sector and industrial sectors in their countries. They use knowledge of geophysics which is a branch of science concerned with the Earth survey. The
survey uses mathematics, physics and the physical properties of the Earth such as the resistivity, conductivity, electric potential, magnetic field and electric field to search for the natural resources.

We create a mathematical model by using magnetometric resistivity method to find the value of magnetic field beneath the Earth surface. In 2003, Chen and Oldenburg[7] assume that the Earth structure consists of horizontally stratified layers having constant conductivity at certain depths except the last layer where the conductivity having the same varying through the rest of the layer. They derived the magnetic field directly by solving a boundary value problem of a horizontally stratified layered Earth with homogeneous layers. However, in the real situation there are cases where the subsurface conductivities vary exponentially, linearly or binomially with depth. There exists a considerable amount of research about mathematical modeling which assumes that the Earth structure consists of horizontally stratified multilayer with one or more layers having exponentially, linearly or binomially varying conductivities at certain depths except the last layer where the conductivity having the same varying through the rest of the layer. Stoyer and Wait[28] studied the problem of computing apparent resistivity for a structure with a homogeneous overburden overlying a medium whose resistivity varies exponentially with depth. Banerjee et al.[1] gave expressions for apparent resistivity of a multilayered Earth with a layer having exponentially varying conductivity. Kim and Lee[14] derived a new resistivity kernel function for calculating apparent resistivity of a multilayered Earth with layers having exponentially varying conductivities. Siew and Yooyuanyong[29] studied the electromagnetic response of a thin disk beneath an inhomogeneous conductive overburden and expressions for the electric fields in the overburden. Ketchanwit[15] studied the Earth surface layers using time-domain electromagnetic field by constructing three mathematical models having exponentially varying and constant varying conductivities. Sripunya[30] derived solutions of the steady state magnetic field due to a DC current source in a layered Earth with some layer having exponentially or binomially or linearly varying conductivity.

In this paper, mathematical model is presented by using numerical techniques for finding approximate solutions. The finite element method (FEM) is used to find the numerical solutions of the magnetic field under the Earth surface. We assume that the Earth structure contains only one layer having exponential conductivity \( \sigma(z) = \sigma_0 + (\sigma_1 - \sigma_0)e^{-bz} \). This method is different from the Hankel transform approach which is difficult to solve for some complex problems such as all the research mentioned above. There are a few research using FEM by applying the Galerkin’s method of Weighted Residuals to find the solution of the magnetic field. For instance, Lee[16] presented a numerical method of computing the electromagnetic response of two-dimensional Earth models to an oscillating magnetic dipole. Velinsky and Martince[36] intro-
duced a time-domain method to solve the problem of geomagnetic induction in a heterogeneous Earth excited by variations of the ionospheric and magnetospheric currents with arbitrary spatiotemporal characteristics. Mitsuhata and Uchida[20] presented a finite element algorithm for computing magnetic field response for 3D conductivity structures. Therefore we are interested approximate techniques in finding the magnetic field beneath the Earth by using the Galerkin’s method of Weighted Residuals.

2 Mathematical Formulations

In this section, we use finite element method (FEM) for constructing approximate solutions of our problems. Assuming that the Earth structure contains only one layer having exponential conductivity ($\sigma(z) = \sigma_0 + (\sigma_1 - \sigma_0)e^{-bz}$) and there are a source providing a DC voltage and a receiver on the ground surface which picks up the signal from $r = 10$ m to $r = 190$ m as shown in Figure 2.1.

![Figure 2.1: Geometric model of the Earth structure.](image)

We define $z$ as the depth of an object from the Earth surface (meter), $r$ as the distance between source and receiver of magnetic field on the Earth surface (meter) and $\sigma(z)$ as the conductivity of the medium which is a function of $z$ (S/m), where $\sigma_0, \sigma_1$ and $b$ are positive constants.

From Maxwell’s equations, the relationship between the electric and magnetic fields[29,30,31,37] written in cylindrical coordinates $(r, \phi, z)$ is as follows.

\[ \nabla \times \vec{E} = \vec{0} \]  
(2.1)

and

\[ \nabla \times \vec{H} = \sigma \vec{E}, \]  
(2.2)

where $\vec{E}$ is the electric field vector, $\vec{H}$ is the magnetic field vector, $\sigma$ is the
conductivity of the medium and $\nabla$ is the gradient operator in cylindrical coordinates $(r, \phi, z)$ \cite{17, 27} defined by

$$\nabla = \frac{\partial}{\partial r} e_r + \frac{1}{r} \frac{\partial}{\partial \phi} e_\phi + \frac{\partial}{\partial z} e_z,$$

where $e_r$ is the unit vector in radial direction $(r)$, $e_\phi$ is the unit vector in the direction of $\phi$, $e_z$ is the unit vector in the direction of $z$.

Substituting equation (2.2) into (2.1), we obtain

$$\nabla \times \frac{1}{\sigma} (\nabla \times \vec{H}) = \vec{0}. \quad (2.3)$$

Substitute equation $\nabla \times \frac{1}{\sigma} (\nabla \times \vec{H})$ in cylindrical coordinates $(r, \phi, z)$ \cite{17, 27} into (2.3), we obtain

$$\begin{align*}
&\frac{1}{r} \left[ \frac{\partial}{\partial \phi} \left( \frac{1}{\sigma} \frac{\partial (r H_\phi)}{\partial r} - \frac{1}{\sigma} \frac{\partial H_r}{\partial \phi} \right) \right] - \frac{1}{\sigma} \left[ \frac{\partial}{\partial \phi} \left( \frac{1}{\sigma} \frac{\partial H_r}{\partial \phi} - \frac{1}{\sigma} \frac{\partial H_z}{\partial \phi} \right) \right] e_r \\
&+ \frac{\partial}{\partial z} \left( \frac{1}{r} \frac{\partial (r H_z)}{\partial r} - \frac{1}{\sigma} \frac{\partial H_r}{\partial z} \right) - \frac{1}{\sigma} \left[ \frac{\partial}{\partial \phi} \left( \frac{1}{\sigma} \frac{\partial H_r}{\partial \phi} - \frac{1}{\sigma} \frac{\partial (r H_\phi)}{\partial \phi} \right) \right] e_\phi \\
&+ \frac{1}{r} \left[ \frac{\partial}{\partial r} \left( \frac{1}{\sigma} \frac{\partial H_r}{\partial r} - \frac{1}{\sigma} \frac{\partial H_z}{\partial r} \right) - \frac{1}{\sigma} \left[ \frac{\partial}{\partial \phi} \left( \frac{1}{\sigma} \frac{\partial (r H_\phi)}{\partial r} \right) \right] e_z = \vec{0}, \quad (2.4)
\end{align*}$$

where $H_r, H_\phi$ and $H_z$ are the components of $\vec{H}$ in $e_r, e_\phi$ and $e_z$ directions, respectively. Since the magnetic field is axisymmetric, it depends only on $r$ and $z$ and not on the azimuth $\phi$ and from electromagnetic theory, we know the magnetic field has only the azimuthal component, i.e. $\vec{H} = H_\phi (r, z) e_\phi$ \cite{19}.

Simplifying equation (2.4) yields

$$\frac{1}{\sigma} \frac{\partial^2 H}{\partial z^2} + \frac{\partial H}{\partial z} \frac{\partial}{\partial z} \left( \frac{1}{\sigma} \right) + \frac{1}{\sigma} \left[ \frac{1}{r} \frac{\partial^2 (r H)}{\partial r^2} + \frac{\partial}{\partial r} \left( \frac{1}{r} \right) \frac{\partial (r H)}{\partial r} \right] = 0.$$

We denote conductivity $\sigma$ as a function of depth $z$ only, i.e. $\sigma = \sigma(z)$, and we now have

$$\frac{\partial^2 H}{\partial z^2} + \frac{\partial H}{\partial z} \frac{\partial}{\partial z} \left( \frac{1}{\sigma} \right) + \frac{\partial^2 H}{\partial r^2} + \frac{1}{r} \frac{\partial H}{\partial r} - \frac{1}{r^2} H = 0. \quad (2.5)$$

Since the Laplace equation and the problem is axisymmetric, our problem becomes

$$\Delta H + \sigma \frac{\partial H}{\partial z} \frac{\partial}{\partial z} \left( \frac{1}{\sigma} \right) - \frac{1}{r^2} H = 0. \quad (2.6)$$

The next step, we use finite element method to establish a numerical solution of our problem. We apply the Galerkin’s Method of Weighted Residuals
to equation (2.6).

We transform equation (2.6) into weak formulation to find \( H \in H^1 \). Let 
\[ V = \{ v \in H^1 : v \text{ is a continuous function on } \Omega, \frac{\partial v}{\partial r} \text{ and } \frac{\partial v}{\partial z} \text{ are piecewise continuous on } \Omega \text{ and } v = 0 \text{ on } \partial \Omega \} \].

The weak formulation of equation (2.6) is denoted by
\[
(\Delta H, v) + \left( \sigma \frac{\partial H}{\partial z} \frac{\partial}{\partial z} \left( \frac{1}{\sigma} \right), v \right) - \left( \frac{1}{r^2} H, v \right) = 0, \quad v \in V
\]
or
\[
\int_{\Omega} \Delta H v d\Omega + \int_{\Omega} \sigma \frac{\partial H}{\partial z} \frac{\partial}{\partial z} \left( \frac{1}{\sigma} \right) v d\Omega - \int_{\Omega} \frac{1}{r^2} H v d\Omega = 0. \tag{2.7}
\]

By Green’s identity[32] and \( v = 0 \) on \( \partial \Omega \), we obtain
\[
- \int_{\Omega} \nabla H \cdot \nabla v d\Omega + \int_{\Omega} \sigma \frac{\partial H}{\partial z} \frac{\partial}{\partial z} \left( \frac{1}{\sigma} \right) v d\Omega - \int_{\Omega} \frac{1}{r^2} H v d\Omega = 0. \tag{2.8}
\]

Using cylindrical co-ordinates \((r, \phi, z)\) [21] and since the problem is axisymmetric and \( H \) has only the azimuthal component in cylindrical coordinate, equation (2.8) becomes
\[
- \int_{\tilde{\Omega}} r \nabla H \cdot \nabla v dr dz + \int_{\tilde{\Omega}} r \sigma \frac{\partial H}{\partial z} \frac{\partial}{\partial z} \left( \frac{1}{\sigma} \right) v dr dz - \int_{\tilde{\Omega}} \frac{1}{r} H v dr dz = 0, \tag{2.9}
\]
where \( \tilde{\Omega} \) is the 2D cross-section of domain \( \Omega \) (\( \phi \) is fixed), i.e. \( \tilde{\Omega} = \{(r, z), 10 \leq r \leq 190, 0 \leq z \leq 180 \} \).

Next we consider the two dimensional domain of equation (2.9). By dividing the domain into rectangular elements, we discretize \( r \) into 9 subintervals equally, discretize \( z \) into 9 subintervals equally and \((r_i, z_j)\) is a node of \( \Omega \) on the non overlapping rectangles such that the horizontal and vertical edges of these rectangles are parallel to the \( r \) and \( z \) coordinate axes, respectively, i.e.
\[
r_i = 10 + 20i, \quad i = 0, \ldots, 9,
\]
\[
z_j = 20j, \quad j = 0, \ldots, 9.
\]

Since the form of equation (2.9) suggests that the finite elements can have an arbitrary shape and position in space computing integrals over their element domains is a bit tricky. To overcome this difficulty, one uses a projection method which maps the coordinates of a well known reference element to the coordinates of an arbitrary element in space by a mapping the values range from -1 to +1, and the reference coordinates are as \((\xi_1, \eta_1) = (-1, -1), (\xi_2, \eta_2) = (1, -1), (\xi_3, \eta_3) = (1, 1), (\xi_4, \eta_4) = (-1, 1)\) that represent in Figure 2.2(b).
For simplicity and to avoid any confusion, we use $H(X_i)$, $i = 1, 2, \ldots, 100$ for $H_i$, $i = 1, 2, \ldots, 100$. In other words, we define nodes $X_i$, $i = 1, 2, \ldots, 100$ for $(r_i, z_j)$, $i, j = 0, 1, \ldots, 9$ as shown in Figure 2.3. For each $i = 1, 2, \ldots, 100$ define $\varphi_j$ as basis function such that

$$\varphi_j(X_i) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}.$$  

A function $v \in V$ can be written in the form of linear combination of basis function $\varphi_i$

$$v(X) = \sum_{i=1}^{100} \alpha_i \varphi_i(X).$$

We obtain $v(X_j) = \alpha_j$ by choosing appropriate values for $\alpha_j$. Equation (2.9) becomes

$$-(r \nabla H, \nabla \varphi_i) + \left( r \sigma \frac{\partial H}{\partial z} \frac{1}{\sigma} \right) \varphi_i - \left( \frac{1}{r} H, \varphi_i \right) = 0.$$
Substituting \( \sigma(z) = \sigma_0 + (\sigma_1 - \sigma_0)e^{-bz} \) where \( \sigma_0, \sigma_1 \) and \( b \) are positive constants, we obtain

\[
-(r \nabla H, \nabla \varphi_i) + \left( br \left( \frac{(\sigma_1 - \sigma_0)e^{-bz}}{\sigma_0 + (\sigma_1 - \sigma_0)e^{-bz}} \right) \frac{\partial H}{\partial z}, \varphi_i \right) - \left( \frac{1}{r} H, \varphi_i \right) = 0,
\]

for \( i = 1, 2, \ldots, 100 \).

Next, we consider the solution in the form of linear combination of basis function \( \varphi_j \)

\[
H(X) = \sum_{j=1}^{100} H_j \varphi_j(X),
\]

when \( H_j \) is the unknown constants to be found.

Then equation (2.9) can be written in the form of linear combination as follows

\[
\sum_{j=1}^{100} H_j \left[ -\int_{\tilde{\Omega}} r(\nabla \varphi_j \cdot \nabla \varphi_i) dr dz + \int_{\tilde{\Omega}} \left( \frac{b(\sigma_1 - \sigma_0)}{\sigma_0 + (\sigma_1 - \sigma_0)e^{-bz}} \right) \frac{\partial \varphi_j}{\partial z} \varphi_i dr dz - \frac{1}{\tilde{\Omega}} \int_{\tilde{\Omega}} \frac{1}{r} \varphi_j \varphi_i dr dz \right] = 0,
\]

(2.10)

for each \( i = 1, 2, \ldots, 100 \).

We need a transformation from an original element to a reference element as shown in Figure 2.2

\[
\begin{align*}
    r &= r_k + \frac{h}{2} (1 + \xi), \quad dr = \frac{h}{2} d\xi, \\
    z &= z_k + \frac{h}{2} (1 + \eta), \quad dz = \frac{h}{2} d\eta.
\end{align*}
\]

(2.11)

where \( k = 0, 1, \ldots, 8 \).

The basis functions can be written in the form of \( \xi \) and \( \eta \) as follows

\[
\begin{align*}
    N_1(\xi, \eta) &= \frac{1}{4} (1 - \xi)(1 - \eta), \\
    N_2(\xi, \eta) &= \frac{1}{4} (1 + \xi)(1 - \eta), \\
    N_3(\xi, \eta) &= \frac{1}{4} (1 + \xi)(1 + \eta), \\
    N_4(\xi, \eta) &= \frac{1}{4} (1 - \xi)(1 + \eta).
\end{align*}
\]

(2.12)

We transform \( r, z \) to \( \xi, \eta \) by using the transformation equation (2.11) to-
together with basis functions equation (2.12), so equation (2.10) becomes

\[
\sum_{j=1}^{100} H_j \left[ -\int_{-1}^{1} \int_{-1}^{1} \left( r_k + \frac{h}{2}(1 + \xi) \right) \left[ \frac{\partial \hat{\phi}_j}{\partial \xi} \frac{\partial \hat{\phi}_i}{\partial \xi} + \frac{\partial \hat{\phi}_j}{\partial \eta} \frac{\partial \hat{\phi}_i}{\partial \eta} \right] d\xi d\eta \\
+ \frac{b h}{2} \int_{-1}^{1} \int_{-1}^{1} \left( r_k + \frac{h}{2}(1 + \xi) \right) \left( \frac{(\sigma_1 - \sigma_0)e^{-b(z_k + \frac{h}{2}(1+\eta))}}{\sigma_0 + (\sigma_1 - \sigma_0)e^{-b(z_k + \frac{h}{2}(1+\eta))}} \right) \frac{\partial \hat{\phi}_j}{\partial \eta} \hat{\phi}_i d\xi d\eta \\
- \frac{h^2}{4} \int_{-1}^{1} \int_{-1}^{1} \left( r_k + \frac{h}{2}(1 + \xi) \right) \hat{\phi}_j \hat{\phi}_i d\xi d\eta \right] = 0,
\]

(2.13)

for each \( i = 1, 2, \ldots, 100 \).

The value of equation (2.13) can be written in the form of matrix as follows

\[
(-A + B - C) u_i = (a + d + c),
\]

where \((-A + B - C)\), the stiffness matrix is an \(100 \times 100\) matrix, \(u_i\) and \((a + d + c)\) are \(100\)-vectors for all \( i = 1, 2, \ldots, 100 \).

### 3 Numerical Results

Since the Galerkin’s Method of Weighted Residuals was applied to equation (2.5), we obtained the values of magnetic field at various positions of the earth’s structure with one layer having exponentially decreasing conductivity \( \sigma = \sigma_0 + (\sigma_1 - \sigma_0)e^{-bz} \) when \( \sigma_0 < \sigma_1 \). There is a source providing a DC voltage and a receiver on the ground surface which picks up the signal from \( r = 10 \, m \) to \( r = 190 \, m \). We discrete the depth into 9 subintervals equally of the size \( h = 20 \, m \), i.e. we consider \( z = 0, 20, \ldots, 180 \, m \). We use constant \( \sigma_1 = 1.5 \, S/m\), \( \sigma_0 = 0.5 \, S/m\) and \( b = 0.01, 0.05, 0.1, 0.2 \) and \( 0.3 \, m^{-1} \). The numerical solutions of the magnetic field at each node is calculated by using Matlab program.

Graphs of the relationship between magnetic field and distance of receiver from source at various depths.
Figure 3.1: The relationship between magnetic field and distance of receiver from source when $z$ is fixed for (a) $b = 0.01 \text{ m}^{-1}$ (b) $b = 0.05 \text{ m}^{-1}$ (c) $b = 0.1 \text{ m}^{-1}$ (d) $b = 0.2 \text{ m}^{-1}$ (e) $b = 0.3 \text{ m}^{-1}$

From Figure 3.1(a) to (e), when $b = 0.01, 0.05, 0.1, 0.2$ and $0.3 \text{ m}^{-1}$, respectively, we can see that the value of magnetic field decreases exponentially as $r$ increases and it decreases as $z$ increases as well. The value of magnetic field is highest when $b = 0.01 \text{ m}^{-1}$ and it decreases when $b = 0.05 \text{ m}^{-1}$ and $b = 0.1 \text{ m}^{-1}$, respectively and increases slowly when $b = 0.2 \text{ m}^{-1}$ and $b = 0.3 \text{ m}^{-1}$, respectively. These results are caused by the conductive of the ground and the vertically location of its.

Figure 3.2: The relationship between magnetic field and distance of receiver from source when $b$ varies from $0.01, 0.05, 0.1, 0.2$ and $0.3 \text{ m}^{-1}$ and $z$ is fixed. (a) $z=20 \text{ m}$ (b) $z=60 \text{ m}$ (c) $z=100 \text{ m}$ (d) $z=140 \text{ m}$
Figure 3.2(a) to (d) represents the value of magnetic field which are plotted against $r$ when $b$ varies and $z$ is fixed at 20, 60, 100 and 140 m, respectively. We can see that the value of magnetic field where $b = 0.01, 0.05, 0.1, 0.2$ and $0.3 \text{ m}^{-1}$ decreases exponentially as $r$ increases and it has similar values when $z$ increases because the value of magnetic field decreases to zero and has value near zero when $z$ increases as $b$ varies. From Figure 3.2(b), (c) and (d), the three curves representing the value of magnetic field when $b = 0.1, 0.2$ and $0.3 \text{ m}^{-1}$ have a different behavior from the others. The value of magnetic field is greater than that when $b = 0.05 \text{ m}^{-1}$, as we can see the curves cross those two lines for $b = 0.05 \text{ m}^{-1}$. These results take place according to the conductive of ground and the spacing distance between source and receiver.

Contour graphs of the relationship between magnetic field and distance of receiver from source at various depth are shown as in Figure 3.3.

![Contour graphs of magnetic field at different distances of receiver from source and different depths](image)

*Figure 3.3: Contour graphs of magnetic field at different distances of receiver from source and different depths when (a) $b = 0.01 \text{ m}^{-1}$ (b) $b = 0.05 \text{ m}^{-1}$ (c) $b = 0.1 \text{ m}^{-1}$ (d) $b = 0.2 \text{ m}^{-1}$ (e) $b = 0.3 \text{ m}^{-1}$*

From Figure 3.3(a) to (e), when $b = 0.01, 0.05, 0.1, 0.2$ and $0.3 \text{ m}^{-1}$, the red color shows the area when the value of magnetic field is high and the blue color shows the area when the value of magnetic field is low. The value of magnetic field decreases from $b = 0.01$ to $b = 0.1 \text{ m}^{-1}$ and it increases slowly from $b = 0.1$ to $b = 0.3 \text{ m}^{-1}$, as we can see in Figure 3.3(a) to (e).

Turning to the case of increasing conductivity $\sigma = \sigma_0 + (\sigma_1 - \sigma_0)e^{-bz}$ when $\sigma_0 > \sigma_1, \sigma_1 = 0.5$ and $\sigma_0 = 1.5 \text{ (S/m)}$, the numerical solutions of the magnetic field at each node is calculated by using Matlab program as shown in Figure.
Graphs of the relationship between magnetic field and distance of receiver from source at various depths.

![Graphs](image)

Figure 3.4: The relationship between magnetic field and distance of receiver from source at various depths when (a) $b = 0.01 \, m^{-1}$ (b) $b = 0.05 \, m^{-1}$ (c) $b = 0.1 \, m^{-1}$ (d) $b = 0.2 \, m^{-1}$ (e) $b = 0.3 \, m^{-1}$

From Figure 3.4(a) to (e), when $b = 0.01, 0.05, 0.1, 0.2$ and $0.3 \, m^{-1}$, respectively, we can see that the value of magnetic field decreases exponentially as $r$ increases and it decreases as $z$ increases as well. The value of magnetic field increases when $b$ increases and it decreases slowly from $b = 0.2 \, m^{-1}$ to $b = 0.3 \, m^{-1}$. This results is caused by the conductive of the ground and the vertically location of its which is as same as the case of decreasing conductivity profile.

![Graphs](image)

Figure 3.5: The relationship between magnetic field and distance of receiver from source when $b$ varies from $0.01, 0.05, 0.1, 0.2$ and $0.3 \, m^{-1}$ and $z$ is fixed. (a) $z=20 \, m$ (b) $z=60 \, m$ (c) $z=100 \, m$ (d) $z=140 \, m$
Figure 3.5(a) to (d) represents the values of magnetic field which are plotted against $r$ when $b$ varies and $z$ is fixed at 20, 60, 100 and 140 m, respectively. We can see that the value of magnetic field where $b = 0.01, 0.05, 0.1, 0.2$ and $0.3 \text{ m}^{-1}$ decreases exponentially as $r$ increases and it has similar values when $z$ increases because the value of magnetic field decreases to zero and has value near zero when $z$ increases as $b$ varies. From Figure 3.5(b), (c) and (d), the three curves representing the values of magnetic field which are plotted against $r$ when $b = 0.1, 0.2$ and $b = 0.3 \text{ m}^{-1}$ have a different behavior from the others. The value of magnetic field is smaller than that when $b = 0.2 \text{ m}^{-1}$, as we can see the curve cross the line for $b = 0.2 \text{ m}^{-1}$. These results occur according to the conductive of ground and the spacing distance between source and receiver.

Contour graphs of the relationship between magnetic field and distance of receiver from source at various depths.

![Contour graphs](image)

Figure 3.6: Contour graphs of magnetic field at different distances of receiver from source and different depths when (a) $b = 0.01 \text{ m}^{-1}$ (b) $b = 0.05 \text{ m}^{-1}$ (c) $b = 0.1 \text{ m}^{-1}$ (d) $b = 0.2 \text{ m}^{-1}$ (e) $b = 0.3 \text{ m}^{-1}$

From Figure 3.6(a) to (e), when $b = 0.01, 0.05, 0.1, 0.2$ and $0.3 \text{ m}^{-1}$, the red color shows the area when the value of magnetic field is high and the blue color shows the area when the value of magnetic field is low. The value of magnetic field increases when $b$ increases and it decreases slowly from $b = 0.2 \text{ m}^{-1}$ to $b = 0.3 \text{ m}^{-1}$, as we can see in Figure 3.6(a) to (e).
4 Conclusions

A mathematical model of the magnetic fields is conducted by using partial differential equations. Numerical solutions of partial differential equations are computed by using finite element method (FEM) to find the value of the magnetic field at various locations of the ground. In our models, the ground have an exponentially decreasing and increasing conductivity profiles. The Galerkin’s Method of Weighted Residuals is applied in finite element method. There is a source providing a DC voltage and a receiver on the ground surface which picks up the signal from \( r = 10 \) m to \( r = 190 \) m. We begin our study with the relationship between magnetic and electric fields. The vector equations for computing magnetic and electric field is very difficult. Thus, in most research, we change the vector equations into scalar equations by considering the components of the vector. We finally arrive with partial differential equation describing the system and it is surprisingly that our equation is independent from the electric current. The numerical results is calculated and plotted by using MATLAB R2008b program to show the behavior of magnetic field at different depths and distances from source point. In our research, the behavior of magnetic field decreases to zero when the depth of soil increases. As well as the case of increasing the space between source-receiver, the magnetic field decreases to zero too. The value of \( b \) is an important role for the conduction of ground and effect to the magnetic field quantities as well. The comparision of the quantities of magnetic field for the case of \( \sigma \) as an increasing function is higher than the case of \( \sigma \) as a decreasing function according to the advantage of the DC source that better reflex at very high depth than on the ground surface.

Acknowledgements. The authors would like to thank the Department of Mathematics, Faculty of Science, Silpakorn University and Centre of Excellence in Mathematics for continuous financial and equipments support.

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Received: February 23, 2015; Published: March 27, 2015