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New Approach to Periodicity of Neutral Type Delay Difference Equations

S. Nalini

Department of Mathematics
Narasu's Sarathy Institute of Technology
Salem, Tamilnadu - 636 305, India

S. Mehar Banu

Department of Mathematics
Govt. Arts college for women, Salem
Tamilnadu - 636 008, India

Abstract

Artificial neural networks have been developed as generalizations of mathematical models of biological nervous system. In this paper we investigate periodicity of neutral typed delay difference equations of unbounded activation functions via fixed point theory and some new techniques; various sufficient conditions for the existence of positive periodic solutions are established.

Keywords: Periodicity, fixed point theory, neutral, delay, difference equations

1 Introduction

Artificial neural networks have been developed as generalizations of mathematical models of biological nervous system. Artificial neurons parallel the brain behavior and consist of one or several layers of neurons connected by links. Each artificial neuron computes a scalar output from a linear combination of inputs, using a given scalar function, which is assumed the same for all neurons. The differences between two neurons are due to either the number of input components or to their associated weights. Since the neural function is given, only the structure (links) and the weights are learned using well known learning methods, as the back propagation method. The applications of neural networks of nonlinear systems are in speech and signal processing, pattern recognition, system modeling, and servomechanism control, etc. The non-linear dynamics is fundamental for understanding higher level brain functions. Difference equations or discrete dynamical systems are various fields which crash almost every branch of pure and applied mathematics. Recently, many researchers show their interest in difference equations systems. One of the reasons for this is a necessity for some techniques which can be used in investigating equations arising in mathematical models describing real life situations in population biology, economic, probability theory, genetics, psychology, ...etc. Many researchers investigate stability, globally asymptotic stability, exponentially stability of neural networks. Very few papers discussed periodicity of delay difference equations [1] - [9].In this paper we consider the following neutral type delay difference equations:

$$u_i(t+1) = a_i(t)u_i(t) + \sum_{j=1}^{\infty} b_{ij}(t)f_j\left(u_j(t)\right) + \sum_{j=1}^{\infty} c_{ij}\Delta\left(u_j(t-\tau)\right)$$

$$+ \sum_{j=1}^{\infty} d_{ij}(t)f_j\left[\sum_{\eta=1}^{\infty} h_j(\eta)\, u_j(t-\eta)\right]$$

$$+ I_i(t) \qquad (1)$$

Where $i \in \mathcal{N} = \{1,2 \dots m\}$.

Our aim in this paper to study the periodicity of neutral type delay difference equations of (1) via Krasnoselkii's fixed point theory.

2 Preliminaries

Let us consider the w - periodic sequences X_w defined on $\mathbb{Z}$, where $w \geq 1$.

Then X_w is a Banach space when it is endowed with the norm

$$\|u\| = \max_{i\in\mathcal{N}}\left\{\operatorname{Sup}_{x\epsilon[0,w]} |u_i(x)|\right\}$$

Let us consider the set of all continuous bounded functions

$$\psi(x) = (\psi_1(x), \psi_2(x), \dots, \psi_m(x))^w : C(-\infty, 0]_z \to R^m.$$

(A1): The connection weights of the neural networks a_i, b_{ij}, d_{ij}, I_i are ϖ – periodic functions defined on $\mathbb{Z}$ and the activation function $f_j(.)$ is non decreasing and unbounded. Let

$$F_i(n,q) = \prod_{x=q+1}^{n+\varpi-1} a_i(x)\left[1 - \prod_{x=n}^{n+\varpi-1} a_i(x)\right]^{-1}, q\epsilon[n, n+\varpi-1] \qquad (2)$$

Where $[1 - \prod_{x=n}^{n+\varpi-1} a_i(x)]^{-1}$ is not zero since $0 < a_i(n) < 1$ for $n\epsilon[0, \varpi - 1]$.

Also, Let

$$m = min\{F(n,u): n \geq 0, u \leq \varpi\} = F(n,n) > 0 \qquad (3)$$

$$M = max\{F(n,u): n \geq 0, u \leq \varpi\} = F(n, n+\varpi-1) = F(0, n-1) > 0 \qquad (4)$$

Lemma 2.1: If (A1) hold. Any sequence $\{u(n)\}\epsilon S_T^m$ is a solution of (1) if and only if

$$u_i(n) = \sum_{j=1}^{\infty} c_{ij} u_j(q-\tau) + \sum_{q=n}^{n+w-1} F_i(n,q)\left[\sum_{j=1}^{\infty} b_{ij}(q) f_j\left(u_j(q)\right) + \sum_{j=1}^{\infty} d_{ij}(q) f_j\left[\sum_{\eta=1}^{\infty} h_j(\eta)\, u_j(q-\eta)\right] + I_i(q) - \sum_{j=1}^{\infty} c_{ij} u_j(q-\tau)\ \left(1 - a_i(q)\right)\right] \qquad (5)$$

Where $F_i(n,q) = \prod_{x=q+1}^{n+\varpi-1} a_i(x)[1 - \prod_{x=n}^{n+\varpi-1} a_i(x)]^{-1}, q\epsilon[n, n+\varpi-1]$ (6)

Lemma 2.2: (Krasnoselskii's theorem)

Assume that F is a closed bounded convex subset of a Banach space X. Furthermore assume that T_1 and $T_2$are mappings from F into X such that

1. $T_1(x) + T_2(y) \in F \ for\ all\ x, y \in F,$
2. T_1 is a contraction,
3. T_2 is a continuous and compact

Then $T_1 + T_2$ has a fixed point in F.

3 Main results

In this section we consider a closed convex and bounded subset of the banach space ς_w and define the set

$$\mathbb{F} = \{\varphi \epsilon \varsigma_w : \ \alpha \leq \varphi \leq \beta\}. \tag{9}$$

Assumption (A2): For each $i, j \epsilon \mathcal{N}, c_{ij} \geq 0, b_{ii}(n) > 0\ and$ assume that for all $u \epsilon \mathbb{Z}$ and $\rho \epsilon \mathbb{M}, 0 \leq \widehat{c_i} := \sum_{j=1}^{\infty} c_{ij} \leq 1$.Moreover there exist a non negative constant α and a positive constant β such that for all $i \epsilon \mathcal{N}$

$$\frac{1 - \widehat{c_i}}{m_i w}\alpha + b_{ii}(n) f_i(\alpha) - \big(1 - a_i(n)\big)\widehat{c_i}\beta \leq f(u, \rho) \\ \leq \frac{1 - \widehat{c_i}}{M_i w}\beta + b_{ii}(n) f_i(\beta) \\ - \big(1 - a_i(n)\big)\widehat{c_i}\alpha \tag{10}$$

Where m and M are defined by (3) and (4) respectively and assume $0 \leq \widehat{c_i} := \sum_{j=1}^{\infty} c_{ij} \leq 1$

And also assume that

$$\sum_{j=m}^{\infty} c_{ij} + \sum_{j=m}^{\infty} b_{ij}(n) f_j(.) \\ + \sum_{j=m}^{\infty} d_{ij}(n) f_j[.] + I_i(n) \geq 0 \tag{11}$$

To apply Krasnoselskii's theorem we will need to construct two mappings (i.e., one is contraction and the other is compact).

For this we define the contraction map under the supremum norm $T_1 : \mathbb{F} \to \varsigma_w$ by

$$(T_1\varphi)(n) = \sum_{j=1}^{\infty} c_{ij}\, u_j(n-\tau) \qquad (12)$$

and the compact map $T_2 : \mathbb{F} \to \varsigma_w$ by

$$(T_2\varphi)(n) = \sum_{q=n}^{n+w-1} F_i(n,q) \left[\sum_{j=1}^{\infty} b_{ij}(q) f_j\left(u_j(q)\right) + \sum_{j=1}^{\infty} d_{ij}(q) f_j \left[\sum_{\eta=1}^{\infty} h_j(\eta)\, u_j(q-\eta) \right] + I_i(q) - \sum_{j=1}^{\infty} c_{ij} u_j(q-\tau)\ \left(1 - a_i(q)\right) \right] \qquad (13)$$

Lemma3.1:

Under the basic assumption (A1) and (A2), the operator T_2 is completely continuous on$\mathbb{F}$.

Proof:

For $n\epsilon[0, w-1]$ and for$\varphi\epsilon\mathbb{F}$, we have by (10) that

$$|(T_2\varphi)(n)| \le \left| \sum_{q=n}^{n+w-1} F_i(n,q) \left[\sum_{j=1}^{\infty} b_{ij}(q) f_j\left(u_j(q)\right) + \sum_{j=1}^{\infty} d_{ij}(q) f_j \left[\sum_{\eta=1}^{\infty} h_j(\eta)\, u_j(q-\eta) \right] + I_i(q) - \sum_{j=1}^{\infty} c_{ij} u_j(q-\tau)\ \left(1 - a_i(q)\right) \right] \right|$$

$$\leq \left| \sum_{q=n}^{n+w-1} F_i(n,q) \left[b_{ii}(q) f_i(\beta) + \sum_{j\neq i}^{m} b_{ij}(q) f_j(.) \right.\right.$$

$$\left.\left. + \sum_{j=1}^{m} d_{ij}(\varphi) f_j[.] + I_i(q) - \sum_{j=1}^{m} c_{ij}\alpha \left(1 - a_i(q)\right) \right] \right|$$

$$\leq \left| \sum_{q=n}^{n+w-1} F_i(n,q) \left[b_{ii}(q) f_i(\beta) - \hat{c_i}\alpha \left(1 - a_i(q)\right)\right] \right|$$

$$\leq M_i w \frac{1 - \hat{c_i}}{M_i w} \beta = (1 - \bar{c_i})\, \beta$$

From the above, $\|T_2\varphi\| \leq \beta$ this shows that $T_2\varphi$ is uniformly bounded. Together with the continuity of $T_2\varphi$, for any bounded sequence $\{\psi_{nk}\}$ in T_1. We know that any sub sequence $\{\psi_{nk}\}$ in T_1 such that $T_2(\psi_{nk})$ is convergent in $T_2\varphi(T_1\varphi)$. Therefore $T_2\varphi$ is compact on $T_1\varphi$. This completes the proof.

Theorem 3.2: Under the basic assumption the equation (1) has a positive periodic solution z satisfying $\alpha \leq z \leq \beta$.

Proof: Let$\varphi,\mu\epsilon\mathbb{F}$ .It follows lemma2 and assumption 2, we have that

$$(T_1\varphi)n+(T_2\varphi)n$$

$$=\sum_{j=1}^{\infty} c_{ij}\, u_j(n-\tau)$$

$$+\sum_{q=n}^{n+w-1} F_i(n,q)\left[\sum_{j=1}^{\infty} b_{ij}(q) f_j\left(u_j(q)\right)\right.$$

$$+\sum_{j=1}^{\infty} d_{ij}(q) f_j\left[\sum_{\eta=1}^{\infty} h_j(\eta)\, u_j(q-\eta)\right]+I_i(q)$$

$$\left.-\sum_{j=1}^{\infty} c_{ij} u_j(q-\tau)\ \left(1-a_i(q)\right)\right]$$

$$\leq \widehat{c_i}\beta+(1-\widehat{c_i})\beta=\beta$$

And also,

$$(T_1\varphi)n+(T_2\varphi)n$$

$$=\sum_{j=1}^{\infty} c_{ij}\, u_j(n-\tau)$$

$$+\sum_{q=n}^{n+w-1} F_i(n,q)\left[\sum_{j=1}^{\infty} b_{ij}(q) f_j\left(u_j(q)\right)\right.$$

$$+\sum_{j=1}^{\infty} d_{ij}(q) f_j\left[\sum_{\eta=1}^{\infty} h_j(\eta)\, u_j(q-\eta)\right]+I_i(q)$$

$$\left.-\sum_{j=1}^{\infty} c_{ij} u_j(q-\tau)\ \left(1-a_i(q)\right)\right]$$

$\geq \hat{c}_i \alpha + (1 - \hat{c}_i)\alpha = \alpha$

This shows that $T_1\varphi + T_2\varphi \epsilon \mathbb{F}$. All the hypotheses of the theorem stated in lemma2.2 are satisfied and therefore equation (1) has a positive periodic solution. This completes the proof. Similarly we can prove for the case $-1 \leq \hat{c}_i \leq 0$.

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