

Bayesian Estimation of Generalized Exponential Distribution under Progressive First Failure Censored Sample

Chansoo Kim

Department of Applied Mathematics
Kongju National University, Gongju, Korea

Keunhee Han

Department of Applied Mathematics
Kongju National University, Gongju, Korea
(Corresponding author)

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Abstract

In this paper, we consider the maximum likelihood (ML) and Bayes estimation of the parameters of the generalized exponential distribution based on progressive first failure censored samples. We also consider the problem of predicting an independent future order statistics from the same distribution. However, since Bayes estimator do not exist in an explicit form for the parameters, Markov Chain Monte Carlo (MCMC) method is used to generate samples from the posterior distribution. Importance sampling is applied to estimate the parameters and to predict the future observations. Simulation data are analyzed for illustrative purpose.

Keywords: Generalized exponential distribution, Progressive first failure censoring, Importance sampling

1 Introduction

The two-parameter generalized exponential distribution (which is denoted by $GE(\alpha, \lambda)$) was introduced by Gupta and Kundu (1999). It was observed that the GE distribution can be used in situations where a skewed distribution for a nonnegative random variable is needed.

The probability density function (pdf), cumulative distribution function (cdf) are given respectively by

$$\begin{aligned} f(x|\alpha, \lambda) &= \alpha\lambda(1 - e^{-\lambda x})^{\alpha-1} e^{-\lambda x}, x > 0, \alpha > 0, \lambda > 0, \\ F(x|\alpha, \lambda) &= (1 - e^{-\lambda x})^\alpha \end{aligned} \quad (1.1)$$

where α is the shape parameter and λ is the reciprocal of a scale parameter. When $\alpha = 1$, the GE distribution coincides with the exponential distribution.

For this model, Gupta and Kundu (1999) considered different estimation procedures and compared their performances through numerical simulations. Raqab and Ahsanullah (2001) and Raqab (2002) studied the properties of order and record statistics, and their inferences, respectively. Gupta and Kundu (2003) used the ratio of the maximized likelihoods in discriminating between the Weibull and GE distributions. Raqab and Madi (2005) considered the Bayesian estimation and prediction for GE distribution, using informative priors. Kundu and Gupta (2008) consider the estimation problem of GE using Bayesian viewpoints. Kim and Song (2010) considered Bayesian estimation of the parameters of the GE distribution based on doubly censored sample. They also used the Gibbs sampling to provide sample-based estimates for predictive density function of the future items under doubly censored sample.

In this paper, we consider the ML and Bayes estimations of the parameters of $GE(\alpha, \lambda)$ based on progressive first failure censored sample. The Bayes estimates can not be obtained in closed form. Therefore, we use the Gibbs and Metropolis sampler to generate MCMC samples and obtain the Bayes estimates of the parameters and sample-based estimates for predictive density function of the future observations using Importance sampling. In Section 2, we briefly describe the progressively first failure censoring scheme. In Section 3 and 4, we consider the ML and Bayes estimators of the unknown parameters. MCMC methods are used to provide sample-based estimates for the parameters and Bayes prediction for future order statistics. Finally, an illustrative example is provided in Section 5.

2 Progressive first-failure-censoring scheme

The first-failure censoring which introduced by Wu and Kus (2009) is combined with progressive censoring. Suppose that n independent groups with k items

within each group are put in a life test. R_1 groups and the group in which the first failure is observed are randomly removed from the test as soon as the first failure, say $X_{1:m:n:k}^R$, has occurred, R_2 groups and the group in which the second failure is observed are randomly removed from the test as soon as the second failure, say, $X_{2:m:n:k}^R$ has occurred, and finally R_m groups and the group in which the m th failure is observed are randomly removed from the test as soon as the m th failure, say $X_{m:n:m:k}^R$, has occurred. Then $X_{1:m:n:k}^R < X_{2:m:n:k}^R < \dots < X_{m:n:m:k}^R$ are called progressively first-failure censored order statistics with the progressive censoring scheme $\mathbf{R} = (R_1, R_2, \dots, R_m)$.

For simplicity, the m ordered observed failure times denoted by X_1, X_2, \dots, X_m are called progressively first-failure censored order statistics and $f(x)$ and $F(x)$ are its corresponding probability density function and cumulative distribution function, respectively. Then, the joint probability density function for X_1, X_2, \dots, X_m is given by

$$L(x_1, x_2, \dots, x_m) = Ck^m \prod_{i=1}^m f(x_i)(1 - F(x_i))^{k(R_i+1)-1} \tag{2.1}$$

where $C = n(n - R_1 - 1)(n - R_1 - R_2 - 2) \dots (n - \sum_{i=1}^{m-1} R_i - m + 1)$. Note that if $k = 1$ then, this sampling scheme reduces to the progressively type II censoring. Statistical inferences for several distributions of the related topics are studied by Wu and Kus (2009), Soliman et. al. (2011), Soliman et al. (2012) and Abou-Elheggag (2013).

3 Maximum likelihood estimation

Let $X_i, i = 1, 2, \dots, m$, be the progressively first-failure censored order statistics with censoring scheme $\mathbf{R} = (R_1, R_2, \dots, R_m)$ from generalized exponential distribution. In accordance with (1.1) and (2.1), the likelihood function of α and λ is given by

$$L(\alpha, \lambda|\mathbf{x}) = Ck^m \lambda^m \exp\left(-\lambda \sum_{i=1}^m x_i + w_\lambda(x)\right) \times \alpha^m \exp(-\alpha w_\lambda(x)) \prod_{i=1}^m (1 - u_i^\alpha)^{k(R_i+1)-1} \tag{3.1}$$

where C is defined in (2.1), $u_i = 1 - e^{-\lambda x_i}$ and $w_\lambda(x) = -\sum_{i=1}^m \log(u_i)$.

Assuming that the parameters α and λ are unknown, the MLE $\hat{\alpha}_M$ and $\hat{\lambda}_M$ of α and λ can be obtained by solving the following likelihood equations

$$\frac{\partial l(\alpha, \lambda|\mathbf{x})}{\partial \alpha} = \frac{m}{\alpha} - w_\lambda(\mathbf{x}) - \sum_{i=1}^m [k(R_i + 1) - 1] \frac{u_i^\alpha \log(u_i)}{1 - u_i^\alpha} \tag{3.2}$$

and

$$\begin{aligned} \frac{\partial l(\alpha, \lambda | \mathbf{x})}{\partial \lambda} &= \frac{m}{\lambda} - \sum_{i=1}^m x_i + (\alpha - 1) \sum_{i=1}^n \frac{x_i e^{-\lambda x_i}}{u_i} \\ &\quad - \sum_{i=1}^m [k(R_i + 1) - 1] \frac{\alpha x_i e^{-\lambda x_i} u_i^{\alpha-1}}{1 - u_i^\alpha} \end{aligned} \quad (3.3)$$

where $l(\alpha, \lambda | \mathbf{x})$ denotes the log-likelihood function of α and λ . Obtaining the closed forms for $\hat{\alpha}_M$ and $\hat{\lambda}_M$ are not possible. The solutions can be obtained by using Newton-Raphson method.

The asymptotic variance-covariance matrix of the MLE for parameters α and λ is given by the elements of the Fisher information matrix

$$I_{ij} = -E \left(\frac{\partial^2 l(\alpha, \lambda | \mathbf{x})}{\partial \alpha \partial \lambda} \right), \quad i, j = 1, 2.$$

But, there is a difficulty to get the exact expressions of the above expectation. Therefore, we will take the approximate asymptotic variance-covariance matrix for MLE. The asymptotic variance-covariance matrix is given by

$$\hat{\Sigma} = \left[\begin{array}{cc} -\frac{\partial^2 l(\alpha, \lambda | \mathbf{x})}{\partial \alpha^2} & -\frac{\partial^2 l(\alpha, \lambda | \mathbf{x})}{\partial \alpha \partial \lambda} \\ & -\frac{\partial^2 l(\alpha, \lambda | \mathbf{x})}{\partial \lambda^2} \end{array} \right]_{(\alpha, \lambda) = (\hat{\alpha}_M, \hat{\lambda}_M)}^{-1} = \left[\begin{array}{cc} \hat{\sigma}_\alpha^2 & \hat{\sigma}_{\alpha, \lambda} \\ & \hat{\sigma}_\lambda^2 \end{array} \right]$$

with

$$\begin{aligned} \frac{\partial^2 l(\alpha, \lambda | \mathbf{x})}{\partial \alpha^2} &= -\frac{m}{\alpha^2} - \sum_{i=1}^m [k(R_i + 1) - 1] \frac{u_i^\alpha (\log(u_i))^2}{(1 - u_i^\alpha)^2}, \\ \frac{\partial^2 l(\alpha, \lambda | \mathbf{x})}{\partial \lambda^2} &= -\frac{m}{\lambda^2} - (\alpha - 1) \sum_{i=1}^m \frac{x_i^2 e^{-\lambda x_i}}{u_i^2} - \sum_{i=1}^m [k(R_i + 1) - 1] \\ &\quad \times \frac{\alpha x_i^2 u_i^\alpha [e^{\lambda x_i} (u_i^\alpha - 1) + \alpha]}{(e^{\lambda x_i} - 1)^2 [1 - u_i^\alpha]^2} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial^2 l(\alpha, \lambda | \mathbf{x})}{\partial \alpha \partial \lambda} &= \sum_{i=1}^m \frac{x_i e^{-\lambda x_i}}{u_i} - \sum_{i=1}^m [k(R_i + 1) - 1] \frac{x_i u_i^\alpha}{(e^{\lambda x_i} - 1) [1 - u_i^\alpha]^2} \\ &\quad \times [\alpha \log(u_i) + 1 - u_i^\alpha] \end{aligned}$$

where $u_i = 1 - e^{-\lambda x_i}$

The asymptotic normality of the MLE can be used to compute the approximate confidence intervals for the parameters α and λ , which become

$$\left(\hat{\alpha} \pm z_{\alpha/2} \sqrt{\hat{\sigma}_\alpha^2}, \quad \hat{\lambda} \pm z_{\alpha/2} \sqrt{\hat{\sigma}_\lambda^2} \right)$$

where $z_{\alpha/2}$ is an upper $(\alpha/2)100$ pth percentile of the standard normal variate.

4 Bayes estimation and Prediction

We present the posterior densities of the parameters α , λ and Bayesian prediction for the future failure times based on the observed progressive first failure censored data. Since α and λ are both unknown, a natural choice for the prior distributions of α and λ would be to assume that the two quantities are independent gamma distribution as the following forms;

$$\pi(\alpha) = \frac{b^a}{\Gamma(a)}\alpha^{a-1}e^{-b\alpha} \quad \text{and} \quad \pi(\lambda) = \frac{d^c}{\Gamma(c)}\lambda^{c-1}e^{-d\lambda} \tag{4.1}$$

where a, b, c and d are chosen to reflect prior knowledge about α and λ .

By combining (3.1) and (4.1), the joint posterior density of α and λ is proportional to

$$\begin{aligned} \pi(\alpha, \lambda|\mathbf{x}) \propto & \alpha^{m+a-1}\lambda^{m+c-1}exp\left(-\lambda\left[d + \sum_{i=1}^m x_i\right] + w_\lambda(\mathbf{x})\right) \\ & \times exp\left(-\alpha[b + w_\lambda(\mathbf{x})] \prod_{i=1}^m (1 - u_i^\alpha)^{k(R_i+1)-1}\right) \end{aligned} \tag{4.2}$$

where $u_i = 1 - e^{-\lambda x_i}$ and $w_\lambda(\mathbf{x}) = -\sum_{i=1}^m \log(u_i)$.

Therefore, the Bayes estimate of any function of α and λ , say $g(\alpha, \lambda)$ under SEL is the following forms

$$\hat{g}_B(\alpha, \lambda) = \frac{\int_0^\infty \int_0^\infty g(\alpha, \lambda)L(\alpha, \lambda|\mathbf{x})\pi(\alpha)\pi(\lambda)d\alpha d\lambda}{\int_0^\infty \int_0^\infty L(\alpha, \lambda|\mathbf{x})\pi(\alpha)\pi(\lambda)d\alpha d\lambda} \tag{4.3}$$

In general, the ratio of the two integrals given by (4.3) can not obtained in a closed form. Markov Chain Monte Carlo method is used to approximate (4.3).

4.1 MCMC method

We propose to approximate (4.3) by using importance sampling technique as suggested by Chen and Chao (1999) and also construct the corresponding credible intervals. From (4.2), the posterior density function of α and λ given the data can be written as

$$\pi(\alpha, \lambda|\mathbf{x}) \propto g_1(\alpha|\lambda, \mathbf{x})g_2(\lambda|\mathbf{x})h(\alpha, \lambda|\mathbf{x}) \tag{4.4}$$

where $g_1(\alpha|\lambda, \mathbf{x})$ is a gamma density function with the shape and scale parameters as $m + a$ and $b + w_\lambda(\mathbf{x})$, respectively. $g_2(\lambda|\mathbf{x})$ is a proper density function given by

$$g_2(\lambda|\mathbf{x}) \propto \frac{1}{[b + w_\lambda(\mathbf{x})]^{m+a}}\lambda^{m+c-1}exp\left(-\lambda\left[d + \sum_{i=1}^m x_i\right] + w_\lambda(\mathbf{x})\right) \tag{4.5}$$

Moreover,

$$h(\alpha, \lambda | \mathbf{x}) = \prod_{i=1}^m [1 - (1 - e^{-\lambda x_i})^\alpha]^{k(R_i+1)-1}. \quad (4.6)$$

Therefore, the Bayes estimate of any function of α and λ , say $g(\alpha, \lambda)$ under SEL is the following forms

$$\hat{g}_B(\alpha, \lambda) = \frac{\int_0^\infty \int_0^\infty g(\alpha, \lambda) g_1(\alpha | \lambda, \mathbf{x}) g_2(\lambda | \mathbf{x}) h(\alpha, \lambda | \mathbf{x}) d\alpha d\lambda}{\int_0^\infty \int_0^\infty g_1(\alpha | \lambda, \mathbf{x}) g_2(\lambda | \mathbf{x}) h(\alpha, \lambda | \mathbf{x}) d\alpha d\lambda}. \quad (4.7)$$

It is not possible to obtain in a closed form. We consider the importance sampling technique to compute the Bayes estimates and also to construct the credible interval of $g(\alpha, \lambda) = \theta$ using Algorithm 1.

Algorithm 1:

Step 1. Generate α from $g_1(\alpha | \lambda, \mathbf{x})$

Step 2. Generate λ from $g_2(\lambda | \mathbf{x})$ using Metropolis-Hastings algorithm

Step 3. Repeat Step 1 and Step 2 and obtain $(\alpha^{(t)}, \lambda^{(t)})$, $t = 1, 2, \dots, N$

Step 4. An approximate Bayes estimate of θ under SEL can be obtained as

$$\hat{g}_B(\alpha, \lambda) = \hat{\theta}_B = \frac{\frac{1}{N-M} \sum_{i=M+1}^N \theta_i h(\alpha_i, \lambda_i | \mathbf{x})}{\frac{1}{N-M} \sum_{i=M+1}^N h(\alpha_i, \lambda_i | \mathbf{x})} \quad (4.8)$$

where M is burn-in.

4.2 Prediction of future observations

Let $X_1 < X_2 < \dots < X_m$ and $Y_1 < Y_2 < \dots < Y_n$ represent a progressive first-failure censored sample of size m , and a future ordered sample of size n , respectively. It is further assumed that the two samples are independent and each of their corresponding random samples is drawn from the generalized exponential distribution, defined by (1.1). Two-sample prediction problem involves the prediction and associated inference of the order statistics $Y_1 < Y_2 < \dots < Y_n$ of a future sample from the same distribution function. In this section, we consider the Bayes prediction of the k th ordered observation in a future sample of size n . We also construct a symmetric $100(1 - \beta)\%$ prediction interval of Y_s .

Let us consider a future sample $Y_1 < Y_2 < \dots < Y_n$ of size n , independent of the informative sample $X_1 < X_2 < \dots < X_m$ and let $Y_1 < \dots < Y_s < \dots < Y_n$ be the order statistics of the future sample of size n . The probability density function of the s th order statistic of the future sample is of the form

$$z(y_s | \alpha, \lambda) = \frac{n!}{(s-1)!(n-s)!} [F(y_s | \alpha, \lambda)]^{s-1} [1 - F(y_s | \alpha, \lambda)]^{n-k} f(y_s | \alpha, \lambda) \quad (4.9)$$

where $f(\cdot|\alpha, \lambda)$ and $F(\cdot|\alpha, \lambda)$ are given in (1.1).

Substituting (1.1) in (4.9), the density function and the cumulative distribution function of y_s , for given α and λ is given by respectively

$$z(y_s|\alpha, \lambda) = s \binom{n}{s} \alpha \lambda e^{-\lambda y_s} \sum_{j=0}^{n-s} \binom{n-s}{j} (-1)^j (1 - e^{-\lambda y_s})^{\alpha(s+j)-1} \quad (4.10)$$

and

$$Z(y_s|\alpha, \lambda) = s \binom{n}{s} \sum_{j=0}^{n-s} \binom{n-s}{j} (-1)^j \frac{(1 - e^{-\lambda y_s})^{\alpha(s+j)}}{s+j}. \quad (4.11)$$

The Bayes predictive density function of Y_s is obtained by

$$z(y_s|\mathbf{x}) = \int_0^\infty \int_0^\infty z(y_s|\alpha, \lambda) \pi(\alpha, \lambda|\mathbf{x}) d\alpha d\lambda \quad (4.12)$$

where $\pi(\alpha, \lambda|\mathbf{x})$ is the joint posterior density function of α and λ given by (4.2). It can not be expressed in closed form and therefore, it can not be evaluated analytically.

Using MCMC sampling technique as described in previous section, the sample-based estimator of predictive density function, $z(y_s|\mathbf{x})$, and predictive distribution function, $Z(y_s|\mathbf{x})$, can be obtained as respectively

$$\hat{z}(y_s|\mathbf{x}) = \sum_{i=M+1}^N w_i z(y_s|\alpha_i, \lambda_i) \quad (4.13)$$

and

$$\hat{Z}(y_s|\mathbf{x}) = \sum_{i=M+1}^N w_i Z(y_s|\alpha_i, \lambda_i) \quad (4.14)$$

where

$$w_i = \frac{h(\alpha_i, \lambda_i|\mathbf{x})}{\sum_{i=M+1}^N h(\alpha_i, \lambda_i|\mathbf{x})}$$

and $\{(\alpha_i, \lambda_i), i = 1, 2, \dots, N\}$ are MCMC samples obtained from the posterior density function of α and λ .

Let us consider a symmetric $100\beta\%$ predictive interval for Y_s . L and U denote the lower and upper bounds. Then the predictive intervals for L and U can be obtained by solving the following nonlinear equations:

$$\begin{aligned} P(Y_s > L|\mathbf{x}) &= \frac{1 + \beta}{2} = 1 - Z(L|\mathbf{x}) \text{ and } Z(L|\mathbf{x}) = \frac{1 - \beta}{2} \\ P(Y_s > U|\mathbf{x}) &= \frac{1 - \beta}{2} = 1 - Z(U|\mathbf{x}) \text{ and } Z(U|\mathbf{x}) = \frac{1 + \beta}{2}. \end{aligned} \quad (4.15)$$

Since it is not possible to obtain the solutions analytically, we need to apply a suitable numerical method for solving these non-linear equations.

Table 1. Simulated data from generalized exponential distribution with $(\alpha, \lambda) = (3, 0.5)$.

1.2287	2.7280	0.5658	3.3657	2.6919
4.2204	2.3505	4.6242	1.0332	0.0997
7.6248	2.8022	1.9372	1.7934	3.8645
3.1840	2.6368	1.3942	6.1415	9.1586
6.2548	2.9094	10.504	2.6873	1.1539
2.0286	2.7172	3.0385	3.1776	3.7822
5.6766	3.0613	1.4214	3.2554	4.9095
11.468	3.2333	8.8425	1.9836	3.1056
4.0872	2.5488	7.0584	1.8420	5.4390
1.6604	1.3057	2.3309	6.3031	4.6612
3.5678	2.3555	1.6567	0.6786	2.8464
3.3109	6.6377	2.3886	1.1554	2.6389

5 Illustrative Example

To illustrate the estimation methods in the previous sections, a data set consisting of 60 items is generated from generalized exponential distribution with parameters $(\alpha, \lambda) = (3, 0.5)$. The generated data are given in Table 1.

We assume that the data are randomly grouped into 30 groups with $k = 2$ items within each group. The groups are as follows: $\{1.6604, 9.1586\}$, $\{0.0997, 2.6368\}$, $\{2.3505, 4.0872\}$, $\{3.5678, 3.1840\}$, $\{2.3555, 1.7934\}$, $\{2.728, 3.3109\}$, $\{3.2554, 5.439\}$, $\{2.3309, 10.504\}$, $\{4.6242, 6.3031\}$, $\{1.1539, 2.6873\}$, $\{1.2287, 6.1415\}$, $\{1.9372, 5.6766\}$, $\{0.5658, 1.3057\}$, $\{3.0385, 4.6612\}$, $\{1.6567, 6.2548\}$, $\{2.0286, 4.2204\}$, $\{2.5488, 2.8464\}$, $\{1.0332, 4.9095\}$, $\{3.7822, 8.8425\}$, $\{2.6919, 2.8022\}$, $\{0.6786, 1.4214\}$, $\{1.3945, 6.6377\}$, $\{3.1776, 7.6248\}$, $\{1.1554, 3.1056\}$, $\{3.0613, 1.9836\}$, $\{1.842, 2.9094\}$, $\{2.6389, 11.468\}$, $\{2.3886, 2.7172\}$, $\{3.3657, 7.0584\}$, $\{3.2333, 3.8645\}$. Suppose that the predetermined progressively first-failure censoring plan is applied using progressive censoring scheme $R = (2, 1, 1, 2, 0, 0, 2, 2, 0, 2, 0, 2, 0, 1, 0)$. The following progressively first-failure censored data of size $m = 15$ out of 30 groups are obtained as $(X_1, X_2, \dots, X_{15}) = (0.0997, 0.5658, 0.6786, 1.0332, 1.1539, 1.1554, 1.2287, 1.3057, 1.3942, 1.6567, 1.7934, 1.9372, 2.0286, 2.3505, 3.0613)$.

For this case, 15 groups are censored and 15 first-failure times are observed. Under the progressively first-failure censored sample, the MLE of α and λ are 2.555 and 0.549, respectively. The approximate 90% confidence intervals for α and λ are (1.248, 3.862) and (0.275, 0.822), respectively. Very small values are given to the prior parameters to reflect vague prior information. Therefore, we assume that $a = b = c = d = 0.1$. According to Algorithm 1, we generate 10,000 Markov Chain Monte Carlo samples and discarded the

Table 2. Two sample prediction intervals for the future order statistics.

Y_s	90% HPD credible intervals for Y_s
Y_1	(0.2960, 2.4582)
Y_2	(0.7866, 3.5306)
Y_3	(1.3070, 4.7789)
Y_4	(1.9268, 6.6105)
Y_5	(2.8341, 10.558)

first 5000 samples as 'burn-in'. The resulting Bayes estimates for α and λ are found to be $\hat{\alpha}_B=2.608$ and $\hat{\lambda}_B=0.544$, respectively. The Bayesian prediction intervals for future order statistics is given in Table 2.

Figure 1 shows the estimated predictive density function of Y_1 and Y_3 , respectively. It is seen that the estimated predictive distribution is unimodal and positively skewed.

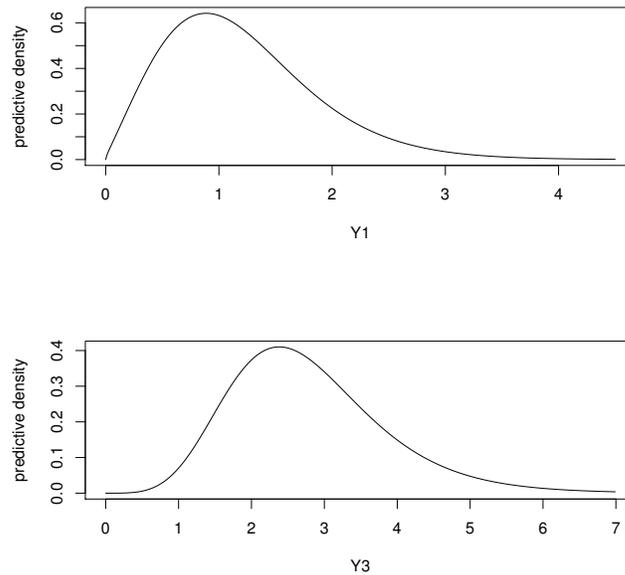


Figure 1: Estimated predictive density function of Y_1 and Y_3 .

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