Accelerated Life Testing and

Age-Replacement Policy under Warranty on

Exponentiated Pareto Distribution

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Abstract

In This Paper, we focus on estimating the expected total cost and expected cost rate for age replacement under warranty policy. We describe how to use design of Accelerated life testing (ALT) plans by assuming constant stress and life times are independently distributed as Exponentiated Pareto Distribution (EP) for predicting the cost of age replacement under warranty policy. This problem is studied when maximum likelihood estimators (MLE) are obtained. Also, the Age Replacement Policy under Pro- rate Rebate Warranty is displayed. Finally, numerical examples are carried out to illustrate theoretical results.

Keywords: Accelerated Life Testing, Age- Replacement policy, Warranty, Exponentiated Pareto Distribution

1- Introduction

Manufacturers need to collect claims data for using it in prediction of future claims and analysis the cost of production. Due to competition and marketing strategies, producers present warranty for their products. Therefore, one of the main problem face any manufacturer is the estimation of the life cycle cost of their components or products. This paper describes how to use design of ALT plans for predicting the cost of age replacement under warranty policy.
Accelerated life testing (ALT) is a widely used approach for reliability and prediction of components or systems’ reliabilities at normal operating conditions using data obtained at accelerated condition. The main reasons for needing ALT are the long life times of today's products, the small time period between design and release, and the challenge of testing products that are used continuously under normal conditions. ALT is used to provide quickly the information about the life distributions of products. ALT is conducted by one of two approaches, the first, accelerated failure time, which means ALT is conducted for the product or component by experiencing at normal conditions but more intensively than normal, and this is suitable for products or components that are used on continuous time basis such as toasters, light bulbs, heaters, and tires. The second is the accelerated stress, which means ALT is conducted by using the product or component a stress level higher than the normal use stress.

Also, there are researches combining ALT and Warranty models such as: Guangbin Yang, (2010), described a method for predicting the warranty cost, and its confidence interval for a product population that will experience varying stress levels using the accelerated life tests conducted in the early stages of the product life cycle.

The main aim of this paper is estimating the expected total cost and expected cost rate for age replacement under warranty policy. Assuming constant stress and life times are independently distributed as EP. This problem is studied when the constant stress of the components are independent identical distributed. Maximum likelihood estimator (MLE) are obtained. Also, the estimating the expected total cost and expected cost rate for age replacement under pro-rate rebate warranty policy.

The rest of the paper is organized as follows. In Section 2, Accelerated Life test plan on Exponentaited Pareto distribution are discussed. In Section 3, Maximum Likelihood Estimation for EP Parameters are done. In Section 4, The Age Replacement Policy under Pro- rate Rebate Warranty is displayed. Numerical examples are carried out to illustrate theoretical results in Section 5.

Notation:

\[ \theta, \alpha \] Parameters of Exponentiated Pareto Distribution

\[ C, P \] Parameters of power rule model

\[ w \] Warranty period

\[ \tau \] Age of replacement of the product
Cd Down cost for each failure of a product
Cp Purchasing cost for product
\( C(\tau) \) Cycle cost when age of replacement is \( \tau \geq w \geq 0 \)
\( R(t) \) Refund proportion during the warranty period
\( CR(\tau) \) The expected cost rate

2- The ALT Plan on Exponentiated Pareto Distribution

When designing an ALT plan, we need to determine: the stress types to be used in the experiment, the stress applications method, the stress levels for each stress type selected, and the proportion of test units to be allocated to each stress level.

There several studies dealt with constant stress such as: Abdel-Ghaly et al. (1998), El-Dessouky (2001), Attia et al., (2011), and Attia et al., (2013).

In this section, we Suppose that there are \( K \) levels of high stresses \( V_j, j = 1, 2, \ldots, K \) and assume that \( V_u \) is the normal use condition where \( V_u < V_1 < V_2 < \ldots < V_K \) and there are \( n_j \) items put on test at each stress level \( V_j, j = 1, 2, \ldots, K \). When complete sample is adopted at each stress level, the experiment terminates once the all number of items \( n_j \) failures. The life time at stress \( V_j, t_{ij}, I = 1, 2, \ldots, n_j \) and \( j = 1, 2, \ldots, K \) is assumed to have Exponentiated Pareto distribution with density function given by:

\[
f(t_{ij}; \alpha_j, \theta) = \alpha_j \theta \left[ (1 + t_{ij})^\theta - 1 \right]^\theta \left( 1 + t_{ij} \right)^{-(\alpha_j + 1)} \quad t_{ij} > 0, \alpha_j, \theta > 0 \tag{1}\]

The cumulative function is:

\[
F(t_{ij}; \alpha_j, \theta) = \left[ 1 - (1 + t_{ij})^{-\alpha_j} \right]^\theta \quad t_{ij} > 0, \alpha_j, \theta > 0 \tag{2}\]

The Reliability function is:

\[
R(t_{ij}; \alpha_j, \theta) = 1 - \left[ 1 - (1 + t_{ij})^{-\alpha_j} \right]^\theta \tag{3}\]

Recently, exponentiated Pareto distribution has been discussed by many authors such as, Shawky and Abu-Zinah (2009), considered the maximum likelihood estimation of the different parameters of an exponentiated Pareto distribution and five other estimation procedures and compare their performances through numerical simulations. Masoom and Jungsoo (2010), obtain the MLE of the tail-probability of the exponentiated Pareto distribution and considered MLE of reliability in two independent exponentiated Pareto distributions. Afify W.M, (2010), presented Bayes and classical estimators for two parameters exponentiated
Pareto distribution when sample is available from complete, type I and type II
censoring scheme. Mahmoud et al. (2012), the best linear unbiased and the
maximum Likelihood estimation of Exponentiated Pareto distribution based on
progressively Type-II right censored order statistics are derived. A comparison
between the two methods of estimation are made using simulation study. Hassan
and Basheikh (2012), dealt with the Bayesian and non-Bayesian estimation of
reliability of an -out-of- system with identical component strengths which are
subjected to a common stress. Assuming that both stress and strength are assumed
to have an exponentiated Pareto distribution with known and unequal shape
parameters.

It is assumed that the stress $V_j$ affects only the shape parameter of the
Exponentiated Pareto distribution, $\alpha_j$ through a certain acceleration function
considered to be the power rule model that takes the following form:

$$\alpha_j = cS_j^{\gamma_p} \quad c > 0, p > 0 \text{ and } S_j = V_j/V^* \quad j=1,2,\ldots,k$$  \hspace{1cm} (4)

Where

$$V^* = \prod_{j=1}^{k} V_j^{b_j} \quad \text{and} \quad b_j = n_j/\sum_{j=1}^{k} n_j$$  \hspace{1cm} (5)

3- Maximum Likelihood Estimation for EP Parameters

We now develop the likelihood function of an observation $t$ (time to failure) at a
single stress level $V_j$ as follows:

The likelihood function of the $k$ independent samples is given by:

$$L(C, p, \theta; t_{ij}) = \prod_{j=1}^{k} \prod_{i=1}^{n_j} cS_j^{-\gamma_p} \theta \left[ -\left(1 + t_{ij}ight)^{-cS_j^{\gamma_p}} \left(1 + t_{ij}ight)^{-cS_j^{\gamma_p} + 1} \right]$$  \hspace{1cm} (6)

By taking the logarithm of both sides,

$$\ln L(C, p, \theta; t_{ij}) = \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[ \ln \frac{c - p \ln S_j + \ln \theta + (\theta - 1) \ln \left[ 1 - \left(1 + t_{ij}\right)^{-cS_j^{\gamma_p}} \right] - (cS_j^{\gamma_p} + 1) \ln \left(1 + t_{ij}\right) }{1 - \left(1 + t_{ij}\right)^{-cS_j^{\gamma_p}}} \right]$$  \hspace{1cm} (7)

So, we need to estimate the three parameters $c, p$ and $\theta$. The first derivatives of
the logarithm of likelihood function (7) with respect to $c, p$ and $\theta$ are given by:

$$\frac{\partial \ln L}{\partial c} = \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[ c - p \ln S_j + \ln \theta + (\theta - 1) \ln \left[ 1 - \left(1 + t_{ij}\right)^{-cS_j^{\gamma_p}} \right] - (cS_j^{\gamma_p} + 1) \ln \left(1 + t_{ij}\right) \right]$$  \hspace{1cm} (8)
\[
\frac{\partial \ln L}{\partial \rho} = \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[ \ln S_j - \frac{(\theta - 1)(1 + t_y)^{-cs_{ij}^\rho} \ln S_j \ln(1 + t_y)}{1 - (1 + t_y)^{-cs_{ij}^\rho}} + cs_{ij}^\rho \ln S_j \ln(1 + t_y) \right]
\]

(9)

and

\[
\frac{\partial \ln L}{\partial \theta} = \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[ \frac{1}{\theta} \ln\left[ 1 - (1 + t_y)^{-cs_{ij}^\rho} \right] \right]
\]

(10)

The maximum likelihood estimators of \( c, \rho \) and \( \theta \) could be obtained by equating the equations (8), (9) and (10) by zero and solving them simultaneously using an iterative technique. We obtain the approximate variance covariance matrix by replacing expected values by their maximum likelihood estimators and inverting the fisher – information matrix, defined by:

\[
\mathbf{I} = - \begin{bmatrix}
\frac{\partial^2 \ln L}{\partial c^2} & \frac{\partial^2 \ln L}{\partial c \partial \rho} & \frac{\partial^2 \ln L}{\partial c \partial \theta} \\
\frac{\partial^2 \ln L}{\partial \rho \partial c} & \frac{\partial^2 \ln L}{\partial \rho^2} & \frac{\partial^2 \ln L}{\partial \rho \partial \theta} \\
\frac{\partial^2 \ln L}{\partial c \partial \theta} & \frac{\partial^2 \ln L}{\partial \rho \partial \theta} & \frac{\partial^2 \ln L}{\partial \theta^2}
\end{bmatrix}
\]

(11)

where the second derivatives of the logarithm of likelihood function defined in equation (7) are given as follows:

\[
\frac{\partial^2 \ln L}{\partial c^2} = \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[ \frac{1}{c^2} - \frac{(\theta - 1)(1 + t_y)^{-cs_{ij}^\rho} \ln S_j \ln(1 + t_y)}{1 - (1 + t_y)^{-cs_{ij}^\rho}} \right]
\]

(12)

\[
\frac{\partial^3 \ln L}{\partial c \partial \rho} = \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[ \frac{(\theta - 1)(1 + t_y)^{-cs_{ij}^\rho} \ln S_j \ln(1 + t_y)}{1 - (1 + t_y)^{-cs_{ij}^\rho}} \right]
\]

(13)

\[
\frac{\partial^3 \ln L}{\partial c \partial \theta} = \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[ \frac{(1 + t_y)^{-cs_{ij}^\rho} \ln(1 + t_y)}{1 - (1 + t_y)^{-cs_{ij}^\rho}} \right]
\]

(14)

\[
\frac{\partial^2 \ln L}{\partial \rho^2} = \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[ - \frac{(\theta - 1)(1 + t_y)^{-cs_{ij}^\rho} \ln S_j \ln(1 + t_y)}{1 - (1 + t_y)^{-cs_{ij}^\rho}} \right]
\]

(15)

\[
\frac{\partial^2 \ln L}{\partial \rho \partial \theta} = \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[ \frac{(1 + t_y)^{-cs_{ij}^\rho} \ln(1 + t_y)}{1 - (1 + t_y)^{-cs_{ij}^\rho}} \right]
\]

(16)

and
\[
\frac{\partial^2 \ln L}{\partial \theta^2} = \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ \frac{-1}{\theta^2} \right]
\]  

(17)

Therefore the MLE \( \hat{c}, \hat{p} \) and \( \hat{\theta} \) have an asymptotic variance-covariance matrix obtained by inverting the Fisher-information matrix defined in (11).

4- The Age Replacement Policy under Pro-rate Rebate Warranty

Under this policy, a non-repairable product is replaced at a certain time age (\( \tau \)) or upon failure, which occurs first. When the product fails at \( t \leq \tau \) a failure replacement are performed with a downtime cost \( C_D > 0 \) and purchasing cost \( C_P > 0 \). If the product fails during the warranty period (\( w \)), the customer is refunded a proportion of the sales price \( C_P \). Where the rebate function under the pro-rata warranty is:

\[
R(t) = \begin{cases} 
C_P(1 - \frac{t}{w}) & 0 \leq t \leq w \\
0 & t > w 
\end{cases}
\]

(18)

The age-replacement policy is proposed by many researchers. Recently, Chien and Chen (2007a), considered an age-replacement model with minimal repair based on a cumulative repair cost limit and random lead time for replacement delivery. A general cost model is developed for the average cost per unit time based on the stochastic behavior of the assumed system, reflecting the costs of both storing a spare and of system downtime. The minimum-cost policy time is derived, its existence and uniqueness is shown, and structural properties are presented. Chien and Chen (2007b), the effects of a renewing free-replacement warranty (RFRW) on the age replacement policy for a repairable product with a general failure model are discussed. Huang et al. (2008), dealt with the problem of estimating the expected warranty cost for the case where the item usage is intermittent and of heterogeneous usage intensity over the product life cycle when sales occur continuously. The failure of the item is dependent on the number of times, the duration the unit has been used and the usage intensity. Also, the product sales depend on product price and design quality. They considered repairable and nonrepairable items and obtain results for the free-replacement warranty (FRW) and Pro-rata Warranty (PRW) policy.

Chien (2010), presented the effects of salvage value on the optimal age-replacement policy for non-repairable products sold with a pro rata rebate warranty (PRRW). Cost models from the customer’s perspective are developed for both warranted, and non warranted products. The corresponding optimal replacement ages are derived such that the long-run expected cost rate is minimized. Under the increasing failure rate assumption, the existence and uniqueness of the optimal age for preventive replacement are shown, and the impacts of a PRRW on the optimal age-replacement policy are investigated analytically. Chien at al. (2014), presented the effects of salvage value on the optimal
Accelerated life testing and age-replacement policy under warranty

age-replacement policy for non-repairable products sold with a renewing free-replacement warranty (RFRW). Na and Sheng (2014), focused on analyzing the impact of warranty periods on the optimal age-replacement from the consumers’ perspectives. First they constructed the mathematical formulations for age-replacement model. After optimizing they found there exists a unique optimal replacement age based the long-run expected cost rate is minimized.

The main assumptions:
- Product is replaced at time of failure (corrective replacement), or at age $\tau$ (preventive replacement), which occurs first.
- Products are sold with pro-rata rebate warranty policy.
- There is no salvage value for preventive replaced product.
- The age of replacement ($\tau$) is greater than warranty period ($w$).

When the product age reaches $\tau$, then the preventive replacement carried out with cost $C_p$ only because it is a planned preventive maintenance action.

The total cost incurred in a renewal cycle for this policy is:

$$C(t) = \begin{cases} C_d + C_p - R(t) & 0 \leq t \leq w \\ C_d + C_p & w < t < \tau \\ C_p & t \geq \tau \end{cases} \quad (19)$$

Therefore, the expected total cost under this policy as Chein (2010) and Chein et al. (2014) is:

$$E(C(t)) = C_d \int_0^w F(u)du + C_p \int_w^\tau \frac{F(u)}{w}du$$ \quad (20)

And, the expected cost rate is

$$E(CR(t)) = \frac{E(C(t))}{\int_0^\tau F(u)du}$$ \quad (21)

Where $\int_0^\tau F(u)du$ is the expected cycle time which donated by $E(T(\tau))$.

**Under Exponentiated Pareto distribution:**

The expected total cost can be gotten by substituting in the equation (20) as:

$$F(\tau) = [1 - (1 + \tau)^{-\alpha}]^\theta \quad \tau > 0, \alpha, \theta > 0$$

$$\int_0^\tau F(u)du = w - IB(\psi_w, \theta + 1, -1/\alpha) \quad (22)$$

Where

$IB$ is donated to incomplete Beta function with parameters $(\psi_w, \theta + 1, -1/\alpha)$.
1764

Eman A. El-Dessouky

and \[ \psi_w = 1 - (1 + w)^{-\alpha} \] (23)

Also, we can get the expected cost rate by substituting in equation (21) as:

\[ \int_0^\tau F(u)\,du = \tau - \frac{IB(\psi', \theta + 1, -1/\alpha)}{\alpha} \] (24)

Where \[ \psi' = 1 - (1 + \tau)^{-\alpha} \]

5- Numerical Results

In this section, we will present a numerical investigation of the maximum likelihood estimation for the parameters of Exponentiated Pareto distribution \((\alpha, \theta)\). Where \(\alpha\) is is the shape parameter which is affected by the stress by using the power rule model defined by equation (4). We estimate the three parameters \(c, p\) and \(\theta\).

5.1 Estimation of the Three Parameters Using Maximum Likelihood Estimation

We need to estimate the three parameters \(c, p\) and \(\theta\) by using the maximum likelihood method. So, we will need to solve the three non-linear equations of logarithm likelihood function (8), (9) and (10) simultaneously using Newton-Raphson method. The iterative technique, can be applied as follows:

\[ x_{m+1} = x_m - B_m^{-1} A_m \]

where

\[ x_{m+1} = \begin{bmatrix} \hat{c}_{m+1} \\ \hat{p}_{m+1} \\ \hat{\theta}_{m+1} \end{bmatrix}, \quad x_m = \begin{bmatrix} \hat{c}_m \\ \hat{p}_m \\ \hat{\theta}_m \end{bmatrix}, \quad A_m = \begin{bmatrix} \frac{\partial \ln L}{\partial c_m} \\ \frac{\partial \ln L}{\partial p_m} \\ \frac{\partial \ln L}{\partial \theta_m} \end{bmatrix} \]

and

\[ B_m = \begin{bmatrix} \frac{\partial^2 \ln L}{\partial c_m^2} & \frac{\partial^2 \ln L}{\partial c_m \partial p_m} & \frac{\partial^2 \ln L}{\partial c_m \partial \theta_m} \\ \frac{\partial^2 \ln L}{\partial p_m \partial c_m} & \frac{\partial^2 \ln L}{\partial p_m^2} & \frac{\partial^2 \ln L}{\partial p_m \partial \theta_m} \\ \frac{\partial^2 \ln L}{\partial \theta_m \partial c_m} & \frac{\partial^2 \ln L}{\partial \theta_m \partial p_m} & \frac{\partial^2 \ln L}{\partial \theta_m^2} \end{bmatrix}. \]
Accelerated life testing and age-replacement policy under warranty

Assuming initial values for each of \(c, p\) and \(\theta\), the Newton-Raphson iterative procedure is continued until either the number of iterations will be (100) or when \(|X_m - X_{m+1}| < 5 \times 10^{-5}\). We carry out this experiment 500 times. The unknown parameter \(c, p\) and \(\theta\) is estimated by the mean of its estimates at 500 times. After the estimates of \(c, p\) and \(\theta\) are obtained, we can substitute their values in equation (4), to get the estimates of the shape parameter of the life time distribution under the three levels of the stress \(V_1, V_2\) and \(V_3\) defined by \(\hat{\alpha}_1, \hat{\alpha}_2, \text{and} \hat{\alpha}_3\), respectively. We estimate the shape parameter \(\hat{\alpha}_u\) under usual stress \(V_u = 0.5\) at different values of \(c_0\) and \(p_0\). Also, we estimate the reliability function at different values of mission time \((t_0)\), where

\[
\hat{R}_u(t_0) = 1 - \left[1 - (1 + t_0)^{-\alpha_u}\right]^{c_u}
\]

It is assumed that \(K = 3\), i.e. there are only three different levels of stresses \(V_1 = 1, V_2 = 1.5\) and \(V_3 = 2\). At each stress \(V_j, j = 1, 2, 3\), three random samples \(t_{ij}\) \(i = 1, 2, \ldots n_j\) and \(j = 1, 2, 3\) of size \(n_1 = 150, n_2 = 100\) and \(n_3 = 50\) are generated from Exponentiated Pareto distribution.

In the following table, the estimates of unknown parameters and the shape parameter at different levels of stress are shown, the relative bias which is the absolute difference between the estimated parameter and its true value divided by its true value.

And the mean square error (MSE) which is the mean square of the difference between the estimated parameter are presented for all the estimated parameters considering different initial points of the parameters.

<table>
<thead>
<tr>
<th>(\theta_0)</th>
<th>(c_0)</th>
<th>(p_0)</th>
<th>Parameter</th>
<th>Estimator</th>
<th>Relative Bias</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.3</td>
<td>0.8</td>
<td>(C)</td>
<td>0.326704</td>
<td>0.08901338</td>
<td>0.00713104</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(p)</td>
<td>0.8339062</td>
<td>0.04238278</td>
<td>0.001149632</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\theta)</td>
<td>2.129249</td>
<td>0.0642443</td>
<td>0.01670527</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\alpha_1)</td>
<td>0.4026622</td>
<td>0.08950944</td>
<td>0.00129903</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\alpha_2)</td>
<td>0.2871423</td>
<td>0.0769057</td>
<td>0.00487654</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\alpha_3)</td>
<td>0.225868</td>
<td>0.06785765</td>
<td>0.000234972</td>
</tr>
</tbody>
</table>
In Table (1), we can notice that the absolute value of the difference between the true value of the parameter and its estimator is small positive value converges to zero, so these estimators are said to be consistent estimators.

### 5.2 Estimation the shape parameter and the reliability function at normal stress

We estimate the shape parameter at normal stress $V_u=0.5$ and the reliability function at the same normal stress for different values of $c$, $p$, $\theta$ and $t_0$.

<table>
<thead>
<tr>
<th>$c$</th>
<th>$p$</th>
<th>$\theta$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$C_p$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$C_p$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.8</td>
<td>0.5445068</td>
<td>0.00001980854</td>
<td>0.00114962</td>
<td>0.01670533</td>
<td>0.003608431</td>
<td>0.001354602</td>
<td>0.000652706</td>
<td>0.5427257</td>
<td>0.08545136</td>
<td>0.001825484</td>
<td>0.001422552</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>1</td>
<td>0.5427257</td>
<td>0.00001980854</td>
<td>0.00114962</td>
<td>0.01670533</td>
<td>0.003608431</td>
<td>0.001354602</td>
<td>0.000652706</td>
<td>0.5427257</td>
<td>0.08545136</td>
<td>0.001825484</td>
<td>0.001422552</td>
</tr>
<tr>
<td>3.5</td>
<td>0.6</td>
<td>1.1</td>
<td>0.6505048</td>
<td>0.00001980854</td>
<td>0.00114962</td>
<td>0.01670533</td>
<td>0.003608431</td>
<td>0.001354602</td>
<td>0.000652706</td>
<td>0.6505048</td>
<td>0.0841746</td>
<td>0.002550735</td>
<td>0.00149354</td>
</tr>
</tbody>
</table>
In Table (2), we estimate the reliability function at different values of mission time \( t_0 \) and \( \theta_0 \). We can notice that the reliability function decreases as the mission time \( t_0 \) increases and that there is an inverse proportional relationship between the shape parameter \( \alpha_0 \) and the reliability function at the same mission time \( t_0 \). Also, the reliability function decreases as the values of \( c_0 \) and \( p_0 \) increases at the same mission time \( t_0 \).

### 5.3 Estimation the expected total cost and expected cost rate for Age-Replacement under Warranty policy

We estimate the expected total cost \( E(C(\tau)) \), the expected cycle time \( E(T(\tau)) \) and the expected cost rate \( CR(\tau) \) for Age- Replacement under warranty policy on EP distribution. Using equations (20), (21), (22), (23),and (24) at a failure replacement are performed with a downtime cost \( Cd = 50 \) and purchasing cost \( Cp = 1000 \) for different values of warranty periods \( w \), the parameters of EP distribution \( (\alpha) \) and \( (\theta) \) at normal use.
Table (3)
The expected total cost, the expected cycle time and the expected cost rate for Age- Replacement under warranty policy on EP distribution

<table>
<thead>
<tr>
<th>α</th>
<th>θ</th>
<th>w</th>
<th>τ</th>
<th>E(C(τ))</th>
<th>E(T(τ))</th>
<th>CR(τ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>2</td>
<td>5</td>
<td>7</td>
<td>955.8676</td>
<td>6.523962</td>
<td>146.5164</td>
</tr>
<tr>
<td>0.2</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>954.6145</td>
<td>4.750396</td>
<td>200.9547</td>
</tr>
<tr>
<td>0.3</td>
<td>2</td>
<td>5</td>
<td>7</td>
<td>919.478</td>
<td>6.139208</td>
<td>149.7681</td>
</tr>
<tr>
<td>0.3</td>
<td>2</td>
<td>5</td>
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From the above table, we find:
- There is inverse relationship among the value of parameter (α) and the expected total cost and expected time cycle.
- There is direct relationship between value of parameter (α) and the expected cost rate.
- The value of Parameter (θ) is directly relationship with the expected total cost and expected time cycle and inverse relationship with expected cost rate.
- Increasing the warranty period (W) leads to decreasing in expected total cost and expected cost rate but doesn't effect on expected life cycle.
- There is inverse relationship between the age of replacement (τ) and the expected cost rate, while there are direct relationships with the expected total cost and expected time cycle.

References


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