Optimal Estimation of the States of Modulated
Semi-synchronous Integrated Flow of Events
in Condition of its Incomplete Observability

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Abstract

This paper is focused on the problem of optimal estimation of the states (or the problem of filtering the intensity process) of the modulated semi-synchronous integrated flow of events, which is one of the mathematical models for an incoming streams of events (claims) in computer communication networks and which is related to the class of Markovian arrival processes (MAPs). The flow is considered in presence of a constant dead time. The algorithm for optimal estimation of the flow states is proposed. The decision about the flow state is made according to criterion of a posteriori probability maximum. This criterion allows to find the most complete characteristic of the flow state, which could be received from observations of the flow. The criterion minimizes the total probability of making a wrong decision. Also simulation experiments and numerical results are presented.

Mathematics Subject Classification: 60G55
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1 Introduction

Due to the rapid evolution of computing and information technology during the last several decades, a new sphere of queueing theory applications – design and development of computer, telecommunication, peer-to-peer and other networks – has appeared. The use of mathematical methods developed in a queueing theory allows to find the quality characteristics of network components operation for various problems, such as estimation of probabilistic characteristics of switching and routing nodes, analysis of nodes buffer storage, the local and global flows management and so on.

It is worthwhile to note that the conditions of the real objects and systems operation are such that we can assert that the servers parameters are known and stable as time goes, but we can not tell this about the intensity processes of the input streams of events that come to the servers. Moreover, the intensities of the input streams usually vary within time, and frequently their changes are accidental. As a result, it is necessary to consider the mathematical models of doubly stochastic Poisson processes (DSPPs), which are characterized by having the number of events in any given time interval as being Poisson distributed, conditionally to another positive stochastic process called intensity [6, 8, 19, 20].

There are two known classes of doubly stochastic flows of events. The first class contains the flows of events, which intensity process is a continuous random process [8, 19]. The second class contains flows, which intensity is a piecewise constant stationary random process with a finite number of states. These flows are typical for telecommunication networks. The flows of the second type were considered for the first time and independently presented by Basharin, Kokotushkin and Naumov [4] and Neuts [21]. Basharin et al. [4] named these flows as Markov chain (MC) arrival processes; Neuts [21] – as Markov Versatile arrival processes (MVP). Since the early 1990s to date, these flows of events are called as doubly stochastic flows of events or MAP-flows, or MC-flows [10, 23, 26].

As has been mentioned above, in the real situations the intensity process of the input streams of events may vary in time in a random way and it is typically unobservable. In such situations, the use of adaptive queueing systems, when the unknown stream parameters or states are estimated during the system operation and the service procedure is changed correspondingly, seems to be more rational. That is why, the central problems faced when modeling these
processes are: 1) flow states estimation on monitoring the time moments of the events occurrence (filtering of underlying and unobservable intensity process) [14]; 2) flow parameters estimation on monitoring the time moments of the events occurrence [15].

In the recent literature, the problem of estimating the intensity process from observations of doubly stochastic Poisson processes (DSPPs) has been of a great interest, since DSPPs have found applications in many fields such as network theory, peer-to-peer streaming networks and adaptive data streaming, optical communication systems, statistical modeling, quantitative finance, spatial epidemiology, etc. [1, 5, 7, 9, 17, 25]. In the literature, various filtering techniques have been proposed for the models based on DSPPs. Among them functional data analysis (FDA) approach [5], a Monte Carlo approach [7], the Kalman filter [25] and its modifications [11], a principal component analysis (PCA) approach [2, 12, 13, 24] and etc.

This paper is focused on the problem of optimal estimation of the states (or the problem of filtering the intensity process) of the modulated semi-synchronous integrated flow of events, which is related to the class of Markovian arrival processes (MAPs). We propose an algorithm for optimal estimation of the flow states, when decision about the flow state is made according to criterion of a posteriori probability maximum. This criterion allows to find the most complete characteristic of the flow state, which could be received from observations of the flow. The criterion minimizes the total probability of making a wrong decision [18]. Also the flow is considered in condition of its incomplete observability, when the dead time period of a constant duration $T$ is generated after every registered event. This negatively affects the quality of estimation.

It is worth noting, that in most of the cases researchers consider the mathematical models of the flows, where the moments of flow events occurrence are observable. In practice, however, any recording device (server in this context) spends some finite time for event measurement and registration, during which server can not handle the next event correctly. In other words, every event registered by a server causes the period which is called the period of a dead time [22]. During this period no other events are observed (they are lost). We may suppose that this period has a fixed duration (constant dead time). Particularly, we may find examples of this mathematical model in the real computer networks using CSMA/CD (Carrier Sense Multiple Access with Collision Detection) protocol. At the moment of a conflict recording at the in-port of a network node, a jam signal is transmitted across the network. During the signal transmission, calls coming to a node of the network are declined and sent to a source of repeated calls. Here time, during which the network node is closed for calls serving after a conflict recording, can be interpreted as a dead time of a server, which registers the conflict in the network nodes.
The rest of the paper is organized as follows. In Section 2 we present the modulated semi-synchronous integrated flow of events, which provides our modeling framework. Section 3 contains our main contribution, a filtering method based on the algorithm of optimal estimation of the flow states. Here we derive the formulas for a posteriori probability calculation in case of an absence of dead time (Section 3.1) and in case of a constant dead time (Section 3.2). And finally, in Section 4 we present simulation experiments and numerical results.

2 Problem statement

In this paper we consider the modulated semi-synchronous integrated flow of events (further flow of events) [3], which intensity process is a piecewise constant stationary random process \( \lambda(t) \) with two states 1, 2 (first, second correspondingly). In the state 1 \( \lambda(t) = \lambda_1 \) and in the state 2 \( \lambda(t) = \lambda_2 \) (\( \lambda_1 > \lambda_2 \)). The duration of the process \( \lambda(t) \) staying in the first (second) state is distributed according to the exponential law with parameter \( \beta(\alpha) \). If at the time moment \( t \) the process is found in the first (second) state, then at the interval \( [t, t + \Delta t) \), where \( \Delta t \) (hereinafter) is sufficiently small, with probability \( \beta \Delta t + o(\Delta t) \left( \alpha \Delta t + o(\Delta t) \right) \) the sojourn time of the process \( \lambda(t) \) in the first (second) state comes to the end and the process \( \lambda(t) \) transits to the second (first) state. During the time interval when \( \lambda(t) = \lambda_i \), a Poisson flow of events with intensity \( \lambda_i \), \( i = 1, 2 \), arrives. Also at any moment of an event occurrence in state 1 of the process \( \lambda(t) \), the process can change its state to state 2 with probability \( p \) (0 \( \leq p \leq 1 \)) or continue to stay in state 1 with complementary probability \( 1 - p \). I.e., after an event occurrence the process \( \lambda(t) \) can change or not change its state from state 1 to state 2. The transition of the process \( \lambda(t) \) from state 2 to state 1 at the moment of an event occurring in the second state is impossible. At the moment when the process state changes from the second to the first state, an additional event in state 1 is assumed to be initiated with probability \( \delta \) (0 \( \leq \delta \leq 1 \)). I.e., first the transition from state 2 to state 1 is made and thereafter an additional event is initiated or not. Such flows with additional events initiation are called integrated flows. Under the made assumptions we can assert that \( \lambda(t) \) is a Markovian process. So the flow can be characterized by \( \{D_0, D_1\} \), in terms of the rate matrices,

\[
D_0 = \begin{pmatrix} -(\lambda_1 + \beta) & \beta \\ (1 - \delta)\alpha & -(\lambda_2 + \alpha) \end{pmatrix}, \quad D_1 = \begin{pmatrix} (1 - p)\lambda_1 & p\lambda_1 \\ \delta\alpha & \lambda_2 \end{pmatrix}. \tag{1}
\]

Intensities of the process \( \lambda(t) \) transitions from state to state without the event occurrence fill in the matrix \( D_0 \) in (1). Intensities of the process \( \lambda(t) \) transitions from state to state with the event occurrence fill in the matrix \( D_1 \).
Optimal estimation of the flow states

in (1). Diagonal elements of the matrix $D_0$ are intensities of the process $\lambda(t)$ output from its states taken with the opposite signs.

It should be mentioned that, if the process $\lambda(t)$ sojourns in the first state, we assume that the event, occurring in a Poisson process in state 1, goes before the transition of the process $\lambda(t)$ from state to state. If the process $\lambda(t)$ sojourns in the second state, we assume that the transition of the process $\lambda(t)$ from state 2 to state 1 precedes the moment when additional event occurs or does not occur in state 1. All mentioned above details are not essential for analytical results derivation since the moments of the event arrival or transition of the process $\lambda(t)$ from state to state occur simultaneously. But, to obtain the numerical results during simulation procedure we should take the mentioned details into account and fix, what occurs first, the event or transition of the process $\lambda(t)$ from one state into another.

The registration of the flow events is considered in condition of a constant dead time (of incomplete observability). The dead time period of a constant duration $T$ begins after every registered at the moment $t_k$, $k \geq 1$, event. During this period, no other events are observed. When the dead time period is over, the first coming event causes the next interval of dead time of duration $T$ and so on. Fig. 1 shows the possible variant of the flow operation and observation. Here 1, 2 are the states of the process $\lambda(t)$; additional events, that may occur in the first state at the moment of the process $\lambda(t)$ transition from state 2 to state 1, are marked with letter $\delta$; dead time periods of duration $T$ are marked with hatching; unobserved events are displayed as black circles, observed events $t_1, t_2, ..., t_m$ are shown as white circles.

The process $\lambda(t)$ and possible events (events of Poisson flows with intensity $\lambda_i$, $i = 1, 2$, and additional events) are basically unobservable. We register only time moments $t_1, t_2, ..., t_m$ of observable events occurring. Using this information, we should estimate the process $\lambda(t)$ state at the end time of the period of observations. The process $\lambda(t)$ is considered in a steady-state conditions. That is why we neglect transient processes at the interval of observations $(t_0, t)$, where $t_0$ is an instant of beginning the observations, $t$ is an instant of ending the observations (the moment of a decision making). In a steady-state conditions we may take $t_0 = 0$. To have a possibility to make a decision regarding to the state of the process $\lambda(t)$ at the time moment $t$, we derived an explicit formula for a posteriori probability $w(\lambda_i|t) = w(\lambda_i|t_1, ..., t_m, t) = P(\lambda(t) = \lambda_i|t_1, ..., t_m, t)$, $i = 1, 2$ ($m$ is the number of observed events during the time period of duration $t$). Obviously $w(\lambda_1|t) + w(\lambda_2|t) = 1$. At any time moment $t$ the decision about the process $\lambda(t)$ state is made according to criterion of a posteriori probability maximum: if $w(\lambda_j|t) \geq w(\lambda_i|t)$, $i, j = 1, 2$, $i \neq j$, then estimation is $\hat{\lambda}(t) = \lambda_j$.
3 The algorithm of optimal estimation of the flow states

The moment of a decision making \( t \) belongs to some interval \((t_k, t_{k+1})\), \( k = 1, 2, \ldots \), between neighboring events of the observable flow. Consider the interval \((t_k, t_{k+1})\), which length is \( \tau_k = t_{k+1} - t_k \), \( k = 0, 1, \ldots \). The event registered at time \( t_k \) causes the beginning of a dead time period of duration \( T \), so the value \( \tau_k \) can be written as \( \tau_k = T + \eta_k \), where \( \eta_k \) is a duration of interval between the end of the dead time period \( t_k + T \) and the moment \( t_{k+1} \), i.e. interval \((t_k, t_{k+1})\) can be divided into two neighboring intervals: half-interval of the dead time \((t_k, t_k + T]\) and \((t_k + T, t_{k+1})\). Algorithms for a posteriori probability \( w(\lambda_1|t) \) calculation at the half-interval \((t_k, t_k + T]\) and interval \((t_k + T, t_{k+1})\) are fundamentally different. Besides, to find the probability \( w(\lambda_1|t) \) we must know exactly the value \( T \) or we should previously estimate the duration of dead time period. Otherwise, without this information we can not find the value of a posteriori probability \( w(\lambda_1|t) \). Here we propose that the value \( T \) is exactly known.

3.1 The expressions for a posteriori probability \( w(\lambda_1|t) \) in case of an absence of dead time \((T = 0)\)

Let us consider first the case of an absence of dead time \((T = 0)\). In this case the third and the fourth time axes should be excluded (see Fig. 1). Then

![Diagram](Image1)
the moment of a decision making \( t \) belongs to the interval \((t_k, t_{k+1}), k = 0, 1, \ldots,\) (see the second time axis at Fig. 1 where the consecutive numeration of the flow events, marked with white circles, starts with the time moment \( t_1 \)). To derive a formula for a posteriori probability \( w(\lambda_1 | t) \) calculation we use the following approach: first we consider the discrete time moments in equal periods of time of sufficiently small duration \( \Delta t \) and thereafter proceed to the limit \( \Delta t \to 0. \) Suppose that time keeps changing discretely with the step \( \Delta t: t = n\Delta t, n = 0, 1, \ldots. \) First we introduce a bivariate process \((\lambda^{(n)}, r_n)\) where \( \lambda^{(n)} = \lambda(n\Delta t) \) is the value of process \( \lambda(t) \) at the time moment \( n\Delta t \) \((\lambda^{(n)} = \lambda_i, i = 1, 2); r_n = r_n(\Delta t) = r(n\Delta t) - r((n-1)\Delta t) \) is the number of flow events registered at the interval \(((n-1)\Delta t, n\Delta t)\) of duration \( \Delta t, r_n = 0, 1, \ldots. \) Furthermore, denote by \( R_m = (r_0, r_1, \ldots, r_m) \) the sequence of the number of events registered during the time period \((0, m\Delta t)\) at the intervals \(((n-1)\Delta t, n\Delta t)\) of duration \( \Delta t (n = 0, m) \). Here \( r_0 \) is the number of events arrived at the interval \((-\Delta t, 0)\). The value of \( r_0 \) is not defined because there was no observation at that interval. So it is supposed to be equal to zero \((r_0 = 0)\). Denote by \( \Lambda^{(m)} = (\lambda(0), \lambda(1), \ldots, \lambda(m)) \) the sequence of unknown values of the process \( \lambda(n\Delta t) \) at time moments \( n\Delta t \) \((n = 0, m)\); assume that \( \lambda(0) = \lambda(0) = \lambda_i, i = 1, 2. \) Denote by \( w(\lambda^{(m)}|R_m) \) the probability of the value of \( \lambda^{(m)} \) in condition that the sequence of the number of events registered during the time period \((0, m\Delta t)\) is \( R_m. \) Similarly, define the probability \( w(\lambda^{(m+1)}|R_{m+1}) \). The recurrent formula for Markovian random process \((\lambda^{(n)}, r_n)\) has been derived by Gortsev and Shmyrin in [16]. This formula relates a posteriori probabilities \( w(\lambda^{(m)}|R_m) \) and \( w(\lambda^{(m+1)}|R_{m+1}) \):

\[
w(\lambda^{(m+1)}|R_{m+1}) = \frac{\sum_{\lambda^{(m)}=\lambda_1}^{\lambda_2} w(\lambda^{(m)}|R_m) p(\lambda^{(m+1)}|\lambda^{(m)}, r_m) p(\lambda^{(m+1)}|\lambda^{(m)}, r_m)}{\sum_{\lambda^{(m)}=\lambda_1}^{\lambda_2} \sum_{\lambda^{(m+1)}=\lambda_1}^{\lambda_2} w(\lambda^{(m)}|R_m) p(\lambda^{(m+1)}|\lambda^{(m)}, r_m) p(\lambda^{(m+1)}|\lambda^{(m)}, r_m)}
\]

(2)

where \( p(\lambda^{(m+1)}, r_{m+1}|\lambda^{(m)}, r_m) \) is the probability of the process \((\lambda^{(n)}, r_n)\) transition from state \((\lambda^{(m)}, r_m)\) to state \((\lambda^{(m+1)}, r_{m+1})\) in one step \( \Delta t. \) In case of consideration of modulated semi-synchronous integrated flow of events, the process \((\lambda^{(n)}, r_n)\) is a Markovian process, so the formula (2) takes place.

**Lemma 3.1.** During the time period between moments \( t_k \) and \( t_{k+1}, k = 0, 1, \ldots, \) of arrival of neighboring flow events, a posteriori probability \( w(\lambda_1 | t) \) satisfies a differential equation

\[
w'(\lambda_1 | t) = \alpha(1-\delta) - (\lambda_1 - \lambda_2 + \alpha + \beta - 2\alpha\delta) w(\lambda_1 | t) + (\lambda_1 - \lambda_2 - \alpha\delta) w^2(\lambda_1 | t),
\]

\[
t_k \leq t < t_{k+1}, k = 0, 1, \ldots \quad (3)
\]

**Proof.** The transition probability \( p(\lambda^{(m+1)}, r_{m+1}|\lambda^{(m)}, r_m) \), which is included into formula (2), can be written as

\[
p(\lambda^{(m+1)}, r_{m+1}|\lambda^{(m)}, r_m) = p(\lambda^{(m+1)}|\lambda^{(m)}, r_{m+1}) p(r_{m+1}|\lambda^{(m)}, \lambda^{(m+1)});
\]
\[ \lambda^{(m)} \lambda^{(m+1)} = \lambda_1, \lambda_2. \]  

Taking into account that
\[ w(\lambda^{(m)}|R_m) = w(\lambda^{(m)}|R_m(t)) = w(\lambda^{(m)}|t), \]
\[ w(\lambda^{(m+1)}|R_{m+1}) = w(\lambda^{(m+1)}|R_{m+1}(t)) = w(\lambda^{(m+1)}|t + \Delta t), \]
considering (4) and assuming for simplicity in (2) that \( \lambda^{(m+1)} = \lambda_1 \), we receive the formula (2) in the following form
\[ w(\lambda^{(m+1)}|R_{m+1}) = \frac{\sum_{s=1}^{2} w(\lambda_s|t) p(\lambda_1|\lambda_s) p(r_{m+1}|\lambda_s, \lambda_1)}{\sum_{j=1}^{2} \sum_{s=1}^{2} w(\lambda_s|t) p(\lambda_j|\lambda_s) p(r_{m+1}|\lambda_s, \lambda_j)}. \]  

It is obvious from the definition of the flow that there are only two values of \( r_{m+1} \): \( r_{m+1} = 0 \) or \( r_{m+1} = 1 \). Here we consider the behavior of probability \( w(\lambda_1|t) \) at the half-interval \([t_k, t_{k+1}]\) between neighboring events of the flow, i.e. \( t_k \leq t < t_{k+1}; t_k \leq t + \Delta t < t_{k+1} \). Thus \( r_{m+1} = 0 \) in (5), and taking into account the matrix \( D_0 \) (see (1)), we receive the expressions for transition probabilities (4) at the half-interval \([t, t + \Delta t) = [m\Delta t, (m+1)\Delta t)\) in the following form
\[ p(\lambda_1|\lambda_1) p(r_{m+1} = 0|\lambda_1, \lambda_1) = 1 - (\lambda_1 + \beta)\Delta t + o(\Delta t), \]
\[ p(\lambda_2|\lambda_2) p(r_{m+1} = 0|\lambda_2, \lambda_2) = 1 - (\lambda_2 + \alpha)\Delta t + o(\Delta t), \]
\[ p(\lambda_1|\lambda_2) p(r_{m+1} = 0|\lambda_2, \lambda_1) = \alpha(1 - \delta)\Delta t + o(\Delta t), \]
\[ p(\lambda_2|\lambda_1) p(r_{m+1} = 0|\lambda_1, \lambda_2) = \beta\Delta t + o(\Delta t). \]

Substituting (6)–(9) into (5) and considering that \( w(\lambda_2|t) = 1 - w(\lambda_1|t)\), we find a numerator \( A_0 \) and denominator \( B_0 \) for (5):
\[ A_0 = (1 - \delta)\alpha\Delta t + [1 - (\lambda_1 + \beta)\Delta t - \alpha(1 - \delta)\Delta t] w(\lambda_1|t) + o(\Delta t), \]
\[ B_0 = 1 - \Delta t [\lambda_2 + \alpha\delta + (\lambda_1 - \lambda_2 - \alpha\delta) w(\lambda_1|t)] + o(\Delta t). \]

Substituting \( A_0 \) and \( B_0 \) into (5) and considering that
\[ B_0^{-1} = 1 + \Delta t [\lambda_2 + \alpha\delta + (\lambda_1 - \lambda_2 - \alpha\delta) w(\lambda_1|t)] + o(\Delta t) \]  

(since \((1 - x)^{-1} = 1 + x + o(x)\) for sufficiently small \( x > 0 \)), we receive
\[ w(\lambda_1|t + \Delta t) - w(\lambda_1|t) = \Delta t[\alpha(1 - \delta) - (\lambda_1 - \lambda_2 + \alpha + \beta - 2\alpha\delta) w(\lambda_1|t) + \]
\[ + (\lambda_1 - \lambda_2 - \alpha\delta) w^2(\lambda_1|t)] + o(\Delta t). \]

Dividing here both parts of the expression above by \( \Delta t \) and proceeding to the limit \( \Delta t \to 0 \), we obtain (3). Lemma is proved.
Remark. The equation (3) defines the behavior of the probability \( w(\lambda_1|t) \) at the interval \([t_k, t_{k+1})\), \( k = 0, 1, \ldots \), i.e. between the moments of the event occurrence. At that, at the right end of this interval the value \( w(\lambda_1|t_{k+1} - 0) \) takes place. As will be demonstrated in Lemma 3.2 below, this value is used for the probability \( w(\lambda_1|t_{k+1} + 0) \) calculation. In turn \( w(\lambda_1|t_{k+1} + 0) \) is the initial value for the next interval \([t_{k+1}, t_{k+2})\).

Lemma 3.2. A posteriori probability \( w(\lambda_1|t) \) at the moment \( t_k \), \( k = 1, 2, \ldots \), of an event occurrence is defined by the formula (11), which we call as “conversion formula”:

\[
w(\lambda_1|t_k + 0) = \frac{\alpha \delta + [\lambda_1 (1 - p) - \alpha \delta] w(\lambda_1|t_k - 0)}{\lambda_2 + \alpha \delta + (\lambda_1 - \lambda_2 - \alpha \delta) w(\lambda_1|t_k - 0)}, \quad k = 1, 2, \ldots \quad (11)
\]

Proof. Suppose that at the interval \((t, t + \Delta t)\) the event arrives at the time moment \( t_k \) \((t < t_k < t + \Delta t)\) \((r_{m+1} = 1)\). In this case we have two neighboring intervals \((t, t_k)\) and \((t_k, t + \Delta t)\) of duration \( t_k - t = \Delta t' \) and \( t + \Delta t - t_k = \Delta t'' \) correspondingly. Then \( w(\lambda_s|t) = w(\lambda_s|t_k - \Delta t'), s = 1, 2; w(\lambda_1|t + \Delta t) = w(\lambda_1|t_k + \Delta t'') \) and the expression (5) is equal to

\[
w(\lambda_1|t_k + \Delta t'') = \frac{\sum_{s=1}^{2} w(\lambda_s|t_k - \Delta t') p(\lambda_1|\lambda_s) p(r_{m+1}|\lambda_s, \lambda_1)}{\sum_{s=1}^{2} w(\lambda_s|t_k - \Delta t') p(\lambda_j|\lambda_s) p(r_{m+1}|\lambda_s, \lambda_j)}, \quad (12)
\]

Taking into account the matrix \( D_1 \) (see (1)), we receive expressions for probabilities (4) at the interval \((t, t + \Delta t) = (m\Delta t, (m+1)\Delta t)\) in the following form

\[
p(\lambda_1|\lambda_1) p(r_{m+1} = 1|\lambda_1, \lambda_1) = (1 - p)\lambda_1 \Delta t + o(\Delta t), \quad (13)
\]

\[
p(\lambda_2|\lambda_2) p(r_{m+1} = 1|\lambda_2, \lambda_2) = \lambda_2 \Delta t + o(\Delta t), \quad (14)
\]

\[
p(\lambda_1|\lambda_2) p(r_{m+1} = 1|\lambda_2, \lambda_1) = \alpha \delta \Delta t + o(\Delta t), \quad (15)
\]

\[
p(\lambda_2|\lambda_1) p(r_{m+1} = 1|\lambda_1, \lambda_2) = p \lambda_1 \Delta t + o(\Delta t). \quad (16)
\]

Substituting (13)–(16) into (12), we obtain the formulas for numerator \( A_1 \) and denominator \( B_1 \) for (12):

\[
A_1 = \Delta t [(1 - p)\lambda_1 w(\lambda_1|t_k - \Delta t') + \alpha \delta w(\lambda_2|t_k - \Delta t')] + o(\Delta t),
\]

\[
B_1 = \Delta t [\lambda_1 w(\lambda_1|t_k - \Delta t') + (\lambda_2 + \alpha \delta) w(\lambda_2|t_k - \Delta t')] + o(\Delta t).
\]

Substituting \( A_1 \) and \( B_1 \) into (12), dividing the numerator and denominator by \( \Delta t \), considering that \( w(\lambda_2|t_k - \Delta t') = 1 - w(\lambda_1|t_k - \Delta t') \), and proceeding to the limit \( \Delta t \to 0 \) \((\Delta t' \text{ and } \Delta t'' \text{ converge to zero simultaneously})\), we obtain the formula (11). Lemma is proved.
Remark. A posteriori probability $w(\lambda_1 | t)$ at the moment $t_k$, $k = 1, 2, ...$, of an event occurrence undergoes a discontinuity of the first kind (a finite jump takes place). The solution of the equation (3) depends on the initial condition at the time moment $t_k$, i.e. it depends on $w(\lambda_1 | t_k + 0)$, $k = 1, 2, ...$. In turn $w(\lambda_1 | t_k + 0)$ depends on the value of $w(\lambda_1 | t_k - 0)$, of probability $w(\lambda_1 | t)$ at the moment $t_k$, when $w(\lambda_1 | t)$ defined in (3) changes at the half-interval $[t_{k-1}, t_k)$ preceding the half-interval $[t_k, t_{k+1})$, $k = 1, 2, ...$. Thereby, all prehistory of the flow observations from the time moment $t_0 = 0$ to $t_k$ is concentrated in the value $w(\lambda_1 | t_k + 0)$. As an initial condition $w(\lambda_1 | t_0 + 0) = w(\lambda_1 | t_0 = 0)$ at the half-interval $[t_0, t_1)$ in (3) we select an a priori stationary probability of the first state of the process $\lambda(t)$:

$$\pi_1 = \frac{\alpha}{\alpha + \beta + p\lambda_1},$$

which can be derived from the evident equations $\pi_1 + \pi_2 = 1$, $\pi_1(\beta + p\lambda_1) = \pi_2\alpha$.

Lemmas 3.1, 3.2 allow to formulate the following theorem.

**Theorem 3.3.** A posteriori probability $w(\lambda_1 | t)$ behavior at the half-intervals $[t_k, t_{k+1})$, $k = 1, 2, ...$, is determined with the explicit formula:

$$w(\lambda_1 | t) = \frac{w_1[w_2 - w(\lambda_1 | t_k + 0)] - w_2[w_1 - w(\lambda_1 | t_k + 0)]}{w_2 - w(\lambda_1 | t_k + 0) - [w_1 - w(\lambda_1 | t_k + 0)]} e^{-b(t-t_k)}$$

(18)

where $w_1 = \frac{\lambda_1-\lambda_2+\alpha+\beta-2\alpha\delta-b}{2(\lambda_1-\lambda_2-\alpha\delta)}$, $w_2 = \frac{\lambda_1-\lambda_2+\alpha+\beta-2\alpha\delta+b}{2(\lambda_1-\lambda_2-\alpha\delta)}$.

$b = \sqrt{(\lambda_1 - \lambda_2 - \alpha + \beta)^2 + 4\alpha\beta(1-\delta)}$ and $t_k \leq t < t_{k+1}$, $k = 0, 1, ...$: $w(\lambda_1 | t_0 + 0) = w(\lambda_1 | t_0 = 0) = \pi_1$, $\pi_1$ is defined in (17), $w(\lambda_1 | t_k + 0)$ is defined in (11).

**Proof.** The equation (3) specified in Lemma 3.1 can be expressed as

$$[(w(\lambda_1 | t) - w_1)^{-1} - (w(\lambda_1 | t) - w_2)^{-1}] \, dw(\lambda_1 | t) = (\lambda_1 - \lambda_2 - \alpha\delta) \, dt$$

(19)

where $w_1$ and $w_2$ are defined in (18). Integrating (19) between $t_k + 0$ and $t$, we obtain (18). The theorem is proved.

**Case 1.** $\lambda_1 - \lambda_2 - \alpha\delta = 0$. In this case the formula (18) turns into:

$$w(\lambda_1 | t) = \frac{\alpha(1-\delta)}{\alpha(1-\delta) + \beta} + \left[ w(\lambda_1 | t_k + 0) - \frac{\alpha(1-\delta)}{\alpha(1-\delta) + \beta} \right] e^{-(\alpha(1-\delta)+\beta)(t-t_k)}$$

where $t_k \leq t < t_{k+1}$, $k = 0, 1, ...$. The formula (11) remains the same. If to add the additional constraint $(1-p)\lambda_1 - \alpha\delta = 0$ to $\lambda_1 - \lambda_2 - \alpha\delta = 0$, then the formula (11) can be written as: $w(\lambda_1 | t_k + 0) = \alpha\delta/((\lambda_2 + \alpha\delta)$, $k = 1, 2, ...$, i.e.
in this case a posteriori probability \( w(\lambda_1|t) \) does not depend on prehistory, it depends on its value at the moment \( t_k \).

**Case 2.** If \( p = 1, \delta = 0 \), then \( w(\lambda_1|t_k+0) = 0, k = 1, 2, \ldots \), i.e. in this case a posteriori probability \( w(\lambda_1|t) \) also does not depend on prehistory, it depends on its value at the moment \( t_k \).

### 3.2 The expressions for a posteriori probability \( w(\lambda_1|t) \)

**in case of a constant dead time \( (T \neq 0) \) (in condition of the flow incomplete observability)**

Let us return to the case of a constant dead time, \( T \neq 0 \) (see Fig. 1). In this case we can use formula (18) for a posteriori probability calculation at the interval \( (t_k + T, t_{k+1}) \). The initial condition for \( w(\lambda_1|t) \) refers to the moment \( t_k + T \), i.e. we should replace in formula (18) \( w(\lambda_1|t_k+0) \) by \( w(\lambda_1|t_k+T) \) and take \( t_k + T \leq t < t_{k+1}, k = 1, 2, \ldots \) There is no modification in formula (11) as it is intended for \( w(\lambda_1|t) \) calculation at the moment \( t_k \) of the event arrival, which causes the beginning of dead time period.

Consider the half-interval \( (t_k, t_k + T], k = 1, 2, \ldots \). At this interval the event takes place at the frontier point \( t_k \), and there is no other events at the interval itself.

**Theorem 3.4.** A posteriori probability \( w(\lambda_1|t) \) behavior at the half-intervals \( (t_k, t_k + T], k = 1, 2, \ldots \) is determined with the explicit formula:

\[
\begin{align*}
    w(\lambda_1|t) &= \pi_1 - [\pi_1 - w(\lambda_1|t_k + 0)] e^{-(\alpha + \beta + \rho \lambda_1)(t-t_k)} \\
    &= w(\lambda_1|t_k) e^{-\rho \lambda_1 T} e^{-(\alpha + \beta + \rho \lambda_1)(t-t_k)}
\end{align*}
\]

where \( t_k < t \leq t_k + T, k = 1, 2, \ldots \) ; \( w(\lambda_1|t_k+0) \) is defined by (11); \( \pi_1 \) is defined by (17).

**Proof.** Let us define an a posteriori probability \( w(\lambda_1|t+\Delta t) \) as a probability that process \( \lambda(t) \) sojourns in the first state at the time moment \( t + \Delta t \) \( (t_k < t + \Delta t < t_k + T) \), where \( \Delta t \) is sufficiently small. Suppose that at the moment \( t \) the process \( \lambda(t) \) sojourns in the first state and at the half-interval \( [t, t + \Delta t] \) it does not change its state to the second one. The probability of this event is \( w(\lambda_1|t)(1 - \beta \Delta t - p \lambda_1 \Delta t) + o(\Delta t) \). Now suppose that at the moment \( t \) the process \( \lambda(t) \) is situated in the second state and at the half-interval \( [t, t + \Delta t] \) it changes its state to the first one. The probability of this event is \( w(\lambda_2|t)\alpha \Delta t + o(\Delta t) \). The probability of the other possible variations is equal to \( o(\Delta t) \). Then, \( w(\lambda_1|t+\Delta t) = w(\lambda_1|t)(1 - \beta \Delta t - p \lambda_1 \Delta t) + w(\lambda_2|t)\alpha \Delta t + o(\Delta t) \). Similarly, we obtain that \( w(\lambda_2|t+\Delta t) = w(\lambda_2|t)(1 - \alpha \Delta t) + w(\lambda_1|t)(\beta \Delta t + p \lambda_1 \Delta t) + o(\Delta t) \).

Doing here some transformations and proceeding to the limit \( \Delta t \to 0 \), we have the system of differential equations for a posteriori probabilities \( w(\lambda_1|t) \)
and \( w(\lambda_2|t) \):
\[
\frac{dw(\lambda_1|t)}{dt} = -(p\lambda_1 + \beta)w(\lambda_1|t) + \alpha w(\lambda_2|t),
\]
\[
\frac{dw(\lambda_2|t)}{dt} = (p\lambda_1 + \beta)w(\lambda_1|t) - \alpha w(\lambda_2|t),
\]
with initial conditions: \( w(\lambda_1|t = t_k) = w(\lambda_1|t_k + 0) \), \( w(\lambda_2|t = t_k) = w(\lambda_2|t_k + 0) = 1 - w(\lambda_1|t_k + 0) \), \( k = 1, 2, \ldots \). These conditions follow from the fact that at the time interval \( (t_{k-1} + T, t_k) \), which is neighboring to the half-interval \( (t_k, t_k + T] \), \( k = 2, 3, \ldots \), the probability \( w(\lambda_1|t) \) is computed according to formula (18), where we use \( w(\lambda_1|t_k + T) \) instead of \( w(\lambda_1|t_k + 0) \). At the point \( t = t_k \) we calculate first the probability \( w(\lambda_1|t_k - 0) \), after that we recalculate the probability at this point using formula (11), so the value \( w(\lambda_1|t_k + 0) \) becomes the boundary condition for the system of equations (21). For the initial half-interval \( [t_0, t_1] \) the probability \( w(\lambda_1|t) \) is calculated according to formula (18) with the following recalculation at the point \( t = t_1 \) using formula (11). Solving the system (21), we find (20). Note that \( \lim w(\lambda_1|t) \to \pi_1 \) in case of \( t \to \infty \). The theorem is proved.

Let us note that it follows from (20) that probability \( w(\lambda_1|t) \) at the point \( t_k + T \) is calculated according to the formula
\[
w(\lambda_1|t_k + T) = \pi_1 - [\pi_1 - w(\lambda_1|t_k + 0)] e^{-(\alpha + \beta + p\lambda_1)T}.
\]

The analytical formulas obtained for \( w(\lambda_1|t) \) calculation allow us to define the algorithm for a posteriori probability \( w(\lambda_1|t) \) calculation and algorithm for decision making about the process \( \lambda(t) \) state at any time moment \( t \) (the algorithm of optimal estimation of the flow states):
1) at the start point of observations \( t_0 = 0 \) we put \( w(\lambda_1|t_0 + 0) = w(\lambda_1|t_0 = 0) = \pi_1 \);
2) for \( k = 0 \), a posteriori probability \( w(\lambda_1|t) \) is calculated according to formula (18) at any time moment \( t \) \( (0 < t < t_1) \), where \( t_1 \) is the arrival moment of the first observable event;
3) for \( k = 0 \), the probability \( w(\lambda_1|t_1) = w(\lambda_1|t_1 - 0) \) is calculated according to formula (18);
4) \( k \) is incremented, and for \( k = 1 \), the probability \( w(\lambda_1|t_k + 0) \) is calculated according to conversion formula (11), \( w(\lambda_1|t_k + 0) \) is the initial value for \( w(\lambda_1|t) \) in formula (20);
5) for \( k = 1 \), a posteriori probability \( w(\lambda_1|t) \) is calculated according to formula (20) at any time moment \( t \) \( (t_k < t < t_k + T) \);
6) for \( k = 1 \), the probability \( w(\lambda_1|t_k + T) \) is calculated according to formula (22), \( w(\lambda_1|t_k + T) \) is the initial value for \( w(\lambda_1|t) \) at the next step of the algorithm;
7) for \( k = 1 \), the probability \( w(\lambda_1|t) \) is calculated according to the formula
\[
w(\lambda_1|t) = \frac{w_1 [w_2 - w(\lambda_1|t_k + T)] - w_2 [w_1 - w(\lambda_1|t_k + T)] e^{-b(t-t_k-T)}}{w_2 - w(\lambda_1|t_k + T) - [w_1 - w(\lambda_1|t_k + T)] e^{-b(t-t_k-T)}} \tag{23}
\]
(\( w_1, w_2 \) and \( b \) are defined in (18)) at any time moment \( t \) \((t_k + T < t < t_{k+1})\), where \( t_{k+1} \) is the moment of arrival of the next observable event;
8) for \( k = 1 \), the probability \( w(\lambda_1|t_{k+1}) = w(\lambda_1|t_{k+1} - 0) \) is calculated according to formula (23);
9) the algorithm goes to step 4, after that steps 4–8 are repeated for \( k = 2 \) and so on.

During the process of probability \( w(\lambda_1|t) \) calculation, decision about the process \( \lambda(t) \) state at any time moment \( t \) is made according to criterion of a posteriori probability maximum: if \( w(\lambda_1|t) \geq w(\lambda_2|t) \), then estimation is \( \hat{\lambda}(t) = \lambda_1 \), else \( \hat{\lambda}(t) = \lambda_2 \).

For cases 1 and 2, which were considered above, the algorithm of optimal estimation of the flow states is identical to proposed above for the general case.

4 Numerical Experiments

The described algorithm for optimal estimation of the flow states has created a background for software development and carrying out the numerical experiments. The program is written in C#, Microsoft Visual Studio 2013. The first stage of calculations assumes the flow simulation in conditions of a constant dead time and collecting the actual path of the process \( \lambda(t) \) and time moments \( t_1, t_2, \ldots \) for observable events as a result. This stage is not described here in more details, as there is no special difficulties in its implementation. The second stage is the direct computation of the values of probabilities \( w(\lambda_1|t), t_0 \leq t < t_1; w(\lambda_1|t_k + 0), k = 1, 2, \ldots; w(\lambda_1|t), t_k \leq t < t_{k+1}, k = 1, 2, \ldots, \) according to formulas (11), (18), (20), (22), (23) and plotting the curve of the estimation process \( \hat{\lambda}(t) \).

The calculations were made for the following values of the flow parameters: \( \lambda_1 = 5, \lambda_2 = 1, p = 0.025, \beta = 0.2, \alpha = 0.2, \delta = 0.2, T = 0.5 \) and modelling time was \( T_m = 1000 \) time units. As an illustration, Fig. 2 shows a trajectory of the process \( \lambda(t) \) (an actual path at a top part of Fig. 2), which was obtained as a result of simulation stage, and a trajectory of the process estimation \( \hat{\lambda}(t) \) (a bottom part of Fig. 2).

The decision about the process \( \lambda(t) \) state was made with the step \( \Delta t = 0.001 \). In Fig. 2, intervals where the trajectory of estimation \( \hat{\lambda}(t) \) does not coincide with the actual path of the process \( \lambda(t) \) are marked with plump lines (an area of making an error). Fig. 3 shows the trajectory of a posteriori probability \( w(\lambda_1|t) \) behavior, which corresponds to the sequence of time moments \( t_1, t_2, \ldots \) of observable events occurrence, that were found during the simulation stage.
To find a frequency of making a wrong decision about the process \( \lambda(t) \) state during the flow observation period, the statistical experiment was implemented. It consists of the following steps: 1) Fix \( j = 1 \) and define the set of parameters \( \lambda_1, \lambda_2, p, \beta, \alpha, \delta, T \). For any \( j \) the simulation is made at the time interval \([0, T_m]\); 2) Calculate the probabilities \( w(\lambda_1|t) \) at the interval \([0, T_m]\) according to formulas (11), (18), (20), (22), (23); 3) Estimate a trajectory of the process \( \lambda(t) \) at the interval \([0, T_m]\); 4) Define the total length \( d_j \) of the time intervals where the trajectory of the estimation process \( \hat{\lambda}(t) \) does not coincide with the actual path of the process \( \lambda(t) \); 5) Calculate the fraction of making a wrong decision \( \hat{p}_j = d_j/T_m \); 6) If \( j < N \), then set \( j = j + 1 \) and repeat steps 1–5.

As a result of implementation of the described above procedure we will receive a sample \( (\hat{p}_1, \hat{p}_2, ..., \hat{p}_N) \) of fractions of making a wrong decision for \( N \) experiments. Using this sample, we can find a sample average of unconditional probability of making an error \( \hat{P}_0 = \frac{1}{N} \sum_{j=1}^{N} \hat{p}_j \) and a sample variance \( \hat{D} = \frac{1}{N-1} \sum_{j=1}^{N} (\hat{p}_j - \hat{P}_0)^2 \).

To find out the value of modeling time \( T_m \), at which the flow can be considered as the flow in a steady-state conditions, the first series of experiments was carried out for different values of the flow parameters \( \lambda_1, \lambda_2, p, \beta, \alpha, \delta, T \). The results of calculations, obtained for the following common set of parame-
Optimal estimation of the flow states

Figure 4: Values $\hat{P}_0$ dependence from $T_m$ for parameters $\lambda_1 = 5$, $\lambda_2 = 1.5$, $p = 0.02$, $\beta = 0.025$, $\alpha = 0.025$, $\delta = 0.5$, $N = 100$ and (A) $T = 0$, (B) $T = 1$ correspondingly.

ters $\lambda_1 = 5$, $\lambda_2 = 1.5$, $p = 0.02$, $\beta = 0.025$, $\alpha = 0.025$, $\delta = 0.5$, $N = 100$, but for the different values of the parameter $T$ ($T = 0$ and $T = 1$ correspondingly), are presented in Fig. 4. Here, the abscissa is a value of modelling time period $T_m$ and the ordinate is a value of $\hat{P}_0$.

The analysis of numerous experiments conducted for the different values of the flow parameters, including the results presented in Fig. 4, shows that the value of estimation $\hat{P}_0$ stabilizes and becomes stable enough with an increase of the value of modeling time $T_m$. As a consequence, the value of modeling time $T_m$ for further experiments was chosen equal to 1000 time units.

The results of the second series of experiments are presented in Tables 1–5 below. The first line of the tables contains the values of the dead time period length $T$ ($T = 0$, 0.5, 1, ..., 3.5). The second and the third lines contain the values of $\hat{P}_0$ and $\hat{D}$ for each test. The results are obtained for the following common set of parameters $\lambda_2 = 1.5$, $p = 0.02$, $\beta = 0.025$, $\alpha = 0.025$, $\delta = 0.5$, $T_m = 1000$, $N = 100$. The results presented in Tables 1–5 correspond to the values 3.5, 4, 5, 6, 7 of the parameter $\lambda_1$.

Table 1: $\lambda_1 = 3.5$

<table>
<thead>
<tr>
<th>$T$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{P}_0$</td>
<td>0.1121</td>
<td>0.1564</td>
<td>0.1820</td>
<td>0.1903</td>
<td>0.1989</td>
<td>0.2036</td>
<td>0.2057</td>
<td>0.2135</td>
</tr>
<tr>
<td>$\hat{D}$</td>
<td>0.0004</td>
<td>0.0006</td>
<td>0.0013</td>
<td>0.0019</td>
<td>0.0024</td>
<td>0.0019</td>
<td>0.0021</td>
<td>0.0022</td>
</tr>
</tbody>
</table>

Table 2: $\lambda_1 = 4$

<table>
<thead>
<tr>
<th>$T$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{P}_0$</td>
<td>0.0891</td>
<td>0.1348</td>
<td>0.1612</td>
<td>0.1717</td>
<td>0.1839</td>
<td>0.1862</td>
<td>0.1923</td>
<td>0.1956</td>
</tr>
<tr>
<td>$\hat{D}$</td>
<td>0.0003</td>
<td>0.0006</td>
<td>0.0010</td>
<td>0.0015</td>
<td>0.0022</td>
<td>0.0019</td>
<td>0.0019</td>
<td>0.0028</td>
</tr>
</tbody>
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Table 3: $\lambda_1 = 5$

<table>
<thead>
<tr>
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<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{P}_0$</td>
<td>0.0671</td>
<td>0.1112</td>
<td>0.1323</td>
<td>0.1467</td>
<td>0.1537</td>
<td>0.1616</td>
<td>0.1678</td>
<td>0.1678</td>
</tr>
<tr>
<td>$\hat{D}$</td>
<td>0.0001</td>
<td>0.0004</td>
<td>0.0009</td>
<td>0.0012</td>
<td>0.0010</td>
<td>0.0014</td>
<td>0.0009</td>
<td>0.0018</td>
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</tbody>
</table>

Table 4: $\lambda_1 = 6$

<table>
<thead>
<tr>
<th>$T$</th>
<th>0</th>
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<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{P}_0$</td>
<td>0.0512</td>
<td>0.0915</td>
<td>0.1205</td>
<td>0.1282</td>
<td>0.1347</td>
<td>0.1366</td>
<td>0.1415</td>
<td>0.1421</td>
</tr>
<tr>
<td>$\hat{D}$</td>
<td>0.0001</td>
<td>0.0002</td>
<td>0.0006</td>
<td>0.0008</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0010</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

Table 5: $\lambda_1 = 7$

<table>
<thead>
<tr>
<th>$T$</th>
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<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
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<td>$\hat{P}_0$</td>
<td>0.0404</td>
<td>0.0808</td>
<td>0.1030</td>
<td>0.1125</td>
<td>0.1205</td>
<td>0.1308</td>
<td>0.1301</td>
<td>0.1332</td>
</tr>
<tr>
<td>$\hat{D}$</td>
<td>0.00005</td>
<td>0.0002</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0008</td>
<td>0.0011</td>
<td>0.0007</td>
</tr>
</tbody>
</table>

The results presented in Tables 1–5 evidently demonstrate that the trend of estimation $\hat{P}_0$ is decrescent depending on $\lambda_1$ when the value of dead time duration $T$ is fixed. In case of increasing of the difference $\lambda_1 - \lambda_2$, the conditions of the flow states discernibility become better. At the same time, sample variance $\hat{D}$ is sufficiently small for all types of calculations. The analysis of the obtained results also shows that the value of the estimated probability of making a wrong decision $\hat{P}_0$ increases in case of dead time period value $T$ is incremented. All this is quite natural, since the increase of dead time period duration always means the loss of useful information. This negatively affects the quality of estimation.

5 Conclusion

The filtering method for estimating the intensity process of the modulated semi-synchronous integrated flow of events has been presented. The method is based on the algorithm of optimal estimation of the flow states, which minimizes the total probability of making a wrong decision. The formulas for a posteriori probability calculation, that provide a foundation for the algorithm, were derived in case of an absence of dead time ($T = 0$) and in case of a constant dead time ($T \neq 0$). The formulas for a posteriori probability calculation were found explicitly, so there was no need to use numerical methods. Also some simulation experiments and numerical results were presented. The
obtained results provide the possibility of the states estimation for the modulated semi-synchronous integrated flow of events in conditions of a constant dead time from the observations of the flow.

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**References**


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