Periodic Solutions for Rotational Motion of an Axially Symmetric Charged Satellite

Yehia A. Abdel-Aziz

National Research Institute of Astronomy and Geophysics (NRIAG)
Cairo, Egypt, University of Hail, Department of Mathematics
P.O. Box 2440, Kingdom of Saudi Arabia

Muhammad Shoaib

University of Ha’il, Department of Mathematics
P.O. Box 2440, Kingdom of Saudi Arabia

Copyright © 2014 Yehia A. Abdel-Aziz and Muhammad Shoaib. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract

The present paper is devoted to the investigation of sufficient conditions for the existence of periodic solutions in the vicinity of stationary motion of a charged satellite in elliptic orbit. Lorentz forces which result from the motion of a charged satellite relative to the magnetic field of the Earth are considered. An axial symmetry is assumed about the center of mass of the satellite. In addition to the Lorentz force the perturbation caused by gravitational and magnetic fields of the Earth are considered. The stationary solutions and periodic orbits close to them are obtained using the Lyapunov theorem of holomorphic integral. Numerical results are used to explain the periodic motion of a certain satellite. It is shown that the charge-to-mass ratio has a significant influence on the periodic motion of satellites.

Keywords: Periodic solutions, Lorentz force, Rotational motion, charged spacecraft, gravitational force

1. Introduction

When a spacecraft is moving in geomagnetic field in Low Earth Orbit (LEO) then due to the accumulation of charging on the surface of the spacecraft the Lorentz
force is generated. A detailed analysis of Lorentz Augmented Orbit (LAO) is available in [1]. The rotational motion of a rigid artificial satellite subject to gravitational and magnetic torque was treated in detail in [2-6]. Chen and Liu [7] uses the Poincare map technique to investigate the behavior of periodic and chaotic motion of the spinning gyrostat satellite. The attitude dynamics of a satellite is described by the sixth order Euler- Poisson equations. To understand the periodic behavior of a rigid body Kolosoff’s [8] transformed its equations of motion to a planar system of equations as was done by Lyapunov in [9]. Abdel-Aziz [10] found the periodic solution of a spacecraft using an approximated model of the Earth magnetic field model. Abdel-Aziz and Shoai [15] studied the effects of Lorentz force on the attitude dynamics of an electroystatic spacecraft moving in a circular orbit and used charge to mass ratio as semi-passive control. In this paper we use a similar model of Lorentz force but the spacecraft is considered to be in an elliptic orbit. In [16] Abdel-Aziz and Shoai developed the torque due to Lorentz force as a function of orbital elements and investigated the effects of lorentz force on the existance and stability of equilibrium positions in Pitch-Roll-Yaw directions.

In the present work we consider a satellite moving in an elliptic orbit under the influence geomagnetic field, Lorentz force and gravitational field. Using the isothermal coordinates we reduce the equations of motion of the satellite to the planar system of equations. The existence of periodic orbits in the neighborhood of stationary solutions is obtained with the help of the Lyapunov method. The numerical results are used to show the significance of Lorentz force on the position and stability of periodic orbits.

2. Equations of motion and their reduction of order

Consider an axially symmetric charged satellite under the action of Lorentz force, gravitational and geomagnetic fields. Two Cartesian coordinate systems are introduced. The origin of the coordinate system is taken to be at the center of mass \( O \) of the satellite. The orbital system of coordinate is named as \( \mathbf{OX'Y'Z'} \) and the coordinate system for the body of satellite is named as \( \mathbf{OX_0Y_0Z_0} \). In the orbital system \( \mathbf{OX'}, \mathbf{OY'} \) and \( \mathbf{OZ'} \) are in the orbital direction, normal to the orbit and along the radius vectors of the satellite. The attitude motion of satellite is defined by three Euler angles namely \( \theta, \psi \) and \( \phi \). Here \( \psi \) is the angle of precession and \( \phi \) is the angle of self-rotation. The angle between the \( Z \)-axis of both the coordinate systems is taken to be \( \theta \). The \( Z_0 \)-axis is taken to be the axis of symmetry. The principal moments of inertia of the satellite are taken to be \( A, B, C \). Due to the symmetry about \( Z_0 \)-axis \( A = B \). The unit vectors of the orbital coordinate system are named \( \mathbf{\hat{a}}, \mathbf{\hat{\beta}}, \mathbf{\hat{\gamma}} \).

\[
\mathbf{\hat{a}} = (\alpha_1, \alpha_2, \alpha_3), \mathbf{\hat{\beta}} = (\beta_1, \beta_2, \beta_3) \text{ and } \mathbf{\hat{\gamma}} = (\gamma_1, \gamma_2, \gamma_3),
\]

where

\[
\alpha_1 = \cos \psi \cos \phi - \sin \psi \sin \phi \cos \theta,
\]
\[ \alpha_2 = -\cos \psi \sin \phi, \]
\[ \alpha_3 = -\cos \theta \sin \psi \cos \phi \sin \theta \sin \psi), \]
\[ \beta_1 = \sin \psi \cos \phi + \cos \theta \cos \psi \sin \phi, \]
\[ \beta_2 = -\sin \psi \sin \phi + \cos \theta \cos \phi \cos \phi, \]
\[ \beta_3 = -\sin \theta \cos \psi, \]
\[ \gamma_1 = \sin \theta \sin \phi, \]
\[ \gamma_2 = \sin \theta \cos \phi, \]
\[ \gamma_3 = \cos \theta. \]

The Lagrangian of the system can be written as in Yehia [11]
\[ L = \frac{1}{2} \tilde{\omega}^T I \tilde{\omega} - V_0. \tag{2} \]

where \( V_0 = V_G + V_L + V_M \) is the total potential of the forces acting on the spacecraft. \( V_G, V_L, \text{and} V_M \) are the potentials due to the gravitation of the Earth, Lorentz force and magnetic field respectively, \( I \) is the interia matrix of the spacecraft.

Let \( \tilde{\omega} = (p, q, r) \) and \( \tilde{\omega}_0 = (p_0, q_0, r_0) \) are the angular velocities of the satellite in the inertial and orbital reference frames respectively. As the orbital system rotate in space with an orbital angular velocity \( \Omega \) about the axis, which is perpendicular to the orbital plane. The relation between the angular velocities in the two systems is given by \( \omega = \omega_0 + \Omega \tilde{\beta} \). According to Abdel-Aziz [10], we can write.
\[ (p, q, r) = (\dot{\psi} \sin \theta \sin \phi + \dot{\theta} \cos \phi + \Omega \beta_1, \dot{\psi} \sin \theta \cos \phi - \dot{\theta} \sin \phi + \Omega \beta_2, \dot{\psi} \cos \theta + \dot{\phi} + \Omega \beta_3) \tag{3} \]

The gravitation potential of the Earth is [2]
\[ V_G = \frac{3}{2} \Omega^2 \gamma \bar{I} \bar{r}^T \tag{4} \]

2.1 Potential due to Lorentz force

The magnetic field is expressed as [12]
\[ B = \frac{B_0}{r^3} \left[ 2 \cos \phi \hat{r} + \sin \phi \hat{\phi} + \dot{\theta} \hat{\theta} \right], \tag{5} \]

where \( B_0 \) is the strength of the magnetic field in Wpm. The acceleration in inertial coordinates is given by
\[ \ddot{a} = \frac{\ddot{r}}{m} = -\frac{\mu}{r^3} \hat{r} + \frac{q}{m} (\vec{V}_{rel} \times \vec{B}). \tag{6} \]

The Lorentz force (per unit mass) can be written as
\[ \vec{F}_L = \frac{q}{m} (\vec{V}_{rel} \times \vec{B}), \]  
\[ \vec{V}_{rel} = \vec{V} - \vec{\omega}_e \times \vec{r}, \]

where, \( \vec{V} \) is the inertial velocity of the spacecraft, \( \vec{\omega}_e \) is the angular velocity vector of the Earth and \( \vec{V}_{rel} \) is velocity of the spacecraft relative to the geomagnetic field. In agreement with [12], we have used

\[ \vec{V} = \dot{r} \hat{r} + r \phi \hat{\phi} + r \theta \sin \phi \ \hat{\theta} \]

\[ \vec{r} = r \hat{r}, \]

\[ \vec{\omega}_e = \omega_e \hat{z}, \]

\[ \dot{z} = \cos \phi \ \dot{r} + \sin \phi \ \dot{\phi}. \]

Therefore, the acceleration due to Lorentz force in inertial coordinates is given by

\[ \vec{F}_L = \frac{qB_0}{m} \frac{m}{r^2} \left[ -\left( \theta - \omega_e \right) \left( \sin^2 \phi \ \dot{r} + \sin 2 \phi \ \dot{\phi} \right) + \left( \frac{\dot{r}}{r} \sin \phi - 2 \dot{\phi} \cos \phi \right) \theta \right]. \]

As stated in [16], the Lorentz acceleration experienced by the geomagnetic field is decomposed to the three components \((R, T, N)\) (radial, transversal and normal) respectively as functions of orbital elements as follows:

\[ R = \frac{qB_0}{mr^2} \left( \omega_e (1 - \sin^2 \theta \sin^2 (\omega + f)) - \sqrt{\mu/p^3} \cos \left( 1 + e \cos f \right) \right), \]

\[ T = \frac{qB_0}{m \sqrt{\mu p}} \left( \frac{\dot{r}}{r} \sqrt{\mu/p^3} \cos \left( 1 + e \cos f \right) + 2 \omega_e \sqrt{\mu/p^3} \sin^2 \theta \sin (\omega + f) \cos (\omega + f) \left( 1 + e \cos f \right) \right), \]

\[ N = \frac{qB_0}{m \sqrt{\mu p}} \left( 2 \omega_e [1 - \sqrt{\mu/p^3} \cos \left( 1 + e \cos f \right)] - 2 \sqrt{\mu/p^3} \cos \left( 1 + e \cos f \right) \sqrt{\mu/p^3} \cos \left( 1 + e \cos f \right) + \frac{\dot{r}}{r} \sqrt{\mu/p^3} \frac{\cos \theta}{\sqrt{1 - \sin^2 \theta \sin^2 (\omega + f)}} \left( 1 + e \cos f \right)^2 - 2 \frac{\mu}{p^3} \frac{\sin^2 \theta \sin (\omega + f) \cos (\omega + f)}{1 - \sin^2 \theta \sin^2 (\omega + f)} \left( 1 + e \cos f \right)^4 \right). \]
where, $i$, $\omega$, $f$, $a$, and $e$ are the inclination of the orbit on the equator, argument of the perigee, true anomaly, the semi-major axis and the eccentricity of the satellite orbit respectively.

The final form of the potential due to Lorentz force can be written as below.

$$V_L = \hat{\rho}_0 A^T (R, T, N)^T,$$

where, $\hat{\rho}_0 = (x_0, y_0, z_0)$ is the radius vector of the spacecraft relative to the center of mass of the spacecraft, $A^T = \begin{bmatrix} 0 & 0 & 0 \\ \beta_1 & \beta_2 & \beta_3 \\ 0 & 0 & 0 \end{bmatrix}$.

### 2.2 Potential of the magnetic field of the Earth

Let a dipole magnetic field $\vec{B} = (B_1, B_2, B_3)$, and the magnetic moment $\vec{M} = (0, m_3)$, of the satellite. Therefore the potential of the geomagnetic field is

$$V_M = \vec{M}^T \vec{B}$$

As in Wertz [13] we can write geomagnetic field and the total magnetic moment of the orbital system directed to the tangent of the orbital plane, normal to the orbit, and in the direction of the radius respectively as the following:

$$B_1 = \frac{a^3 M_0}{2 r^3} \sin \theta_m [3 \cos(2f - \alpha_m) + \cos \alpha_m],$$

$$B_2 = -\frac{a^3 M_0}{2 r^3} \cos \theta_m,$$

$$B_3 = \frac{a^3 M_0}{2 r^3} \sin \theta_m [3 \sin(2f - \alpha_m) + \sin \alpha_m].$$

$$m_3 = m_g \cos \theta_m,$$

where, $a^3 M_0 = 7.943 \times 10^7$, $\theta_m = 168.6^\circ$ is co-elevation of the dipole, $\alpha_m = 109.3^\circ$, and $f$ is the true anomaly measured from ascending node and $m_g$ is the magnitude of the total magnetic moment. Hence, we can write the potential due to the Earth magnetic field as follows

$$V_M = m_g B_3.$$ 

Therefore, the final form of the potential of the problem is

$$V_0 = \frac{3}{2} \Omega^2 (C - A) \cos^2 \theta + \frac{1}{2} \Omega^2 \sin \theta \cos \psi - (m_3 B_3 - \Omega f) \sin \theta \cos \psi - z_0 \cos \psi [-R - T \cos \theta + N \sin \theta]$$
It is clear that in addition to the first term of gravitational effects, the main parameters of the potential of the satellite motion is the charge-to-mass ratio and \( z_0 \). The Lagrangian can be written as below.

\[
L = \frac{1}{2} A (p^2 + q^2) + \frac{1}{2} C r^2 - V_0.
\]

(25)

It is clear that \( \phi \) is a cyclic variable, therefore we can use \( L_\phi = C r = f^* \) (arbitrary constant). Using equation (24) and the cyclic integral, we can construct the Routhian as follows

\[
R = L - f^* \frac{\partial \phi}{\partial t} = R_0 + R_1 + R_2,
\]

where

\[
R_2 = \frac{A}{2} [\dot{\psi}^2 \sin^2 \theta + \dot{\theta}^2],
\]

\[
R_1 = \Omega \Lambda \dot{\theta} \sin \psi + [\Omega A \sin \theta \cos \theta \cos \psi + f^* \cos \theta] \dot{\psi},
\]

\[
R_0 = \frac{3}{2} \Omega^2 (C - A) \cos^2 \theta + \frac{1}{2} \Omega^2 A \sin \theta \cos \psi - (m_B - \Omega^2) \sin \theta \cos \psi - z_0 \cos \psi [-R - T \cos \theta + N \sin \theta]
\]

(27)

### 2.3 Reduction of the equations of motion

Using Lyapunov method of the holomorphic integral [9], the system of equation of motion are reduced to another simpler form. Let

\[
\sin x = u,
\]

(28)

\[
y = \sqrt{\frac{A}{C}} \int_0^y \frac{dv}{(1 - v^2) \sqrt{1 + m v^2}},
\]

(29)

\[
dt = \mu d\tau,
\]

(30)

\[
\psi = \arcsin u,
\]

(31)

\[
\theta = \arccos \left( \frac{\sqrt{A/C}}{ \sqrt{1 + m v^2}} \right),
\]

(32)

\[
m^* = \frac{A-C}{C}, \mu = A [ \frac{1 - v^2}{1 + m v^2} ].
\]

(33)

In the new coordinates [14], the Routhian function can now be written as below.

\[
R = \frac{1}{2} \left( \frac{\partial \dot{x}}{\partial \tau} \right)^2 + \left( \frac{\partial \dot{y}}{\partial \tau} \right)^2 + \delta_1 \frac{\partial \dot{y}}{\partial \tau} + \delta_2 \frac{\partial \dot{x}}{\partial \tau} + U,
\]

(34)
where
\[ \delta_1 = -u \Omega (1 + m^* v^2)^{1/2} / [1 - v^2]^{1/2}, \]  
(35)
\[ \delta_2 = (1 - u^2) \left[v \sqrt{A/C} + f^* \frac{(1 + m^* v^2)^{1/2}}{v^2 (1 - u^2)^{1/2}}\right] / [1 - v^2]^{1/2} \]  
(36)
\[ U = \mu \left(-\frac{3}{2} \Omega^2 (C - A) - \frac{v^2}{c(1 + m^* v^2)}\right) - \frac{1}{2} \Omega^2 (1 - u^2 + \frac{1}{A} [m_3 B_3 - \Omega f^*]) [1 - u^2]^{1/2} \]  
(37)
\[ v^2 \left[1 - u^2 \right]^{1/2} + z_0 \left[1 - u^2\right]^{1/2} \left[-R - T \sqrt{C} / 1 + m^* v^2 + N \left[1 - u^2 \right]^{1/2}\right]. \]

The potential function in isothermal coordinates
\[ U(x, y, h, f^*). \]

The equation derived from the Routhian can be written as below.
\[ \frac{d}{d \tau} \left( \frac{\partial R}{\partial x} \right) - \frac{\partial R}{\partial x} = 0, \quad \frac{d}{d \tau} \left( \frac{\partial R}{\partial y} \right) - \frac{\partial R}{\partial y} = 0. \]  
(38)

The above equations are transformed to the following planar system of equations.
\[ \frac{\partial^2 x}{\partial \tau^2} = -\Gamma \frac{\partial y}{\partial \tau} + \frac{\partial U}{\partial x}, \quad \frac{\partial^2 y}{\partial \tau^2} = \Gamma \frac{\partial x}{\partial \tau} + \frac{\partial U}{\partial y} \]  
(39)
Where
\[ \Gamma = \frac{\partial}{\partial x} (\delta_1) - \frac{\partial}{\partial y} (\delta_2) = \frac{\cos^2 x}{(1 - v^2)^2} \left(-\Omega v_1 \cos^{-1} x\right) - \left[\Omega m^* - \left(\frac{f^*}{A}\right) \cos^{-3} x \right] (1 + 2v - v^2), \]  
(40)
\[ v_1 = (1 + m^* v^2)^{1/2}. \]
The Jacobi integral of the new system is
\[ h = \left(\frac{\partial x}{\partial \tau}\right)^2 + \left(\frac{\partial y}{\partial \tau}\right)^2 - 2U. \]  
(41)
The region of possible motions can be determined by the inequality
\[ U(x, y, h, f^*) \geq 0. \]

3. The periodic solutions and numerical results

The potential function in isothermal coordinates [9, 14] can be written as below.
\[ U(x, y) = \mu \left(h + w\right), \]  
(42)
\[ w = \frac{3}{2} \Omega^2 (C - A) \frac{v^2(x, y)}{c v_1} - \frac{\Omega^2}{2(1 - u^2(x, y))} + \frac{1}{A} \left(m_3 B_3 - \Omega f^*\right) (1 - u^2(x, y))^{1/2} \]  
(43)
The stationary solutions of the new system of as given in Yehia [14] are
\[
\frac{\partial U}{\partial x} = 0, \quad \frac{\partial U}{\partial y} = 0, \quad \frac{\partial U}{\partial z} = 0.
\] (44)

Here \( \Gamma, u, v \) are functions of \( x, y \); \( U \) is the force function. The system (44) can be written as
\[
\frac{\partial w}{\partial x} = 0, \quad \frac{\partial w}{\partial y} = 0, \quad h + w = 0.
\] (45)

We assume that \( P_i(x_i, y_i) = P_i(f^+), i = 1, 2, 3, \ldots \) are the stationary solutions and \( h_i = h_i(f^+) \) is the corresponding energy constant. Equations \( \frac{\partial w}{\partial x} = 0 \) and \( \frac{\partial w}{\partial y} = 0 \) will give the values of \( x_i \) and \( y_i \) and \( h + w = 0 \) will determine the energy constant.

To find periodic solutions around the stationary points for some values of \( f^+ \), let
\[
\begin{align*}
\xi &= \sum_{i=1}^{\infty} c^i x^{(i)}(\tau) + c^i h_i, \\
y &= \sum_{i=1}^{\infty} c^i y^{(i)}(\tau),
\end{align*}
\] (48)

Here \( c^i \) are parameters, \( h_i \) are constants and
\[
x^{(i)}(\tau) = x^{(i)}(\tau + T), y^{(i)}(\tau) = (\tau + T).
\] The period \( T \) can also be written as
\[
T = \frac{2\pi}{\vartheta} = \frac{2\pi}{\vartheta_0} (1 + \sum_{j=1}^{\infty} \gamma^j)
\] (49)

To simplify the system, transform the variable \( u \).
\[
u = \lambda \tau.
\] (50)

In the same way as in [14] we approximate \( x^{(i)} \) and \( y^{(i)} \) as follows.
Periodic solutions for rotational motion

\[ x^{(s)} = a_{1s} + \sum_{r=1}^{\infty} (a_{1r}^{(s)} \cos ru + b_{1r}^{(s)} \sin ru), \]
\[ y^{(s)} = a_{2s} + \sum_{r=1}^{\infty} (a_{2r}^{(s)} \cos ru + b_{2r}^{(s)} \sin ru). \] (51)

For \( s = 1 \), equation (46) takes the following form
\[ x = x_0 + c x^{(1)}, \quad y = y_0 + c y^{(1)}, \] (52)

The time \( t \) and the coordinates satisfy the following relationship.
\[ t - t_0 = \int_0^\mu (x(\tau), y(\tau)) d\mu. \] (53)

Therefore equations (41) reduces to:
\[ \lambda_0^2 \frac{d^2 x^{(1)}}{du^2} + \Gamma_0 \lambda_0 \frac{dy^{(1)}}{du} + \zeta + \eta = 0, \]
\[ \lambda_0^2 \frac{d^2 y^{(1)}}{du^2} - \Gamma_0 \lambda_0 \frac{dx^{(1)}}{du} + \zeta + \eta = 0, \] (54)

The zero subscript in the above equations corresponds to the value \( x = x_0, y = y_0 \),
\[ \zeta = -\mu_0 \frac{\partial^2 w}{\partial x^2}, x = x_0, y = y_0, \]
\[ \eta = -\mu_0 \frac{\partial^2 w}{\partial y^2}, x = x_0, y = y_0, \]
\[ \xi = -\mu_0 \frac{\partial^2 w}{\partial x \partial y}, x = x_0, y = y_0. \] (55)

Equation (54) has the following characteristic equation.
\[ \lambda_0 = \frac{1}{\sqrt{2}} \sqrt{\zeta + \eta + \Gamma_0^2 \pm \sqrt{\left( \zeta + \eta + \Gamma_0^2 \right)^2 - 4(\zeta \eta - \xi^2)}}. \] (56)

Let's consider the two possible cases.
1) \( \zeta \eta - \xi^2 > 0 \) (\( w \) has extremum at \( P_0 \)). In this case two different frequencies are possible if the following condition is satisfied.
\[ \Gamma_0^2 > 2 \sqrt{\zeta \eta - \xi^2} - \zeta - \eta. \] (57)

The above inequality holds at all points where \( w \) has a maximum.
2) \( \zeta \eta - \xi^2 < 0 \) (\( w \) has saddle point at \( P_0 \)). The only possible frequency in this case is when the positive sign is taken in equation (56).
In the case of non-commensurable frequencies the values of $h$, $c$, and $T$ can easily be obtained. For a bounded $|c|$ the series will absolutely converge. The following value of $x^{(1)}, y^{(1)}$ and $h$ are obtained from equations (54).

$$x^{(1)} = (\lambda_0^2 - \eta), \quad y^{(1)} = \xi \sin u - \Gamma_0 \lambda_0 \cos u, \quad h_1 = 0.$$  (58)

The system of equations given below is the first approximation of the solution of equation (41).

$$x = x_0 + (\lambda_0^2 - \eta) c \sin u,$$
$$y = y_0 + c(\xi \sin u - \Gamma_0 \lambda_0 \cos u),$$
$$h = h_0.$$  (59)

The periodic solution in equations (59) could be transformed again as a solution in $\theta$ and $\psi$. To aid the understanding of the existence of periodic orbits some numerical examples are given in figures (1-3) for various charge to mass ratios. In figure (1a), the charge to mass ratio is taken to be 0.001. It can be seen here that two types of orbits exist in this special case. One type is the usual elliptical orbits and the other is a figure 8 type of orbit. Figure 8 type of orbits are not very common. The numerical example in figures (1b, 1c) shows the dependence of the position of the periodic orbit around the stationary positions on the charge-to-mass ratio. These figures explains that the Lorentz force have significant influence on the positions of the periodic solutions.

For different values of charge-to- mass ratio figures (1-3) show the oscillation of the energy integral $h$, and the maximum and saddle points of the energy integral as in equation (69).

Figure 1. The phase plane of $\theta-\psi$ at a) $q/m = 0.001$ b) $q/m = 0.02$ c) $q/m = 0.3$. 
Figure 2. The maximum and saddle points of the energy integral $h$ at $q/m = 0.001, 0.1$.

Figure 3. The maximum and saddle points of the energy integral $h$ at $q/m = 0.2, q/m = 0.5$.

4. Conclusions

This paper presented the sufficient conditions for the existence of periodic solutions close the stationary motion of an axially symmetric charged satellite in an elliptic orbit. We investigated the rotational motion of the satellite under the action of the Lorentz forces, gravitational and magnetic fields of the Earth. The Routhian function of the satellite motion is constructed. The equations of motion of the satellite are reduced to the planar equations of motion. The existence of periodic solutions are obtained using Lyapunov method of the holomorphic integral. Numerical
results have shown the effects of Lorentz force, in particularly the charge to mass ratio on the position of periodic solutions and the maximum and saddle points of the energy integral. In addition to the elliptical orbits some very exotic figure 8 orbits have also been identified in the vicinity of stationary motion.

References


Received: June 15, 2015; Published: February 27, 2015