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Currency Band and the Approximations: Fitting of Rouble with 3-Parametric Functions

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Abstract

Experimental data on the time evolution of value of Russian rouble are approximated with elementary functions. Range since 2014.02.20 to 2015 February is considered. Data that are already available for the moment of submission are used. Four 3-parametric approximations are compared. The slow dependence of these approximation on the date of its preparation is interpreted as high predictive ability of such a fitting. The approximations are suggested as a guide to establish the time-dependent currency band. Such a band could be useful for the stabilisation of short time fluctuations of the currency exchange rate.

Mathematics Subject Classification: 65D99 Numerical approximation

Keywords: Fitting of experimental data, approximation with elementary functions

1 Introduction

The currency band refers to some specific interval of values of a national currency, compared with currencies of other countries. This interval indicates that the National Bank sells the national currency each time when it becomes above the upper bound of the currency band, and buy it, as soon as it becomes cheaper than quantity specified in by the lower bound of the currency band. In the simple case, the currency band is specified with two values of the exchange rate, price P_1 and price P_2; with the operations mentioned, the Bank can:

(A) keep the value of the national currency within the specific interval and (B) get some benefit, profit, interest, buying the currency by the price P_1 and selling the currency at price P_2, which is larger than P_1.

Usually, it is assumed, that during some period of time (years, months or weeks), values P_1 and P_2 are constants [1]. At the rapid and strong inflation, the concept of the constant currency band requires frequent reconsideration, modification of values P_1 and P_2. The currency suffer sudden jumps, for the profit of those, who are first to get information about modification of the currency band and, especially, for friends of those, who determine the date of modification of the currency band and new values of P_1 and P_2. These persons become interested in fast and strong oscillation of value of the national currency. I am not economist, but, looking at the behaviour of course of the national currency of countries with strong inflation, I suspect this mechanism to be dominant in generation of fluctuations of the national currency.

The possible improvement of the financial system, while the fast inflation is observed, could be establishing of values $P_1 = p_1(x)$ and $P_2 = p_2(x)$, dependent on time x. The time-dependent currency band should cover, include the smooth trend of variation of the course of the currency. In order to establish the time-dependent band, some kind of forecast of the inflation is necessary.

I consider the example approximations with elementary functions; and take into account only fits with two and with three parameters. The generalisation to more complicated approximations is straightforward. Each fitting, approximation of experimental data, should be considered as a primitive model, as a concept. Following the TORI axioms [2], the important criteria to choose among various concepts are the range of applicability and the simplicity. The applicability in this case includes not only the time interval, for which the approximation is valid, but also range of precision: which deviation from the new experimental data is considered a confirmation of the concept, and which is considered as contradiction. The concept (fit) may be applicable while the precision of one significant figure is sufficient, and not applicable, if the two significant figures are required.

In this article, I describe one example of application of declaration above to the evolution of rate of the Russian rouble since 2014.02.20 to the date of submission (2015.01.09). Here I consider only approximations with elementary functions, and only those with 2 and 3 adjusting parameters. I show, that even with 3-parametric functions, the data for the time interval of order of a year can be approximated with error smaller than the shorttime fluctuations of the currency exchange rate.

Some forecasts of the ratio of currencies are available [4], but the sites do not indicate the way they the estimates are elaborated; therefore, these estimates cannot be used of the scientific analysis. The goal of this paper is to show method, that could be used to predict the trend of currencies and to smooth the fast oscillations. Below, I consider the experimental data for the exchange rate between the Russian rouble and the USA dollar. The analysis of data for the European Euro and the Japanese Yen lead to the similar observations [3]. Usually, the forecasts approximate logarithm of ration of currencies, with goal to predict the short time fluctuations, taking into account, in particular, variations from weekdays to weekends and the holidays [5, 7, 6].

On the first look, with the strong achievement of the forecasting, variations of the courses of currencies are supposed to be well predicted, and many agencies could smooth the fluctuations, and the strong fluctuations cannot take place at all. On the other hand, the strong fluctuations, comparable to the value of a currency, are observed. For this reason, I suggest an alternative approach, based on the simulation of general trend (and ignoring variation of the course from working days to holidays).

Here I use data by [8]. Site [9] gives similar (a little bit more sparse) data, and can be used too.

2 Experimental data and the two-parametric approximations

In this article, time is measured in days since the beginning of the project (2014,10,27), when the first linear approximation for the short interval had been suggested and then happened to be upper bound for the all later time interval [3]. This time x is calculated from the date (year,month,day), specified in [8], with

$$x = \mathrm{daju24}(\mathrm{year}, \mathrm{month}, \mathrm{day}) - \mathrm{daju24}(2014, 10, 27) \tag{1}$$

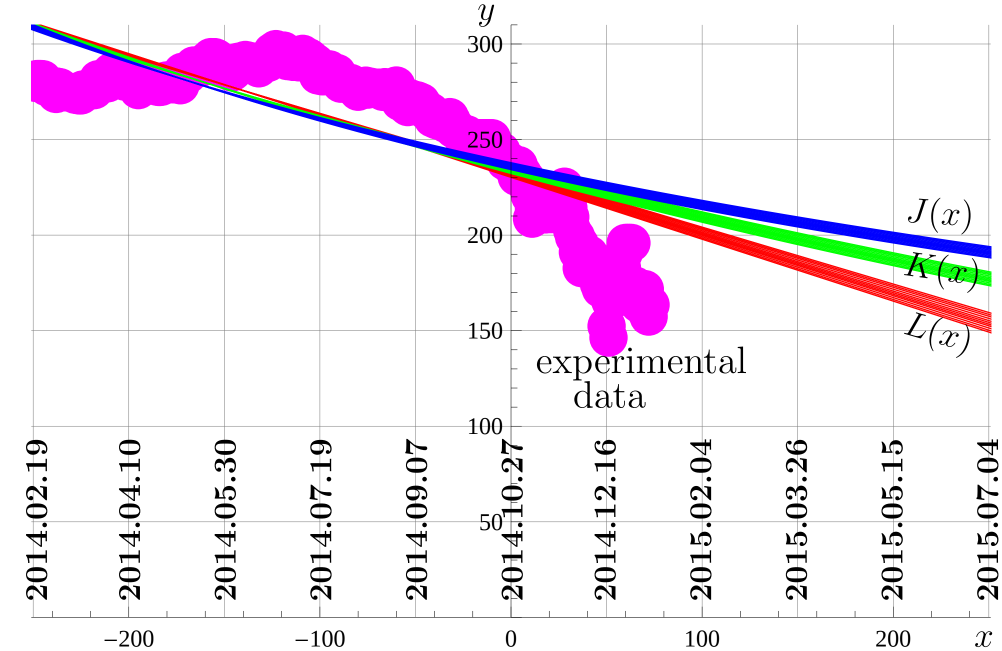
with function daju24 defined below in C++:

```
int daju24(int Y,int M, int D){ int a, y, m;
a = (14-M)/12;  y = Y + 4800 - a;  m = M + 12*a - 3;
return D+(153*m+2)/5+365*y+y/4-y/100+y/400-32045 - 2400000; }
```

In such a way, the dates loaded from [8] are converted to values of the abscissa coordinate x.

The original data correspond to the price of the 1000 USA dollars, expressed in roubles. For the analysis, it is easier to work with value of 100 roubles, expressed in the USA cents. This quantity is calculated from the original data with transform $z \mapsto 100000/z$. This determines the ordinate y, shown in graphics below.

The original data are shown in figures 1 and 2 with circular spots. These spots form the thick pink line.



http://mizugadro.mydns.jp/t/index.php/File:2014rubleDollar2param.png

Figure 1: Data by [8] and the approximations by (2), (3) and (4)

In addition to experimental data, in figure 1, three two-parametric approximations are shown. I denote these approximations with letters J, K and L;

$$y = f_{21}(x) = J(x) \;=\; 1/(a+bx) \tag{2}$$
$$y = f_{22}(x) = K(x) \;=\; \exp(p+qx) \tag{3}$$
$$y = f_{23}(x) = L(x) \;=\; u + vx \tag{4}$$

where parameters a, b, p, q, u, v, depend on the time $X = x_m$, when the approximation is built-up. Values of these parameters are chosen to minimise the mean-square deviation

$$Q = \sqrt{\frac{1}{m}\sum_{n=1}^{m}\left(y_n - f(x_n)\right)^2} \tag{5}$$

where f denotes J, K or L. The same formula is used also below for other f.

For positive values of time x, the two-parametric approximations above remain as the upper bound of the trend. The initial linear approximation $y=240-0.48x$ by [3], made at $x=0$ and not shown in the figure, also remains as the upper bound (at least for $x < 90$, id est, at least to the date of the last revision of this article). It can be considered as a a special case of function L by (4), and it looks optimistic in compare with function L, obtained with minimisation of the mean-square deviation Q.

For $M - 10 \le m \le M$, the mean square deviation is minimised, and the corresponding 11 lines are drown. Positions of these lines depend on the moment of preparation of the approximation; for larger interval for m, the three bunches of these lines overlap. At $m = M$, the mean square deviations for approximations J, K and L are, correspondently, 24.74, 23.57 and 22.18; in such a way, the linear approximation happens to be a little bit better, than the other two two-parametric approximations J and K.

I assume, that the experimental data has some smooth trend, with some additional noise added. In this case, the deviation of each of the approximations above from the original (experimental) data looks large, compared to the noise of the data. This indicates, that the use of three-parametric fits is methodically-correct. Approximations with 3 parameters are considered in the next section.

3 Three parametric approximations

All the three approximations shown in figure 1, have the same defect: their second derivatives are either zero, or positive, while the main trend of the experimental data, contrary, visually has negative second derivative. This property can be taken into account with the three-parametric approximations shown in figure 2.

I am a bit out of single-character names for the functions and their parameters. So, I use the following names for the approximating functions: Bell, Gauss, Quadratic and Ellipse. These functions are plotted in figure 2, in the similar way, as the two parametric approximations are plotted in figure 1. Assuming some dummy parameters a, b, c, the approximations shown in figure 2 can be defined as follows:

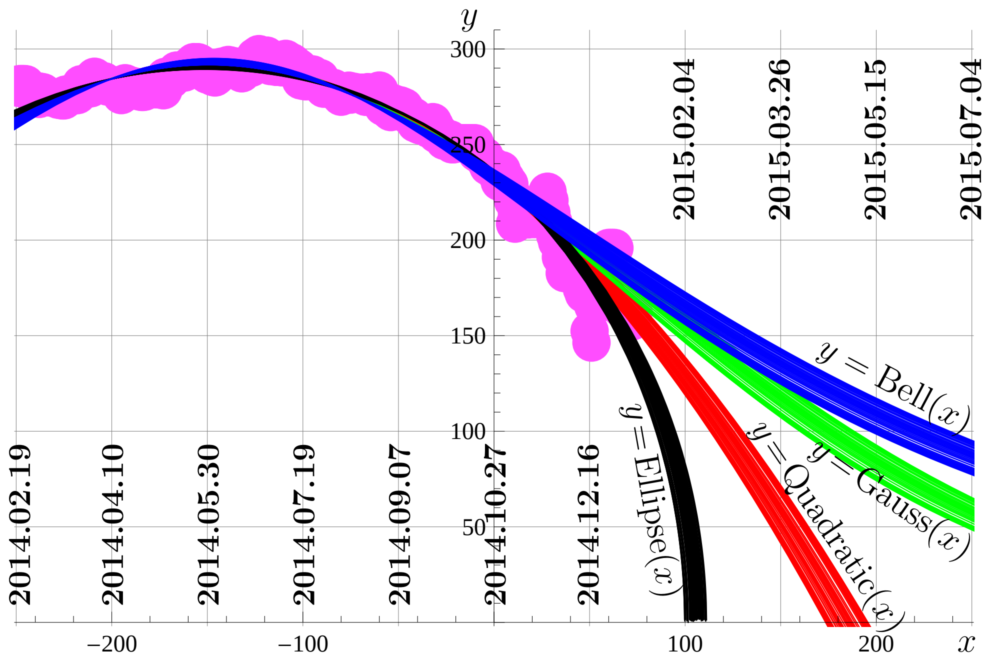
$$\mathrm{Bell}(x) = a/\cosh\left(b(x-c)\right) \tag{6}$$

$$\mathrm{Gauss}(x) = a\exp\left(b(x-c)^2\right) \tag{7}$$

$$\mathrm{Quadratic}(x) = a + bx + cx^2 \tag{8}$$

$$\mathrm{Ellipse}(x) = a\sqrt{(b-x)(c+x)} \tag{9}$$

However, values of parameters a, b, c may be different for the four approximations mentioned. As in the case of two parametric fitting, the mean-square deviation (5) had been minimised for $f = \mathrm{Bell}$, $f = \mathrm{Gauss}$, $f = \mathrm{Quadratic}$, and for $f = \mathrm{Ellipse}$. As in the case of two parametric approximations, for each formula, the set of curves is drawn, corresponding to different values of m. Variations of the 3-parametric approximations are smaller, than variation of two-parametric fits in figure 1; and I could draw 51 lines for each approximation, id est, results of minimisation expression 5 for wider interval $M - 51 \le m \le M$. The curves look as a four strips, well separated at least in



http://mizugadro.mydns.jp/t/index.php/File:2014rubleDollar3param.png

Figure 2: Data by [8] and the approximations by (6), (7), (8) and (9)

the right hand side of the figure. These strips look still thin in comparison to the fluctuations of the experimental data in vicinity of some intuitive trend. At $m = M$, the mean square deviations of these approximations are 8.86 for Bell, 8.00 for Gauss, 7.08 for Quadratic, and 7.16 for Ellipse. The primitive polynomial approximation happens a little bit better, than the 3 other approximations, but this advantage is small and cannot be interpreted as important argument in favour of namely this approximation. For example, if the approximations are constricted at time $x \approx 50$, then, contrary, the approximation Ellipse seems to be better than others.

Several similar three-parametric approximations can be considered in the similar way; they either have mean square deviation bigger, than the four approximations shown in figure 2, or lie between the lines already shown in the figure. In such a way, the four approximations roughly show, what can we expect from the evolution of analysed quantity, and how slowly do these expectations vary while taking into account new and new data.

The four approximations shown in figure 2 have the important common property: Each of them covers the mean trend of the experimental data. The data go up and down, again and again, crossing the approximating function many times. This property seems to be stable: if the experimental curve happens to be well below the curves, then, within some 15 days, it happen well above them, and vice versa. The same property is seen also in the earlier approximation with ellipse with parameters $a\!=\!1$, $b\!=\!123$, $c\!=\!471$, constructed at $t\!=\!10$ for the shorter array of data [3].

The deviation of the approximations from the experimental data can be even reduced, considered fittings with more parameters. In particular, the quasi-regular oscillations in the right hand side of the experimental curve can be taken into account, assign some grown-exponential term with additional four parameters (amplitude, increment, frequency and phase). However, these parameters are not so stable, as a, b, c in the approximations above, and the improvement of the agreement with experimental data is not so drastic, as that due to the change from two-parametric approximations at figure 1 to the three-parametric approximations at figure 2.

The calculus above should be easy to reproduce and to apply for other temporal sequences. The example show the importance of the stability as criterion of choose the formal model. This stability should be considered as one of the most important properties of the model, the prediction should not show fast variation, while new data are taken into account.

4 Conclusion

The approximations of the experimental data for the ratio of two currencies, the Russian rouble and the USA dollar are considered. In order to simplify the labels in the figures, the price y of 100 roubles is evaluated in the USA cents; time x is measured in days. The figures 1 and 2 show, that the smoothed trend of the experimental data can be taken into account by fitting with 3-parametric elementary functions. Parameters of these functions look stable, while the new experimental data are added, at the scale to order of 50 days. The typical scale of strong variation of y counts or order of 15 days. This indicates, that the 3-parametric fits considered could be useful tool for stabilisation of fast variation of the rates of exchange of currency, especially in condition of strong inflation, while the random variation of the value becomes of the same order of magnitude as the value itself.

For 2015 January, it is yet difficult to indicate, which of the four fits (6), (7), (8), (9) will provide better agreement with experimental data in Future. However, each scientific concept has limited range of applicability. In particular, functions Esllipse and Quadratic by (9) and (8) show their limits of applicability explicitly, it is limited with some hundred days since the elaboration.

The method suggested, namely, use of criterion of stability of the approximation, allows to establish the time-dependent currency corridor. This corridor should allow to avoid the short-scale fluctuations of the currency value at the money exchange. In particular, in the case of rouble/dollar in 2014-2015, function $p_1 = \mathrm{Ellipse}$ could be used as the lower bound, and $p_2 = \mathrm{Bell}$ could be used as the upper bound, at least at the scale of order of 30 days, where the strong fluctuations are observed.

The data for ratio rouble/dollar are chosen because they show the strong fluctuations around the smooth trend. The origin of these fluctuations and construction of the appropriate mathematical model may be matter for future research. Other currencies, at the strong inflation, may show similar properties and should allow application of the same methods.

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