Mathematical and Economic Model of Generators’ Strategies on Wholesale Electricity Markets

E.M. Lisin

Department of Economics in Power Engineering and Industry
National Research University “Moscow Power Engineering Institute”
Krasnokazarmennaya 14 – Moscow, Russia

A. Y. Amelina

Department of Economics in Power Engineering and Industry
National Research University “Moscow Power Engineering Institute”
Krasnokazarmennaya 14 – Moscow, Russia

W. Strielkowski

Charles University in Prague, Faculty of Social Sciences, Smetanovo nábř. 6
11001, Praha 1, Prague, Czech Republic

V. K. Lozenko

Department of Economics in Power Engineering and Industry
National Research University "Moscow Power Engineering Institute"
Krasnokazarmennaya 14 – Moscow, Russia

O. V. Zlyvko

Department of Economics in Power Engineering and Industry
National Research University “Moscow Power Engineering Institute”
Krasnokazarmennaya 14 – Moscow, Russia

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Abstract

This paper explores a model that may be used in driving the decisions of generator companies for strategies to be pursued in the competitive sector of the Russian electricity market for day-ahead electric energy and power deliveries. A mathematical economics model of the market was developed to test various strategies for placing offers into the trading system, including a strategy based on marginal cost pricing, strategies involving the exercise of market power by withdrawing available power physically or financially, and a mixed strategy. Our yardstick for choosing the optimum strategy was the daily profit earned by the generator. For computation, the model was fed with a sample of online generating capacity of electric utilities operating within the South Russian Unified Power System operations zone.

Keywords: wholesale electricity and power markets, day-ahead market, marginal pricing, generator strategy, market power, mathematical economics model

1 Introduction

Currently, Russian Federation’s wholesale electricity and power market (commonly known as OREM) comprises a totally new framework of business transactions in the electric utilities industry, with a competitive environment that shares many properties of “perfectly contestable markets”.

The OREM model follows the rules of deregulated electricity markets while paying heed to specific Russian nuances and incorporates the experience of PJM Interconnection, one of the world’s first deregulated electricity markets that has brought together transmission networks operated by utility companies in the states of Pennsylvania, New Jersey, Maryland, Delaware, Virginia and Ohio [1; 2].

The emergence of electricity markets in US and most-advanced European countries in late 20th century was driven by the growing discontent with the quality of services provided by a number of natural-monopoly industries as well as accelerating globalization trends and the birth of unified global markets.

This discontent with poor performance of natural monopolists was reinforced by works of W. Baumol, H. Demsetz, J. Panzar, R. Posner, G. Stigler, R. Willig [3-8] Without denying that some industries may experience efficiency gains when organized into monopolies or oligopolies, William Baumol has devised a new model of economic relations featured by optimum behavior of market players that has become known as a “perfectly contestable market”. In essence this model is a generalization of the perfectly competitive market concept to encompass a broader spectrum of industry arrangements including monopolies and oligopolies [3; 4].

A perfectly contestable market has the following features [3; 4]:

- No entry/exit barriers including cost discrimination against entrants
- No inefficient production in industry-wide equilibrium
• When a market includes two or more sellers, no goods can be sold at a price above or below marginal costs.

Basic principles of perfectly contestable markets have shaped the deregulated organizational model of modern wholesale electricity markets. In particular, marginal pricing is seen as the optimum pricing method in the OREM market. Marginal pricing enables generators to recoup their semi-variable costs that are dependent on the relative increase in the amount of heat (and, therefore, fuel) consumed per unit electricity output. However, as long as the OREM electricity trading sector (known as the day-ahead market or RSV) fails to meet all preconditions for a perfectly contestable market, a gap opens between bid and ask prices (Figure 1) [9-11].

![Figure 1. Centralized auction at the RSV market](image)

The problem identified above may be solved in two ways:
• By improving the existing pricing rules
• By improving operational management of generator companies to adjust for new conditions.

Considering that the RSV market concept has seen no global changes since late 2008 even though electricity market regulations have remained in a state of flux, it would be reasonable for generators to embrace market strategies that would let them adapt their market behavior to new competitive conditions. As the marginal strategy becomes suboptimal for generators in the electricity market in many cases, generators may benefit from market-power mechanisms based on withdrawing power from the electricity market either financially or physically. This paper will focus on the efficiency of these generators’ strategies, invoking a model for benchmarking the latter against the marginal strategy.
2 The mathematical economics model of the wholesale electricity and power market

The wholesale electricity and power market is comprised by three electricity trading sectors:
- Bilateral contract market (RDD/SDD)
- Day-ahead market (RSV)
- Balancing market (BR).

At the bilateral contract market, electricity is traded under regulated (RDD) and free (SDD) bilateral contracts. In the regulated sector, rates for electricity supplied to and bought at the wholesale market are set by the Federal Tariff Service of Russia.

The day-ahead market, RSV, is used to sell/buy excess/lacking amounts of electricity to complement volumes traded in bilateral contracts. Electricity is traded at RSV at prices influenced by supply and demand.

![Offer submission flowchart for VSVGO and RSV](image)

\[ c_b, c_r, P_b, P_c \] are the “prices” and “volumes” of generator and retailer companies, \( P_{\text{min}} \) and \( P_{\text{max}} \) are technically feasible minimum and maximum power outputs, \( c^* \) is the equilibrium price, \( \delta_b \) is a Boolean variable (1 or 0 for the online/offline output condition of generating unit \( b \) of generator \( g \) taking part in VSVGO, at the end of the period in question).

The balancing market, BR, accommodates deviations of actual hourly output/consumption figures from the scheduled electricity trading volumes and serves to balance supply against demand in real time. Rewards are paid to generators who adjust their electricity output in response to System Operator’s (SO) initiative; at the same time, generators reducing their output unilaterally as well as load-increasing consumers are penalized with extra charges [11; 12].
Sales at RSV are preceded by a bidding procedure for selecting generator plant to be brought online, known as VSVGO and carried out a week ahead of the trading day. Prices from offers submitted by generators for VSVGO then serve as ceiling prices for offers submitted to RSV and BR markets throughout the following week. The diagram below (Figure 2) illustrates the influence VSVGO bidding has on RSV trading outcome.

3 The mathematical economics model of price discovery in the day-ahead market

Let us put together a mathematical economics model for assessing the outcome of a generator applying various market strategies at the day-ahead market.

Assuming a power grid comprised by several nodes, let $q$ be the number of generator companies supplying electricity using $n$ numbers of similar plant belonging to various supply point groups (SPGs) so that the number of plant is equal to the number of SPGs. Assume that this equipment has passed the VSVGO procedure.

Let us further assume that there are no limits on the amount of power that can flow between the nodes. In this case the respective counter flows will act to balance prices at nodes (Figure 3). A single price $[9; 11; 13]$ will arise as a result.

![Figure 3. RSV auction without internodal flow constraints](image)

$P_1^*, Q_1^*, P_2^*, Q_2^*$ are equilibrium prices and electricity sales volume at the RSV market for Nodes 1 and 2, respectively

Let generator $A$ have a set of generating plant $i$ ($i = 1...k$) that generates electricity to be sold at the transaction node of the electricity market model in accordance with daily demand pattern. Every hour, the electronic trading system of the electricity market will receive two-stage offers for every generator plant / SPG indicating the desired amount of electricity to be sold and its price per megawatt-hour.

Stage one is a price-taker offer, covering generator’s minimum safe output at zero price $V_{ij}^{\text{min}}$ where $j$ ($j = 0...23$) is the respective hour of the day. Price-
taker offers mean that the generator agrees to sell its electricity at whatever price the market may bid up.

Stage two is offered at price $P_{ij}$ for volume $V_{ij}$ which is determined as follows (1):

$$V_{ij} = V_{ij}^{exc} + V_{ij}^{min},$$  \hspace{1cm} (1)

Where $V_{ij}^{exc}$ is the amount of electricity produced by generator plant $i$ in excess of its minimum safe output.

Every generator plant $i$ will have a particular generation cost $C_{ij}$ (cost of electricity per 1 MWh) that is a function of the amount of electricity produced, following the formula (2):

$$C_{ij} = E_{eeij} / V_{ij},$$  \hspace{1cm} (2)

where $E_{eeij}$ are total electric energy production expenses.

Every change in output is accompanied with a change in the cost of electricity. The cost of electricity can be calculated based on the output performance of the generator plant. Cogeneration heat-and-power plant (CHP) deserves special attention in that regard. Given that CHP plant operates as a price-taker when submitting offers for its electricity output in cogeneration mode (when heat and electricity are produced together), the cost of electricity for CHP plant will be assessed by us with reference to its condensation cycle which is the standard operating mode of thermal power plant that only produces a single type of energy.

Offers are submitted so that the condition $P_{ij} \geq C_{ij}$ is met. The electronic system sorts offers in ascending order by price $P_{ij} \leq P_{2j} \ldots \leq P_{qj}$.

Figure 4 shows a graphical interpretation of the price discovery model for the RSV market. Only in a single potential situation may the consumer act as a price-setter when bidding for electricity (Figure 4a). In all other instances (Figures 4b, 4c), a generator’s offer sets the price [10-12; 14].

![Figure 4](image.png)

**Figure 4.** Price discovery at the RSV market
Assume that the demand side dominates the electricity market and the
demand and supply curves intersect. Further assume that the demand is price-
inelastic. This brings some simplification into the trading model so that daily
demand may be represented as (3):

\[ Q_0 = \sum_{j=0}^{23} Q_{0j}, \quad (3) \]

where \( Q_{0j} \) is the hourly demand for electricity (a constant).

**Figure 5.** Price discovery at the RSV market assuming priceinelastic demand

The intersection point of demand and supply curves determines the
equilibrium market price (\( P_0 = P_{m} \)) at which electricity will be sold in the
transaction node of the market model where \( m (m = 1...n) \) is the index number of
supplier, matching the total number of offers accepted by the market. A price-
setting offer will be understood as the last offer to be accepted by the market.

The equilibrium market will change following daily fluctuation of the
demand for electricity in the transaction node, and so will the price-setting offer.

Let us now determine the amount supplied which should obviously be equal
to the amount demanded, \( V_{0j} = Q_{0j} \). To that end, consider the following cases:

1. The market accepts the entire amount from generator’s price-setting offer
   at the specified price (Figure 5a)
2. The market only accepts a part of generator’s price-setting offer (Figure
   5b).

In the latter case, the amount rejected by the market from the final offer that
Closes price discovery can be determined for every hour of the day \( j \) as follows (4):

\[ \Delta Q_{0j} = \sum_{i=1}^{m} V_{ij}^{\text{exc.}} + \sum_{i=1}^{n} V_{ij}^{\text{min.}} - Q_{0j} \quad (4) \]

**4 The mathematical economics model of generator’s strategy in the day-ahead market**

Let us assume that all generator companies begin with the marginal strategy,
submitting their offers at prices equal to electricity generation cost \( P_{ij} = C_{ij} \) that
only includes variable costs, and further assume that demand exceeds supply i.e. is price-inelastic.

The marginal strategy calls for submitting offers based on marginal costs i.e. the relative increase in production cost per megawatt-hour output. It is known that the optimum utilization of power plant has no relation to its semi-fixed costs. For that reason variable costs are included in the expense function.

A generator may benefit from submitting offers using the marginal cost strategy only when all market players follow the same strategy. Power plant will be loaded most efficiently in this case while the market price and volume will be close to the competitive equilibrium point [12; 15; 16].

Assuming uniform load on turbine generator units, the cost of electricity production behaves as a function of volume (5):

\[ C_{ij} = f \left( \frac{V_{ij}}{n_i'} \right), \]

where \( n_i' \) is the number of turbine units at plant \( i \).

The cost of electricity produced below the minimum safe output level is determined using the formula (6):

\[ C_{ij}^{\text{min}} = f \left( \frac{V_{ij}^{\text{min}}}{n_i'} \right) \]

Let \( V_{ij}' \) be the electricity sales volume for power plant \( i \) in hour \( j \). As a result of competitive bidding the market will reject a part of electricity corresponding to offers submitted at prices above equilibrium.

The following cases are possible:

1. If \( P_{ij} < P_0 \) then \( V_{ij}' = V_{ij}^{\text{exc}} + V_{ij}^{\text{min}} \)
2. If \( P_{ij} = P_0 \) then \( V_{ij}' = V_{ij}^{\text{exc}} + V_{ij}^{\text{min}} - \Delta Q_{ij} \)
3. If \( P_{ij} > P_0 \) then \( V_{ij}' = V_{ij}^{\text{min}} \).

Now let us raise the equilibrium price \( P_0 = P_{m_j} \) to the next price level \( P_{(m+1)j} \). This can be achieved in practice by physical or financial withdrawal.

Physical and financial withdrawal are two market strategies that seek to remove a part of low-priced offers from the market either by raising price or by minimizing underpriced supply. In both instances these strategies should be viewed as exercise of market power [9; 11]. Table 1 shows the potential value that may be obtained by applying them.

**Table 1. Basic strategies available to generators for exercising market power**

<table>
<thead>
<tr>
<th>Strategy name</th>
<th>Application mechanism</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial withdrawal</td>
<td>Place electricity supply offers at a higher price (exceeding marginal electricity production costs in particular).</td>
<td>In a demand-dominated market (with demand becoming inelastic) this should increase the equilibrium market price and potential generator’s revenue as a consequence.</td>
</tr>
</tbody>
</table>
Table 1. (Continued): Basic strategies available to generators for exercising market power

<table>
<thead>
<tr>
<th>Physical withdrawal</th>
<th>Offer reduced amounts of electricity at the market relative to the full output capacity of company’s generator plant.</th>
<th>A part of the supplied power is taken away from the market causing the equilibrium price to go up.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical withdrawal by means of free bilateral contracts</td>
<td>Make free bilateral contracts aiming to reduce the amount of cheap product offered at the market.</td>
<td>This will make the equilibrium market price go higher if electricity consumption increases.</td>
</tr>
</tbody>
</table>

Let $M = \{(P_{ij}, V_{ij}) | i = 1...m; j = 0...23\}$ be a set of generator offers accepted by the market in a particular hour of the day. Let $N = \{(P_{ij}, V_{ij}) | i = 1...n; j = 0...23\}$ be the set of offers accepted by the market from generator $A$. It is obvious that $N \in M$.

Consider three cases of market price application by a generator company corresponding to the following computation algorithms:

1. In the case of financial withdrawal, for every offer of generator $A$ that the market accepts we would raise the quoted electricity price to the next price level $P_{0j} = P_{(m+1)j}$ in a manner that would prevent us from falling out of the market’s trading schedule. Then the permissible price increase for every hour of the day $j$ may be defined as: $\Delta P_{ij} = (P_{(i+1)j} - P_{ij} - 0.01)$ with the condition that $P_{(i+1)j} > P_{ij}$ for all $(P_{ij}, V_{ij}) \in N$.

2. In the case of physical withdrawal, we would seek to raise the equilibrium price to the next level by reducing the desired amount of output in generator $A$’s offers accepted by the market by $\Delta V_{ij} = (\Delta Q_j + 1)R$ with the condition that

$$\sum_{i=1}^{\infty} V_{ij} + (V_{mj} - \Delta Q_j) \geq (\Delta Q_j + r) \text{ for all } (P_{ij}, V_{ij}) \in N.$$ 

3. In the case of mixed strategy, we would apply physical and financial withdrawal mechanisms simultaneously.

For every strategy, we will view generator’s profits as a yardstick of its economic efficiency.

Let $R$ be a set of generator company $A$’s generating assets. In view of strategy choice, generator $A$’s daily revenue $L_d$ can be determined using the formula (7): [9, 12, 17]

$$L_d = \sum_{j=0}^{23} \sum_{i=1; i \in R}^{n} (V_{ij}^p (P_{mj} - C_{ij}^P)),$$

(7)

where $C_{ij}^p$ is the cost of electricity production that behaves as $C_{ij}^p = f(V_{ij}^p / h_i^p)$.
With reference to algorithms described above, let us compute the revenue earned by generator A applying the marginal strategy as well as market-power strategies. For every hour of the day, we will choose a strategy that maximizes the profit, referring to it as the best strategy.

5 Findings from computations using the model

We will now review findings from using our model with computations of value captured by generator companies pursuing individual strategies in the day-ahead market.

For computation purposes, the mathematical economics model devised by us was fed with a sample [18] of online generating capacity of electric utilities operating within the South Russia Unified Power System operations zone (South UPS).

South UPS includes Astrakhan’, Volgograd, Dagestan, Kalmykia, Karachay-Cherkessia, Kabardino-Balkaria, Kuban’, Rostov-on-Don, North Ossetia, Stavropol, Chechnya and Ingushetia regional grids. As of 2014, South UPS had an installed capacity of 17376 MW with annual production of 69229 million kWh. Table 2 breaks down the installed capacity and electricity production [18].

<table>
<thead>
<tr>
<th>South UPS</th>
<th>Installed capacity (MW)</th>
<th>Output (million kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal power plant (TPP)</td>
<td>9570</td>
<td>39116</td>
</tr>
<tr>
<td>Hydroelectric plant (HEP)</td>
<td>5334</td>
<td>20625</td>
</tr>
<tr>
<td>Nuclear power plant (NPP)</td>
<td>2000</td>
<td>8321</td>
</tr>
<tr>
<td>Isolated generating plant</td>
<td>472</td>
<td>1167</td>
</tr>
</tbody>
</table>

Major day-ahead market players responsible for shaping the demand curve include wholesale generator companies (OGKs) and territorial generator companies (TGKs or TGCs). Both are owners of thermal power plant assets. In contrast to OGKs which mainly produce electricity at their condensation plant, TGKs are primarily focused on cogeneration of electricity and heat to meet the need for industrial heat supply and district heating.

Core parameters of our mathematical economics model include:
- The “generating mix” of companies and their generating assets
- Hourly figures making up region’s daily consumption pattern
- Data on cross-flows of power in the grid
- Calculations of generator equipment costs and hourly production rates.

Our computation involved a number of simplifications. It was assumed that there are no systemic bottlenecks in the grid and that there are no price-setters among external power plant supplying the grid with electricity from Center UPS and Mid-Volga UPS. It was also expected that all generating assets of thermal power
plant have passed the VSVGO procedure and should be online during the day.

Table 3 shows simulated daily profit computations for various strategies pursued by South UPS generators at the day-ahead market (RSV), provided that all other companies pursue the marginal strategy.

<table>
<thead>
<tr>
<th>Strategies</th>
<th>OGK 2</th>
<th>OGK 5</th>
<th>OGK 6</th>
<th>TGK 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marginal</td>
<td>29134187</td>
<td>14450860</td>
<td>33994831</td>
<td>26138291</td>
</tr>
<tr>
<td>Financial withdrawal</td>
<td>29134187</td>
<td>14454296</td>
<td>33994831</td>
<td>26906329</td>
</tr>
<tr>
<td>Physical withdrawal</td>
<td>29216765</td>
<td>14222279</td>
<td>33749154</td>
<td>30413384</td>
</tr>
<tr>
<td>Mixed</td>
<td>29216765</td>
<td>14229624</td>
<td>33738296</td>
<td>37918737</td>
</tr>
</tbody>
</table>

The following conclusions can be drawn based on computation using the model.

- For OGK 6, the marginal strategy has proved to be the most efficient. This is because its installed generating capacity is able to produce electricity at a low cost when operated at a high utilization rate. Given that cutting electricity production would raise production costs and eat into resulting profits, strategies based on partial withdrawal of output should be deemed inefficient in this case. Nevertheless, there are several hours in the daily schedule when the generator company may still profit from using physical and financial withdrawal strategies.

- For OGK 5 the financial withdrawal strategy provides the best efficiency. This is because a diverse mix of generating assets at Nevinnomyssk TPP operated by the generator enables this plant to close the supply for a few hours in the day thus shaping the equilibrium price. Nevertheless the increased revenue that results from the application of the financial withdrawal strategy only comprises a minor improvement over what would be earned with the marginal strategy. The explanation is that there is only a modest rise in the equilibrium level.

- For OGK 2, strategies based on physical withdrawal perform the best as any extra generation costs resulting from slashing output would be fully counterbalanced by extra profits earned by bringing the equilibrium price up.

- TGK 8 comprises a number of cogeneration facilities with the consequence that its generation assets are more diverse in numbers and composition. Much like TGK 5, company’s plant closes the supply for a number of hours in the day, shaping the equilibrium price. However, the greatest value would be obtained by pursuing strategies based on physical withdrawal.

To summarize, electricity generator companies may earn extra profit by applying different market strategies whenever there is inelastic demand. For the power grid system under study, computation shows that generators possessing
cost-efficient assets benefit the most from the marginal strategy. In contrast, when generator’s plant closes the supply curve and comprise a diverse mix of assets, market-power strategies provide the greatest benefit. Major factors influencing the efficiency of strategies include the number and size of generator’s installed capacity as well as asset utilization.

6 Conclusions

This paper explored the issue of deciding on a strategy to be pursued by a generator company in the competitive sector of the Russian electricity market for electric energy and power – the day-ahead market. Given that Russian wholesale electricity market is not completely aligned with the principles of perfectly contestable markets, the marginal strategy no longer delivers optimum operating results for generators in the electricity market from the profit maximization standpoint. At the same time, strategies based on the exercise of market power may be resorted to in order to improve company’s bottom line from operations in the electricity market.

A mathematical economics model of the day-ahead market, price discovery systems and generator’s strategy decisions have been developed for surveying the efficiency of strategies based on market power. Our yardstick for choosing the optimum strategy was the daily profit earned by the generator.

For computation, the model was fed with a sample of online generating capacity of electric utilities operating within the South Russia Unified Power System operations zone.

Our study demonstrated that generator companies may earn extra profit by applying different market strategies whenever there is inelastic demand. For the power grid system under study, our computation reveals that generators possessing cost-efficient assets would benefit the greatest from the marginal strategy. In contrast, when generator’s plant closes the supply curve and comprise a diverse mix of assets, market-power strategies provide the greatest benefit. Major factors influencing the efficiency of strategies include the number and size of generator’s installed capacity as well as asset utilization.

Acknowledgements. This paper was partially supported by the Ministry of Education and Science of the Russian Federation, research project No. 26.1795.2014/K.

The research leading to these results has received funding from the People Programme (Marie Curie Actions) of the European Union's Seventh Framework Programme FP7/2007-2013/ under REA grant agreement number 609642.
Mathematical and economic model of generators’ strategies

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Received: October 14, 2015; Published: December 4, 2015