Algorithm for Prediction of the Technical Availability of Medical Equipment

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Abstract

Predicting the technical availability of medical equipment is an issue of great importance for the health sector, mainly in the electromedical activity. Medical technology is not safe from hazards due to poor handling of the operators; the power failures that cause the unexpectedly detention of procedures in progress, as well as other causes of technology itself. All these risks lead medical equipment to present a dynamic behavior of work passing through a finite number of states: running, faulty and broken. This paper presents a probabilistic algorithm supported on Markov chains in discrete time, to predict the dynamics of the technical availability of medical equipments.

Mathematics Subject Classification: 60J20, 65C40, 11K99

Keywords: Medical equipment, technical availability, probabilistic algorithm, Markov chain
1 Introduction

Markov models are statistical methods used to represent processes and general situations. These models are based on the probability of occurrence of distinct stages in the behavior of such processes and in indirect observations of them. Such models are used in segmentation and evaluation of time series (see, for example [1]), in spaces of dissimilarity as an alternative for selection of prototypes (see, for example [2]), for modeling population dynamics, standby systems, inventory control, maintenance, equipment replacement and to support decision making in engineering, medicine, management; applications like these can be consulted in [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. Further, the Markov models are another way to calculate life expectancy of medical equipment, which can pass through a finite number of states in a certain period of time, called cycles: days, months, years, etc [13].

The health sector is one of the areas that must be constantly redirecting its resources to ensure the technical availability of medical equipments in the health units. Medical technology is widely used for the prevention, diagnosis and treatment of various diseases and abnormal physical states. This technology is not without risks, because of the occurrence of failures in clinical practice caused by poor handling of the operator, and typical failures of technology, such as those relating to quality standards, the noncompliance with the procedures established by manufacturers, bad calibration or other related external causes [14]. All these risks cause medical technology to present a dynamic behavior of performance (see Figure 1), passing through a finite number of states: working, defective, broken and poor technical conditions. These statements may be absorbent (working, defective and broken) and not absorbent (poor technical conditions). When broken and poor technical conditions occur, the monitoring process ends. Understandably then the importance anticipating situations of malfunctions that may occur in medical equipment. In the present paper only the absorbent states are considered.

![Figure 1: Dynamic behavior of the way of working of a medical equipment](image)

We present a probabilistic algorithm (DISTEM) supported on Markov chain in discrete time which allows the prediction of the technical availabil-
ity of medical equipment in a health care unit. By obtaining the indicator of technical availability it is possible to properly planning maintenance (corrective, preventive and predictive) according to the classification categories of medical equipment in order to get a proper relationship between productivity and maintenance costs at the equipment level [15]. Coming up next we developed two more sections. In the second one we address the fundamental concepts used in the development of the algorithm, explaining step by step the performance of this algorithm. Moreover, in the third section, we present the sub-module “availability and reliability of equipment”. Also in this section a detailed analysis of the algorithm is performed to show the reliability of the predictions. Finally, we offer some conclusions.

2 Materials and methods

The DISTEM algorithm which performs the calculation of technical availability using a Markov chain, is based on the concepts of stochastic processes and Markov property [1], [16], [17]. A Markov chain corresponds to a specific class of stochastic process in the field of probabilistic models. A stochastic process is defined as a collection of random variables $X_1, X_2, \cdots, X_t$, where the subscript $t$ takes values a given set $T$, and usually $0 \leq t \leq T$. The variable $X$ represents a characteristic of interest that is measured at time $t$. The study of the behavior of a system over a period often leads to analyze a stochastic process with the following structure: at specific instants of time $t$, the system is exactly in the position of a finite number of mutually exclusive and exhaustive states $0; 1; \cdots; S$. Periods of time may be equally spaced or its spread may depend on the overall system behavior in which is immersed the stochastic process. Thus, the mathematical representation of the physical system is a stochastic process $\{X_t\}$, where the random variables are observed at the instants $t = 1; 2; \cdots; T$ and where each random variable can take any of the $S + 1$ integer values $0; 1; 2; \cdots; S$ that characterize the states of whole process. It is said that a stochastic process has the Markov property when from the current state can only be obtained information on the future behavior of the process, i.e. their future states are independent of the past states. In a stochastic process without the Markov property, given a distribution of random variables $\{X_k, k = 1, 2, \cdots, n\}$, a random variable $X_j$ is in the state $x_j$ when $P[X_j = x_j|\{X_k, j \neq k\}]$. This means that the probability that the variable $X_j$ to be in the state $x_j$ depends on the values of all other random variables $X_k$. Markov property states that, being $X(t), t \geq 0$ a non-negative continuous time stochastic process with integers values, it is said that this process is a discrete Markov process if for $n \geq 0$ and at the instants $0 < t_0 < t_1 < \cdots < t_n < t_{n+1}$
and in the states $i_0, i_1, \ldots, i_{n+1}$ it holds that

$$P[X(t_{n+1})] = P[X(t_{n+1}) = i_{n+1}|X(t_n) = i_n]$$

Let us suppose that $X_t$ represents the state of working of a medical equipment in the moment of future time $t$. This defines a stochastic process that corresponds to the sequence $X_1, X_2, \ldots, X_t$ representing the operating level of the equipment over time. The value of $X_t$ constantly depends of the previous values in the sequence. State changes occur in probabilistic terms as time passes. These state changes are represented by the so-called transition probabilities between states. In the case of transitions in one step these state changes correspond to the probability of moving from one state at time $t$ to another state at time $t + 1$.

The Markov model we are following has the assumptions:

- It assumes a finite number of states to describe the dynamic behavior of the medical equipment: working (F); defective (D) and Broken (R).

- It implies a known probability distribution at the start of the study horizon ($t = 0$), reflecting on what state, of the previously defined, belongs the medical equipment of the health institution where it is installed.

- It is assumed that the transition from current state to another state in the future depends only on the current state (the Markov property holds).

- The probability of this transition is independent of the considered time period (stationary property).

To form the chain, we rely on historical data of orders made by electro-medical staff in the health units. These service orders collect a set of data relating to the medical equipment management, within which is the technical condition (F, D or R). The medical equipments are managed according to four factors: functional role, physical risk associated with clinical application, maintenance requirements and background of equipment problems. The first factor has a direct relationship with the equipment, and the orders one with their operating level over time [15]. The chain of states is constructed by laminating equipments by specialty in order to facilitate the search for reports related to its operation over time. Then it proceeds to building the transition matrix from searching the sequence of states of the following occurrences: FF (working to working), FD (working to defective), FR (working to broken), DF (defective to working), DD (defective to defective), DR (defective to broken), RF (broken to working), RD (broken to defective) and RR (broken to broken).
Let us denote $A$ the transition matrix. The probability of changing from state $i$ to the state $j$: $P(j|i)$, is given by the element $a_{ij}$ of the matrix $A$:

$$A = \begin{pmatrix}
a_{11} & a_{12} & \ldots & a_{1j} & \ldots \\
a_{21} & a_{22} & \ldots & a_{2j} & \ldots \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
a_{i1} & a_{i2} & \ldots & a_{ij} & \ldots
\end{pmatrix}$$

The transition probability is defined, due to the Markov property, as follows [1]:

$$a_{ij} = P(X_{n+1} = x | X_n = x_n)$$

The vector of initial probabilities associated with the chain sequences will be represented by a constant row vector $\pi \in \mathbb{R}^{1 \times 3}$, and verifies that $\pi A = \pi$; moreover, the largest eigenvalue of $A$ is always 1.

The state Markov diagram designed to represent the technical availability of a medical equipment presents an ergodic or fully connected architecture (see Figure 2) whereby each state of the model can be reached from any other state in a finite number of steps.

![Figure 2: Ergodic structure of the Markov model for the process of technical availability of a medical equipment](image)

2.1 Proposed algorithm

Predicting the technical availability will be conducted to $n$ steps. Next, the operation of the probabilistic algorithm DISTEM is described (see Table 1). This algorithm has as output a list with prediction of the technical availability of a medical equipment for absorbing states. The input parameters are a list with the sequence of states through which a medical equipment passes during its lifetime, and the instant of time (quantity of months in this case) in that the probabilities are predicted for different absorbing states.
**Algorithm DISTEM**

**Input:**
- `listaSecuenciaEstados`: List with the sequence of states through which passes a medical equipment
- `varCantMeses`: Time predicts probabilities for the various possible values

**Output:**
- `listaDispTecnica`: List with prediction of technical availability of medical equipment for absorbing states

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><code>listaMatrices = ConstruirMatrices (listaSecuenciaEstados)</code></td>
</tr>
<tr>
<td>2</td>
<td><code>varPotencia = 2</code></td>
</tr>
<tr>
<td>3</td>
<td><strong>If</strong> <code>varCantMeses = 1</code> <strong>then</strong></td>
</tr>
<tr>
<td>4</td>
<td><code>listaDispTecnica = MultiplicarMatrices (listaMatrices [0], listaMatrices [1])</code></td>
</tr>
<tr>
<td>5</td>
<td><strong>Else If</strong></td>
</tr>
<tr>
<td>6</td>
<td><code>listaVectorMatriz = {}</code></td>
</tr>
<tr>
<td>7</td>
<td><code>listaAux = listaMatrices [1]</code></td>
</tr>
<tr>
<td>8</td>
<td><strong>End If</strong></td>
</tr>
<tr>
<td>9</td>
<td><strong>While</strong> <code>varPotencia ≤ varCantMeses</code> <strong>then</strong></td>
</tr>
<tr>
<td>10</td>
<td><code>listaVectorMatriz = MultiplicarMatrices (listaMatrices [1], listaAux)</code></td>
</tr>
<tr>
<td>11</td>
<td><code>listaAux = listaVectorMatriz</code></td>
</tr>
<tr>
<td>12</td>
<td><code>listaVectorMatriz = listaDispTecnica</code></td>
</tr>
<tr>
<td>13</td>
<td><code>varPotencia++</code></td>
</tr>
<tr>
<td>14</td>
<td><strong>End While</strong></td>
</tr>
<tr>
<td>15</td>
<td><code>listaDispTecnica = MultiplicarMatrices (listaMatrices [0], listaAux)</code></td>
</tr>
<tr>
<td>16</td>
<td><strong>Return</strong> <code>listaDispTecnica</code></td>
</tr>
</tbody>
</table>

For the calculation of the temporary complexity of the algorithm it was determined the complexity of each step of the algorithms MultiplicarMatrices and ConstruirMatrices, using the rules of addition and multiplication of asymptotic notation ”big O”. The complexity of the algorithm DISTEM is $O(n)$.

### 3 Results and discussion

We have implemented a sub-module that is called *availability and reliability of the equipment* (see Figure 3), which is integrated with the module called *prediction and management of the stock of the management system for clinical engineering and electromedicine*. This sub-module incorporates the algorithm DISTEM, because the technical availability is one of the variables considered in the multivariate model predicting the *stock of spare parts for medical equipment* [18].
In order to validate the effectiveness of the algorithm DISTEM we have used the experimental method. The data used in the experiment are from 30,843 reports made in service orders from all health centers of the country over the years from 2003 to 2014, available on the system Reportech [19]. The experimental framework was limited to a medical equipments belonging to the specialty Electromechanic, but the results can be generalized to the rest of equipments of the population in study:

Medical equipment

- Specialty: Electromechanic.
- Name of equipment: Vacuum cleaner.

With the data of Table 2, we can fine the transition matrix $A$ and the vector of initial probabilities $\pi$:

\[
A = \begin{pmatrix}
149/176 & 27/176 & 0 \\
1/228 & 201/228 & 26/228 \\
25/217 & 0 & 192/217
\end{pmatrix} = \begin{pmatrix}
0.847 & 0.153 & 0 \\
0.004 & 0.882 & 0.114 \\
0.115 & 0 & 0.885
\end{pmatrix}
\]

The columns of the matrix $A$ corresponds to states working, defective and broken respectively. Also it happens with the rows of this matrix.

\[
\pi = \begin{pmatrix}
175/621 & 228/621 & 218/621
\end{pmatrix} = \begin{pmatrix}
0.282 & 0.367 & 0.351
\end{pmatrix}
\]
The first, second and third coordinates of the vector $\pi$ reflect the probability that the equipment be initially working, defective or broken respectively. So, if we want to predict the state of the equipment within three months, we have:

$$\pi \times A^3 = \begin{pmatrix} 0.282 & 0.367 & 0.351 \end{pmatrix} \times \begin{pmatrix} 0.847 & 0.153 & 0 \\ 0.004 & 0.882 & 0.114 \\ 0.115 & 0 & 0.885 \end{pmatrix} = \begin{pmatrix} 0.28 & 0.39 & 0.33 \end{pmatrix}$$

In the Figure 4 we can see the process of technical availability by which transits the equipment. And the resulting vector tells us that the probability that the equipment will be functioning, defective or broken within three months is 0.28, 0.39 or 0.33 respectively.

According to Figure 5 in the next page, the technical availability of medical equipment, for three stages (three months) is 28 percent, with an operating reliability of 50 percent. And in the Figure 6 we can see the relationship between the observed and predicted availability of this equipment (vacuum cleaner).
Algorithm for prediction of the technical availability of medical equipment

<table>
<thead>
<tr>
<th>Working (F)</th>
<th>Defective (D)</th>
<th>Broken (R)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working (F)</td>
<td>149</td>
<td>27</td>
<td>0</td>
</tr>
<tr>
<td>Defective (D)</td>
<td>1</td>
<td>201</td>
<td>26</td>
</tr>
<tr>
<td>Broken (R)</td>
<td>25</td>
<td>0</td>
<td>192</td>
</tr>
<tr>
<td>Total</td>
<td>175</td>
<td>228</td>
<td>218</td>
</tr>
</tbody>
</table>

Table 1: Parameters used in the experimental analysis

Figure 4: Graph representing the process of technical availability of the equipment

Conclusions
The proposed probabilistic algorithm lets face to the elements of uncertainty in predicting the technical availability of a medical equipment. The algorithm facilitates the extension of the sequence of states without changing the model adopted and the temporary complexity of its execution. The model proposed could be extended by using more complex methods, for example, hidden Markov models to measure, using external effects or observations, weak directly observable states.

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Figure 5: Technical availability for the vacuum cleaner, expressed in percent

Figure 6: Contrast between the observed and predicted availability for the vacuum cleaner
Algorithm for prediction of the technical availability of medical equipment

References


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