Uncertain Programming Model for Location Problem of Multi-product Logistics Distribution Centers

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Abstract

This paper mainly investigates the location problem of multi-product logistics distribution centers in uncertain environment. Based on the assumption that the customers’ demands and the transportation times are uncertain variables, an uncertain programming model is constructed. On the precondition of satisfying the capacity constraint and time window constraint, the objective of the model is to minimize the mean yearly total cost which includes construction cost, management cost, inventory cost and transportation cost. At last, a numerical example is given to verify the feasibility of the proposed model, and Lingo software is employed to find the optimal solution.

Keywords: Location problem, Multi-product distribution centers, Uncertainty theory, Uncertain variable, Uncertain programming model
1 Introduction

Logistics distribution center is an indispensable part which links suppliers with customers, and it can directly affect the satisfaction of customers. This means that the location problem of logistics distribution center has an important platform in logistics network design. In the process of commodity sales, the distribution centers order products from suppliers, then provide customers with a certain number of products after receiving the customers’ specific requirements. In order to ensure that the logistics company can operate effectively, logistic enterprise needs to make a location decision under the condition of satisfying all customers’ demands. Actually, this decision problem involves two aspects, the first one is selecting the distribution centers locations from some potential sites, and the second one is determining which distribution center serves which customers.

Location problem was first proposed by Weber [1] in 1929. After that, this problem has attracted much attention, and many models have been presented. With the development of location problem, there are three types of basic location problems. The simplest one is \(p\)-median problem [2], which describes how to select \(p\) facilities to minimize the total weighted distances under the condition of meeting customers’ demands. The second one is \(p\)-center problem [3], in which decision makers select \(p\) facilities to minimize the maximum distance between any given customer and its nearest facility. The third one is covering problem, which includes set covering problem and maximum coverage problem. The set covering problem [4] aims at establishing the minimum number of facilities. And maximum coverage problem [5] describes how to choose \(p\) facilities to maximize the acceptable service demand. Based on the previous three basic location problems, many extended location problems were studied by considering other factors. Considering different objectives, the problem was extended by minimizing the total costs [6], maximizing the expectant benefits [7] or satisfying multiple objectives at the same time [8]. Considering different constraint conditions, this problem was extended as uncapacitated facility location problem [9], capacitated facility location problem [10] and so on. The specific facility location models and applications can be referred to the review article presented by Klose and Drexl [11].

The above mentioned papers were studied under the condition of single product or deterministic environment. However, we usually encounter the situation that a customer needs various kinds of products at the same time. In addition, decision makers often face some indeterministic factors in this kind of location problem. In this situation, the traditional models will not suit to use. In order to deal with this problem, some researchers studied the indeterminate multi-product logistics network location problem by probability theory. For example, Ramezani et al. [12] presented a stochastic multi-objective model for
forward and reverse logistics network design. And the goal of this model is to maximize the profit, customer responsiveness and quality. Qin et al. [13] considered the customers’ demands as stochastic variables, and apportioned the cost to every day, then the objective is to minimize the average daily total cost that includes construction cost, inventory cost and transportation cost. Gzara et al. [14] proposed linear location-inventory models for service parts logistics network design, in which time-based service level constraints were considered and the yearly total cost was minimized.

As we know, the premise of applying probability theory is that there are enough previous data to estimate probability distribution function. However, in our daily life, we often lack of observed data in many situations. Such as the location problem is a decision making problem that makes a plan for future. It is not easy for us to get the exact values of some parameters. So that no samples are available to estimate a probability distribution. In this case, we have to invite some domain experts to give the belief degree that each event will occur. Liu [15] declared that it is inappropriate to model belief degrees by probability theory because it may lead to counterintuitive results, and presented some counterexamples. Under this condition, it will be not suitable for us to deal with some indeterministic parameters by probability theory. In order to rationally deal with belief degrees, uncertainty theory was founded by Liu [16] based on normality, self-duality, subadditivity and product axioms in 2007 and perfected by Liu [17] in 2010, which has become a branch of axiomatic mathematics and has gained considerable achievement.

Up to now, theory and practice have shown that uncertainty theory is an efficient tool to deal with non-deterministic information. These years, uncertainty theory has been employed to deal with facility location problem in uncertain environment. Such as single commodity’s logistics distribution center location problem [18], single facility location problems [19], facility location problem for reverse logistics network [20, 21], capacitated facility location-allocation problem with uncertain demands [22] and uncapacitated facility location problem with uncertain customers positions [23].

Different from other previous articles, this paper proposes an uncertain programming model to investigate the multi-product logistics distribution centers location problem. In this model, both customers’ demands and transportation times are described as uncertain variables, and the objective is to minimize the yearly total cost which includes construction cost, management cost, transportation cost and inventory cost. Each project has its planning horizon which will affect the total cost. In order to keep the consistency of all costs in unit time, we allocate construction cost to each year and the objective is to minimize the mean yearly total cost. However Wu and Peng [18] without considering the planning horizon. In addition, we assume that all the products needed by one customer are transported by only one distribution center. Besides the
capacity limitation of distribution centers, we also consider time window as constraint conditions which were ignored by Wu and Peng [18]. Technically, the model is transformed into its equivalent model and Lingo software is used to obtain the optimal solution.

The rest of the paper is organized as follows. In Section 2, we introduce some basic concepts and properties of uncertainty theory associated with this paper. In Section 3, we describe the location problem of multi-product logistics distribution centers and construct an uncertain programming model. Then the model is transformed into its deterministic form. In Section 4, a numerical example is given. At last, a brief summary is presented in Section 5.

2 Preliminary

The uncertainty theory proposed by Liu [16] in 2007 is a mathematics tool based on normality, self-duality, countable subadditivity, and product axioms. In order to make readers understand this paper better, in this section we firstly state some basic concepts and results which selected from uncertainty theory.

Let $\Gamma$ be a nonempty set and $\mathcal{L}$ a $\sigma$-algebra over $\Gamma$. Each element $\Lambda \in \mathcal{L}$ is called an event. Each event $\Lambda$ is assigned a nonnegative number $M\{\Lambda\}$ to indicate the belief degree that $\Lambda$ will happen. A set function $M\{\cdot\}$ is called an uncertain measure if it satisfies the following three axioms:

(1) **(Normality Axiom)** $M\{\Gamma\} = 1$ for the universal set $\Gamma$;
(2) **(Duality Axiom)** $M\{\Lambda\} + M\{\Lambda^c\} = 1$ for any event $\Lambda$;
(3) **(Subadditivity Axiom)** $M\left(\bigcup_{i=1}^{\infty} \Lambda_i\right) \leq \sum_{i=1}^{\infty} M\{\Lambda_i\}$ for every countable sequence of events $\{\Lambda_i\}$.

In order to obtain an uncertain measure of compound event, Liu [24] defined the product axiom as follows.

(4) **(Product Axiom)** Let the triplets $(\Gamma_k, \mathcal{L}_k, M_k)(k = 1, 2, \ldots)$ be uncertainty spaces. The product uncertain measure $M$ is an uncertain measure satisfying

$$M\left(\prod_{k=1}^{\infty} \Lambda_k\right) = \bigwedge_{k=1}^{\infty} M_k\{\Lambda_k\},$$

where $\Lambda_k$ are arbitrarily chosen events from $\mathcal{L}_k (k = 1, 2, \ldots)$, respectively.

Uncertain variable is a fundamental concept in uncertainty theory. It is used to represent quantities with uncertainty.

**Definition 1.** (Liu [16]) An uncertain variable $\xi$ is a measurable function from an uncertainty space $(\Gamma, \mathcal{L}, M)$ to the set of real numbers, i.e., for any Borel set of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$$
is an event.

In order to describe an uncertain variable in practice, the uncertainty distribution $\Phi : \mathbb{R} \rightarrow [0, 1]$ of an uncertain variable $\xi$ was defined by Liu [16] as follows

$$\Phi(x) = \mathcal{M}\{\xi \leq x\}, \quad \forall x \in \mathbb{R}.$$ 

If $\Phi(x)$ is a continuous and strictly increasing function with respect to $x$ at which $0 < \Phi(x) < 1$, and

$$\lim_{x \to -\infty} \Phi(x) = 0, \quad \lim_{x \to +\infty} \Phi(x) = 1,$$

then we said $\Phi(x)$ is a regular uncertainty distribution, and the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of $\xi$.

Among some special uncertain variables, zigzag uncertain variable is a type of uncertain variable which is widely used. An uncertain variable $\xi$ is called zigzag if it has a zigzag uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x < a \\ \frac{x - a}{2(b - a)}, & \text{if } a \leq x < b \\ \frac{x + c - 2b}{2(c - b)}, & \text{if } b \leq x < c \\ 1, & \text{if } x \geq c, \end{cases}$$

denoted by $\mathcal{Z}(a, b, c)$, where $a$, $b$ and $c$ are real numbers with $a < b < c$. The inverse uncertainty distribution of $\mathcal{Z}(a, b, c)$ is

$$\Phi^{-1}(\alpha) = \begin{cases} (1 - 2\alpha)a + 2\alpha b, & \text{if } \alpha < 0.5 \\ (2 - 2\alpha)b + (2\alpha - 1)c, & \text{if } \alpha \geq 0.5. \end{cases}$$

**Definition 2.** (Liu [24]) The uncertain variables $\xi_1, \xi_2, \ldots, \xi_n$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{n}\{\xi_i \in B_i\}\right\} = \mathcal{M}\{\xi_i \in B_i\}$$

for any Borel sets $B_1, B_2, \ldots, B_n$.

**Theorem 1.** (Liu [17]) Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If $f(x_1, x_2, \ldots, x_n)$ is strictly increasing with respect to $x_1, x_2, \ldots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots, x_n$, then $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ is an uncertain variable with inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f\left(\Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1 - \alpha), \ldots, \Phi_n^{-1}(1 - \alpha)\right).$$
Expected value represents the size of uncertain variable. The expected value of uncertain variable $\xi$ was defined by Liu [16] as the following form

$$E[\xi] = \int_{0}^{+\infty} M\{\xi \geq r\} \, dr - \int_{-\infty}^{0} M\{\xi \leq r\} \, dr$$

provided that at least one of the two integrals is finite. And Liu [17] proved the formula

$$E[\xi] = \int_{0}^{1} \Phi^{-1}(\alpha) \, d\alpha$$

under the condition that $\xi$ is an uncertain variable with regular uncertainty distribution $\Phi$.

**Example 1. (Liu [17])** If $\xi$ is an uncertain variable with the zigzag uncertainty distribution $Z(a,b,c)$, then the expected value of $\xi$ is

$$E[\xi] = \frac{a + 2b + c}{4}.$$  

**Theorem 2. (Liu [17])** Let $\xi$ and $\eta$ be independent uncertain variables with finite expected values. Then for any real numbers $a$ and $b$, we have

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta].$$

**Theorem 3. (Liu and Ha [25])** Assume $\xi_1, \xi_2, \ldots, \xi_n$ are independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If $f(x_1, x_2, \ldots, x_n)$ is strictly increasing with respect to $x_1, x_2, \ldots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots, x_n$, then uncertain variable $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ has an expected value

$$E[\xi] = \int_{0}^{1} f (\Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1 - \alpha), \ldots, \Phi_n^{-1}(1 - \alpha)) \, d\alpha.$$

**Theorem 4. (Liu [17])** Assume $\xi_1, \xi_2, \ldots, \xi_n$ are independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If the function $f(x_1, x_2, \ldots, x_n)$ is strictly increasing with respect to $x_1, x_2, \ldots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots, x_n$, then the chance constraint

$$\mathcal{M}\{f(\xi_1, \xi_2, \ldots, \xi_n) \leq 0\} \geq \alpha$$

holds if and only if

$$f (\Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1 - \alpha), \ldots, \Phi_n^{-1}(1 - \alpha)) \leq 0.$$
3 Uncertain Location Problem of Multi-product Logistics Distribution Centers

In multi-product logistics network, logistics distribution centers play a role as a bridge to link enterprises and customers. They order products from different suppliers, then transport these products to customers with specified demand quantities. Suppose there are \( N \) customers, \( M \) potential distribution centers and \( L \) kinds of products which provided by suppliers. Figure 1 provides a pictorial description of the process. During designing a multi-product network, in order to reduce operational costs, the decision makers need to determine the number and the sites of distribution centers and determine which distribution center serves which customers. The premise of making this decision is meeting all the customers’ demands, and subjecting to storage capacity limitations of distribution centers and time window limitations.

![Figure 1: Multi-product Logistics Network Schematic](image)

3.1 Assumptions and Notations

In order to model the uncertain location problem of multi-product distribution centers, this section presents some relevant assumptions and notations.

The assumptions which have been widely used in many papers such as Ramezani et al. [12], Qin et al. [13] and Gzara et al. [14] considered in this paper are as follows:

1. All the products needed by one customer are transported by only one distribution center;
2. The locations of suppliers, customers and potential distribution centers are known;
3. The products of each supplier are enough to cover the business requirements, but the distribution centers have some capacity limitations and the capacities of distribution centers are known;
(4) The customers’ demands are assumed to be independent uncertain variables with zigzag uncertainty distributions;
(5) The flows among distribution centers are not allowed.

The notations for the parameters are introduced in tables 1, 2 and 3.

<table>
<thead>
<tr>
<th>Table 1: Index Parameters</th>
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<tbody>
<tr>
<td>$i$ the index of distribution centers, $i \in {1,2,\ldots,M}$;</td>
</tr>
<tr>
<td>$j$ the index of customers, $j \in {1,2,\ldots,N}$;</td>
</tr>
<tr>
<td>$l$ the index of products, $l \in {1,2,\ldots,L}$.</td>
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<table>
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<th>Table 2: Decision Variables</th>
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<tbody>
<tr>
<td>$x_i$ 0-1 variable implies whether the potential distribution center $i$ is chosen or not;</td>
</tr>
<tr>
<td>$y_{ij}$ 0-1 variable implies whether customer $j$ is serviced by the distribution center $i$.</td>
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<table>
<thead>
<tr>
<th>Table 3: Model Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_i$ the construction cost of the distribution center $i$;</td>
</tr>
<tr>
<td>$F_i$ the annual management cost of the distribution center $i$;</td>
</tr>
<tr>
<td>$c_{il}$ the transportation cost of unit product $l$ transported from supplier to distribution center $i$;</td>
</tr>
<tr>
<td>$e_{ijl}$ the transportation cost of transporting unit product $l$ from distribution center $i$ to customer $j$;</td>
</tr>
<tr>
<td>$h_{il}$ unit inventory cost for product $l$ at distribution center $i$;</td>
</tr>
<tr>
<td>$v_i$ the capacity of the distribution center $i$ for product $l$;</td>
</tr>
<tr>
<td>$\xi_{jl}$ the uncertain demand for product $l$ of customer $j$ yearly;</td>
</tr>
<tr>
<td>$\Phi_{jl}$ the uncertainty distribution of $\xi_{jl}$;</td>
</tr>
<tr>
<td>$\tau_{ij}$ the uncertain transportation time from the distribution center $i$ to the customer $j$;</td>
</tr>
<tr>
<td>$\Psi_{ij}$ the uncertainty distribution of $\tau_{ij}$;</td>
</tr>
<tr>
<td>$w_j$ the time window of customer $j$.</td>
</tr>
</tbody>
</table>

### 3.2 Uncertain Programming Model

Under the condition of satisfying the multiple demands of customers, the objective function in our model is to minimize the mean yearly total cost. In which, the total cost include construction cost, management cost, inventory cost and transportation cost. It is obvious that construction cost is invested at the beginning and it is fixed and immutable. However the other costs normally associate with product and need to be invested everyday. In order to keep the consistency of all costs in unit time, construction cost can be allocated to each year in the planning horizon $p$ years, i.e., the average yearly constriction cost
is \( \sum_{i=1}^{M} f_i x_i / p. \) We denote the total annual demand for product \( l \) at distribution center \( i \) as \( D_{il} \), where \( D_{il} = \sum_{j=1}^{N} \xi_{jl} y_{ij}. \) So the total yearly inventory cost is \( \sum_{l=1}^{L} \sum_{i=1}^{M} h_{il} D_{il}. \)

In addition, we take the management cost and the transportation cost into consideration, the total annual cost is a function related to \( x, y \) and \( \xi \), which can be written as

\[
C(x, y, \xi) = \sum_{i=1}^{M} \left( \frac{f_i}{p} + F_i \right) x_i + \sum_{i=1}^{M} \sum_{l=1}^{L} c_{il} D_{il} + \sum_{i=1}^{M} \sum_{l=1}^{L} \sum_{j=1}^{N} e_{ijl} \xi_{jl} y_{ij} + \sum_{l=1}^{L} \sum_{i=1}^{M} h_{il} D_{il}
\]

\[
= \sum_{i=1}^{M} \left( \frac{f_i}{p} + F_i \right) x_i + \sum_{l=1}^{L} \sum_{i=1}^{M} \sum_{j=1}^{N} (c_{il} + e_{ijl} + h_{il}) \xi_{jl} y_{ij},
\]

where \( x = (x_1, x_2, \ldots, x_M) \) and \( y = (y_{11}, \ldots, y_{1N}, \ldots, y_{M1}, \ldots, y_{MN}) \) are decision vectors, and \( \xi = (\xi_{11}, \ldots, \xi_{1L}, \ldots, \xi_{N1}, \ldots, \xi_{NL}) \) is an uncertain vector.

Due to the capacity limitations of distribution centers, the distribution centers need to order products many times in one year. The order frequency and the order quantity of each distribution center are determined by the demand of customers [26]. Suppose each distribution center orders products with a fixed frequency, and \( n \) is the ordering number per year. Then the average order quantity each time from supplier to distribution center \( i \) is \( D_{il} / n \). And we must ensure that the order quantity does not exceed capacity limits of distribution centers with certain confidence levels \( \alpha_i \), i.e.,

\[ \mathcal{M} \{ D_{il} / n \leq v_{il} x_i \} \geq \alpha_i. \]

There are some others constraints need to be considered in the specific application domain. For example, the customers’ demands must be fulfilled, and many customers require to be serviced within a certain time window. Inspired by this idea, we can establish the model as follows:

\[
\begin{aligned}
\text{min} \quad & E [C(x, y, \xi)] \\
\text{s.t.} \quad & \mathcal{M} \{ D_{il} / n \leq v_{il} x_i \} \geq \alpha_i, \quad i = 1, \ldots, M, l = 1, \ldots, L \\
& y_{ij} = 0 \quad \text{if} \quad \mathcal{M} \{ \tau_{ij} \leq w_j \} \leq \beta_j, \quad i = 1, \ldots, M, j = 1, \ldots, N \\
& \sum_{i=1}^{M} y_{ij} = 1, \quad j = 1, \ldots, N \\
& y_{ij} \leq x_i, \quad i = 1, \ldots, M, j = 1, \ldots, N \\
& x_i \in \{0, 1\}, \quad i = 1, \ldots, M \\
& y_{ij} \in \{0, 1\}, \quad i = 1, \ldots, M, j = 1, \ldots, N,
\end{aligned}
\]

where \( \alpha_i \) and \( \beta_j \) are predetermined confidence levels. In this model, the first constraint is capacity constraint. The second constraint implies that the products must
be transported to customers within the time window with the confidence levels $\beta_j$. Otherwise, this customer will not be serviced by this distribution center. The third constraint means that the demand of each customer is serviced by only one distribution center. And the forth one controls that the customer just can be served by the opened distribution center.

### 3.3 The Crisp Equivalent Model

It is difficult to obtain the optimal solution because there are many uncertain variables in the model of the previous subsection, so we need to transform the model into its crisp equivalence form. According to Theorem 3, we can know

$$E[C(x, y, \xi)] = \sum_{i=1}^{M} (f_i/p + F_i)x_i + \sum_{l=1}^{L} \sum_{i=1}^{M} \sum_{j=1}^{N} (c_{il} + e_{ijl} + h_{il})y_{ij} \int_{0}^{1} \Phi_{jl}^{-1}(\alpha) d\alpha. \quad (3)$$

It is clear that $\mathcal{M}\{D_{il}/n \leq v_{il}x_i\} = \mathcal{M}\left\{\sum_{j=1}^{N} \xi_{jl}y_{ij}/n \leq v_{il}x_i\right\}$, and $\sum_{j=1}^{N} \xi_{jl}y_{ij}/n$ is a strictly increasing function. According to Theorem 4,

$$\mathcal{M}\left\{\sum_{j=1}^{N} \xi_{jl}y_{ij}/n \leq v_{il}x_i\right\} \geq \alpha$$

can be turned into

$$\sum_{j=1}^{N} y_{ij} \Phi_{jl}^{-1}(\alpha) \leq nv_{il}x_i. \quad (4)$$

In a similar way,

$$\mathcal{M}\{\tau_{ij} \leq w_j\} \leq \beta$$

can be turned into

$$\Psi_{ij}^{-1}(\beta) \geq w_j. \quad (5)$$

It follows from (3), (4) and (5), we can transform the model (2) into the following form

$$\begin{align*}
\min & \quad \sum_{i=1}^{M} (f_i/p + F_i)x_i + \sum_{l=1}^{L} \sum_{i=1}^{M} \sum_{j=1}^{N} (c_{il} + e_{ijl} + h_{il})y_{ij} \int_{0}^{1} \Phi_{jl}^{-1}(\alpha) d\alpha \\
\text{s.t.} & \quad \sum_{j=1}^{N} y_{ij} \Phi_{jl}^{-1}(\alpha) \leq nv_{il}x_i, \quad i = 1, \ldots, M, l = 1, \ldots, L \\
& \quad y_{ij} = 0 \quad \text{if} \quad \Psi_{ij}^{-1}(\beta_j) \geq w_j, \quad i = 1, \ldots, M, j = 1, \ldots, N \\
& \quad \sum_{i=1}^{M} y_{ij} = 1, \quad j = 1, \ldots, N \\
& \quad y_{ij} \leq x_i, \quad i = 1, \ldots, M, j = 1, \ldots, N \\
& \quad x_i \in \{0, 1\}, \quad i = 1, \ldots, M \\
& \quad y_{ij} \in \{0, 1\}, \quad i = 1, \ldots, M, j = 1, \ldots, N.
\end{align*} \quad (6)$$
4 Application Example

With the rapid development of automobile industry, it is necessary for auto parts distribution center which serves for vehicle manufacturers to provide efficient service and build more advanced auto parts distribution centers. The location of the auto parts distribution center planning is the key of all.

Suppose that there is one auto parts supplier that produces a few kinds of products in city $A$. It supplies three types of parts to 6 vehicle manufacturers in city $B$. In which the vehicle manufacturers are viewed as customers. After the overall evaluation of multiple factors, 5 sites are selected as candidate distribution centers. This paper helps auto parts logistics enterprise to design a location scheme for multi-product logistics distribution centers in the planning horizon 10 years (i.e., $p = 10$). Suppose all the customers would like to be serviced within 4 hours, i.e., $w_j = 4$. Suppose that construction costs are equivalent for all new distribution centers, yearly management costs are undifferentiated for all new distribution centers, and the distribution centers that plan to be constructed have the same capacity for each kind of product. We set $f_i = 500000$, $F_i = 30000$, $n = 10$ and $h_{ij} = 1$.

The unit transportation costs and the capacities of distribution centers are listed in Tables 4-8.

Table 4: Unit Transportation Cost $c_{il}$

<table>
<thead>
<tr>
<th>$l \setminus i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>2.5</td>
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<td>1.9</td>
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<td>2.1</td>
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<td>2</td>
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<td>2.2</td>
<td>1.9</td>
<td>2.3</td>
<td>1.9</td>
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</table>

Table 5: The 1-th Product’s Unit Transportation Cost $e_{ij1}$

<table>
<thead>
<tr>
<th>$i \setminus j$</th>
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<tbody>
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<td>0.21</td>
<td>0.26</td>
<td>0.27</td>
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<td>0.28</td>
<td>0.26</td>
<td>0.32</td>
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Table 6: The 2-th Product’s Unit Transportation Cost $e_{ij2}$

<table>
<thead>
<tr>
<th>$i \setminus j$</th>
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<td>0.33</td>
<td>0.22</td>
<td>0.30</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>0.29</td>
<td>0.29</td>
<td>0.32</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>3</td>
<td>0.30</td>
<td>0.35</td>
<td>0.31</td>
<td>0.23</td>
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<td>0.26</td>
</tr>
<tr>
<td>4</td>
<td>0.33</td>
<td>0.28</td>
<td>0.29</td>
<td>0.27</td>
<td>0.33</td>
<td>0.31</td>
</tr>
<tr>
<td>5</td>
<td>0.27</td>
<td>0.34</td>
<td>0.32</td>
<td>0.33</td>
<td>0.31</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Table 7: The 3-th Product's Unit Transportation Cost $e_{ij3}$

\[
\begin{array}{ccccccc}
  i \backslash j & 1 & 2 & 3 & 4 & 5 & 6 \\
  \hline
  1 & 0.24 & 0.20 & 0.23 & 0.24 & 0.18 & 0.24 \\
  2 & 0.16 & 0.22 & 0.22 & 0.25 & 0.18 & 0.22 \\
  3 & 0.22 & 0.30 & 0.24 & 0.16 & 0.25 & 0.20 \\
  4 & 0.28 & 0.23 & 0.25 & 0.22 & 0.27 & 0.25 \\
  5 & 0.22 & 0.27 & 0.25 & 0.26 & 0.25 & 0.28 \\
\end{array}
\]

Table 8: The Capacities of Three Kinds of Products

\[
\begin{array}{cccc}
  l & 1 & 2 & 3 \\
  \hline
  \text{Capacities} & 9000 & 6300 & 8500 \\
\end{array}
\]

We assume that $\xi_{jl}$ and $\tau_{ij}$ are independent uncertain variables with known zigzag uncertainty distributions which are shown in Tables 9 and 10.

Table 9: The Uncertainty Distributions of $\xi_{jl}$ ($\times 10^3$)

\[
\begin{array}{cccccccc}
  l \backslash j & 1 & 2 & 3 & 4 & 5 & 6 \\
  \hline
  1 & Z(12, 14, 16) & Z(15, 18, 21) & Z(13, 16, 19) & Z(9, 14, 17) & Z(11, 12, 15) & Z(13, 14, 15) \\
  2 & Z(11, 14, 15) & Z(9, 11, 15) & Z(12, 15, 18) & Z(13, 14, 15) & Z(8, 11, 14) & Z(13, 15, 19) \\
  3 & Z(9, 11, 15) & Z(11, 14, 15) & Z(9, 12, 15) & Z(14, 17, 18) & Z(12, 15, 16) & Z(14, 15, 18) \\
\end{array}
\]

Table 10: Uncertainty Distributions of Transportation Time $\tau_{ij}$

\[
\begin{array}{cccccccc}
  i \backslash j & 1 & 2 & 3 & 4 & 5 & 6 \\
  \hline
  1 & Z(2, 2.5, 3.2) & Z(1.2, 2, 3) & Z(3, 3.5, 4) & Z(3.2, 4, 5) & Z(1.5, 2, 2.5) & Z(2, 2.6, 3) \\
  2 & Z(2.6, 3, 3.4) & Z(3.4, 4, 4.2) & Z(1.8, 2, 3.4) & Z(3.4, 3.6, 4) & Z(4, 4.2, 4.6) & Z(4, 4.5, 5) \\
  3 & Z(2, 2.4, 3) & Z(2.4, 3, 3.2) & Z(1.4, 2, 2.2) & Z(2, 2.4, 3) & Z(4, 4.2, 5) & Z(3, 3.4, 4) \\
  4 & Z(3.5, 4, 4.6) & Z(3.4, 4, 4.4) & Z(1.4, 2, 3) & Z(2.2, 2.6, 3) & Z(2.3, 3, 3.3) & Z(4.2, 4.6, 5) \\
  5 & Z(2, 3, 3.4) & Z(3.2, 4, 4.2) & Z(2, 2.4, 3) & Z(3, 3.4, 4) & Z(4, 4.4, 5) & Z(3, 3.4, 4) \\
\end{array}
\]

We set the confidence levels $\alpha_i = 0.95$ and $\beta_j = 0.90$. Based on the above given
data, we transform the model into its crisp equivalent model:

\[
\begin{align*}
\min & \quad 80000 \sum_{i=1}^{5} x_i + \sum_{l=1}^{3} \sum_{i=1}^{5} \sum_{j=1}^{6} (c_{il} + e_{ijl} + 1) y_{ij} \int_{0}^{1} \Phi^{-1}_{jl}(\alpha) d\alpha \\
\text{s.t.} & \quad \sum_{j=1}^{6} y_{ij} \Phi^{-1}_{jl}(0.95) \leq 10v_{il}x_i, \quad i = 1, \ldots, 5, l = 1, 2, 3 \\
& \quad y_{ij} = 0 \quad \text{if} \quad \Psi^{-1}_{ij}(0.90) \geq 4, \quad i = 1, \ldots, 5, j = 1, \ldots, 6 \\
& \quad \sum_{i=1}^{5} y_{ij} = 1, \quad j = 1, \ldots, 6 \\
& \quad y_{ij} \leq x_i, \quad i = 1, \ldots, 5, j = 1, \ldots, 6 \\
& \quad x_i \in \{0, 1\}, \quad i = 1, \ldots, 5 \\
& \quad y_{ij} \in \{0, 1\}, \quad i = 1, \ldots, 5, j = 1, \ldots, 6.
\end{align*}
\]

Then we use Lingo to obtain the optimal solution which is showed as follows:

\[
\begin{align*}
x_1 &= x_4 = 1, \\
y_{11} &= 1, y_{12} = 1, y_{16} = 1, \\
y_{43} &= 1, y_{44} = 1, y_{45} = 1.
\end{align*}
\]

The above result means that the first and the fourth potential distribution centers should be selected and built. The customers 1, 2 and 6 are serviced by the first potential distribution center, and customers 3, 4 and 5 are serviced by the fourth potential distribution center. In this situation the mean yearly total cost is 1024715. The transportation plan is illustrated in Figure 2.

<table>
<thead>
<tr>
<th>Products</th>
<th>Distribution Centers</th>
<th>Customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

Figure 2: Transportation Plan
5 Conclusion

This paper investigated the location problem of multi-product logistics distribution centers in uncertain environment. In which the transportation times and the customers’ demands were supposed to be uncertain variables. In order to seek the approximate best location plan and assignment plan for the distribution centers with smallest yearly total cost, we constructed an uncertain programming model based on the time window constraint and capacity constraint. For the convenience of solving the model, this model was transformed into its equivalence model. At last, the model was tested by a numerical example.

It is worth saying that some of the assumptions can be relaxed to explore more complex facility location problems with more realistic considerations and more uncertain constraints. In addition, inventory control problem has great influence on facility location problem, the study that incorporates inventory control decision into facility location problem may become a new topic in our further research. Furthermore, the research about multi-product reverse logistics network in uncertain environment can be further researched.

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Uncertain location problem of multi-product logistics distribution centers

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