The Relationship between the Home Resources for Learning and Sciences Achievement in TIMSS 2011
A Multilevel Analysis

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Abstract

This study concerns the evaluation of the effect of the Home Resources for Learning index \( HRL \) on the Sciences performance of Moroccan students who participated in TIMSS 2011\(^1\). The study focused on fourth grade students. The objective of this study is to answer the following research questions: (1) Do schools make a difference in Sciences achievement? (2) Do schools make a difference in the home resources for learning index? (3) The schools whose students have a high availability of resources at home have also a high performance? (4) How student’s performance varies as a function of the home resources for learning index at the student’s level and across schools? The results of the multilevel analysis show that about 31.73% of the total variance is attributable to school differences and the remaining 68.27% of the total variance comes from individual differences among students within schools, and the home resources for learning index is positively correlated with student achievement.

Keywords: Multilevel analysis; Home Resources for Learning; PCM model; TIMSS 2011; R software

\(^1\)Trends International Mathematics and Science Study.
1 Introduction

To assess the impact of family background on student performance, TIMSS 2011 collected detailed data on various aspects related to the economic, social and cultural status of their family, including the level of education and professional status of their father and their mother, as well as educational resources available to them at home. In the literature, we find several authors who studied the relationship between the socio-cultural background and student achievement [1, 2]. These studies are often conducted on topics that vary according to the theoretical vision of the author. However, the family background is the common denominator in all this research. Referring to Turkey TIMSS 1999 data, Yayan & Berberoglu [3] noted that the increased in mathematics performance of fourth grade students is related to the increased level of parental education or the number of books at house. According to Astone & McLanahan [4] the cultural contributions of parents is the key to success in school , as stated the author Perron [5]. The report of Coleman [2] published in 1966, concluded that inequalities in educational outcomes among students were due largely, not with the school resources but with the socioeconomic status of their parents. The studies made by Balli et al. [6] show that the school careers and social status of parents strongly influence the academic futures of students. The most educated parents are more familiar with the administrative procedures and school curriculum, have a role more important in school orientation of their children by helping them in their orientation, or their choice of options, but also in the school selection .

Several studies show that the students from advantaged backgrounds followed a curriculum without repetition, while the students from disadvantaged backgrounds accumulated a academic delay. In fact, study produced by Veillette et al. [7] shows several inequalities in school pathways depending on the social origin of the students at the entrance to the College. Foremost, in terms of access to education, students from advantaged areas tend to do better, while those from disadvantaged backgrounds. They have access to college in a proportion of 84.1% compared to 64.9% for students in medium residential areas and 50.3% for those from disadvantaged areas. According to Coleman et al.[2]; Sanders & Horn [8], the students from low socioeconomic backgrounds have more difficulty in school and they accuse educational underachievement than those from wealthy backgrounds.

The purpose of the study is to evaluate the Home Resources for Learning impact on student achievement in Sciences using the data from student’s questionnaire in TIMSS 2011. This evaluation was performed by answering the following questions:

(1) Do schools make a difference in Sciences achievement?
(2) Do schools make a difference in the home resources for learning index?
(3) Schools with high $\text{MEANHRL}^2$ have also a high performance?

\[2 \text{MEANHRL}_j = \frac{1}{n_j} \sum_{i=1}^{n_{ij}} \text{HRL}_{ij}; \quad j = 1, 2, ..., 275; \quad i = 1, 2, ..., 6820; \quad n_j = \text{the number of individual in school } j.\]
(4) How varies the students’ achievement as a function of $HRL$?

The Home Resources for Learning index ($HRL$) is derived from students’ responses to the question whether they have at home, (a) Number of books in the home (b) Number of children’s books in the home (c) Number of home study supports (e) Highest level of education of either parent and (f) Highest level of occupation of either parent [9,10].

This paper is organized as follows: Section 1 provides an introduction addressing the study context, its purpose and research questions. Section 2 describes the methodology. This section includes: a brief overview of the TIMSS survey, a description of the explanatory variables and the dependent variables, missing data processing, and Scaling the $HRL$ index. Section 3 applies multilevel analysis to answer the research questions. Finally, Section 4 discusses the results with a conclusion of the study.

2 Method

Our study relates to 275 schools with 6820 student. We used two statistical software packages in our application: R Core Team [11] and SPSS 20, so we used the IEA$^3$ IDB Analyzer 2009 Version 3. To answer the first and the second question, we used the “One-way random effect ANOVA” model and for the third question, we used the “means as-outcomes” model, and finally to the fourth question the “Intercepts and Slopes as Outcomes Model” [12]. The parameters of these models are estimated by the R $nlme$ package [13], the significance of fixed effects was tested using the t-test, which is implemented in the $lme$ function of the same package and the variance components are tested using the restricted likelihood ratio test (RLRT) or the asymptotic test ($\chi^2$—mixture approximation) as appropriate: if the model have a single variance components, we used the Exact Restricted Likelihood Ratio Test [14], else if the model have 2 variance components and these latter are correlated, we used the asymptotic test ($\chi^2$—mixture approximation), in this case the $RLRT$ statistic will be approximated by 50% — 50% mixture of $\chi^2_1$ and $\chi^2_2$ [15]. These test are realized by using the simulate function from the $nlme$ package. The partial credit model (PCM) [16] was used to scale the $HLR$ index; the latter is represented by the ability parameter $\theta$. The missing data are imputed by the MICE Method (Multivariate Imputation by Chained Equation) [17] with the R $mice$ package [18].

2.1 TIMSS

Trends International Mathematics and Science Study (TIMSS) is an international assessment conducted every four years and the most recent edition is that of 2011,
it’s coordinated by the International Association for the Evaluation of Educational Achievement. The study measures the mathematics and science achievement of students in Grades 4 and 8. TIMSS collects a wider range of background information on students’ learning environments. Teachers on the one hand fill out questionnaires on the teaching methods... On the other hand, surveyed school’s principals provide information on the resources of the school and the climate of learning. Students are also asked about their attitudes toward science and mathematics and school safety. The TIMSS used a stratified cluster sampling with two steps: the first step consists of sampling the schools by using “systematic probability-proportional-to-size(PS) approach” [19], the second involves selecting a sample of whole classes of the targeted level in schools in the first sample; in Morocco, one or two class by school are selected.

2.2 Measures and Covariates

2.2.1 Dependent variable

The dependent variable was the mean score of student’s achievement in Sciences from the five plausible values: ASSSCI01, ASSSCI02 ASSSCI03, ASSSCI04 and ASSSCI05 obtained by the methods of item response theory [20].

2.2.2 Explanatory variables

There is a single variable, this is the index of Home Resources for Learning HRL, this index is based on responses to the following question in the students’ Questionnaire.

(1) Number of books in the home (students) (ASBG04)?
(2) Number of children’s books in the home (parents) (ASBH15A)?
Response options: 1: 0-10; 2: 11-25; 3: 26-50; 4: 51-100; 5: More than 100;
(3) Number of home study supports (students) (ASDG05S)?
Response options: 1: None; 2: Internet connection or own room; 3: Both;
(4) Highest level of education of either parent (parents)(ASDHEDUP)?
1: Finished some primary or lower secondary or did not go to school; 2: Finished lower secondary; 3: Finished upper secondary; 4: Finished post-secondary education; 5: Finished university or higher;
(5) Highest level of occupation of either parent (parents) (ASDHOCCP)?
Response options: 1 Has never worked outside home for pay, general laborer, or semi-professional (skilled agricultural or fishery worker, craft or trade worker, plant or machine operator; 2: Clerical (clerk or service or sales worker); 3: Small business owner; 4: Professional (corporate manager or senior official, professional, or technician or associate professional).
2.3 Missing data

At the presence of missing data, a solution is multiple imputations, is a statistical method for the analysis of incomplete data originally proposed by Rubin [21] and which can produce m matrices’ imputed data. We used the method Multivariate Imputation by Chained Equation (MICE) [17]. This procedure is implemented in R software with the mice package. The data are categorical, for this we proposed to replace each missing value by the median value of 5 imputed values (m=5) in each variable.

2.4 Scaling the HRL index

There are two models for ordinal items responses: the Rating Scale Model (RSM) [22] and the Partial Credit Model (PCM) [16]. Following the nature of the items in our application, we chose the unidimensional PCM model. In this research, we used the adjustment Infit and Outfit indices used in the R TAM package[5], this latter is open source and is completely free for download. The model parameters are estimated by the tam.mml function (Test Analysis Modules: Marginal Maximum Likelihood Estimation). This function was used with the option PCM2 (partial credit model with ConQuest parametrization “item + item*step”). The ability parameter \( \theta \) is estimated by the tam.wle function (Weighted Likelihood Estimation). Items are perfectly adjusted to the model if the values of Infit and Outfit are equal to 1; some authors recommend an interval of critical values that extends from 0.5 to 1.7 which does not indicate a major problem of adjustment [24]. Table 1 represent the Infit and Outfit indexes, we note that the values of these indexes are between 0.92 and 1.12, the reliability of the test (WLE Reliability[6]=0.40; EAP Reliability[7]= 0.53) and the Cronbach’s Alpha=0.57. These results show that the model adequately fits the items.

Table 1: Outfit and Infit indexes for The Home Resources for Learning items.

<table>
<thead>
<tr>
<th>Item</th>
<th>Outfit</th>
<th>Outfit_t</th>
<th>Infit</th>
<th>Infit_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASBG04</td>
<td>1.06</td>
<td>2.32</td>
<td>1.12</td>
<td>4.63</td>
</tr>
<tr>
<td>ASDG05S</td>
<td>0.95</td>
<td>-3.43</td>
<td>0.96</td>
<td>-3.19</td>
</tr>
<tr>
<td>ASBH15A</td>
<td>1.05</td>
<td>1.81</td>
<td>1.01</td>
<td>0.33</td>
</tr>
<tr>
<td>ASDHOCCP</td>
<td>0.95</td>
<td>-2.36</td>
<td>0.95</td>
<td>-3.04</td>
</tr>
<tr>
<td>ASDHEDUP</td>
<td>0.92</td>
<td>-3.81</td>
<td>0.94</td>
<td>-3.29</td>
</tr>
</tbody>
</table>

3  Multilevel analysis

Multilevel modeling is not always necessary for a hierarchical database. Zero ICC (Eq: (4)), indicates that there is no variation between group means. In this case, a single level analysis is justified, instead of a multilevel analysis. But a non-zero value of ICC does not indicate that we can apply multilevel analysis. Particularly for social studies, a value of ICC between .05 and .20 is sufficient to apply the HLM [25, 26, 27, 28, 29]. According to Hox [25]; Muthén [26]; Muthén & Satorra [29], if the value of the Design Effect denoted by $DE$ (Eq: (5)) is greater than 2, we can apply the HLM. Therefore, the use of HLM depends on the intraclass correlation coefficient (ICC) and the Design Effect.

In Hierarchical Linear Models, two forms of level-1 predictors centering are possible: the grand-mean centering, in this case $X_{ij}$ will be centered around the grand-mean (i.e., $X_{ij} - \bar{X}$) and the group-mean centering, for this possibility, $X_{ij}$ is centered around the group-mean (i.e., $X_{ij} - \bar{X}_j$). The centering of level-1 or level-2 predictors depends on the research question. Centering predictor variables in multilevel models resulted in several recent researches that develop the consequences of those choices [12, 30, 31, 32].

3.1  Answers to research questions

3.1.1  Do schools make a difference in Sciences achievement?

The first question is considered as one of the most fundamental research questions in the analysis of the schools performance; it allows us to study the variability of students performance between schools, and consequently if there is necessary to use HLM to address other research questions. To answer this question, we used the “One-way random effect ANOVA” model [12]:

Level 1 model: $ScieAch_{ij} = \beta_0 + \varepsilon_{ij}; \varepsilon_{ij} \sim N(0, \sigma^2), \quad (1)$

Level 2 model: $\beta_{0j} = \gamma_{00} + \mu_{0j}; \mu_{0j} \sim N(0, \tau_{00}^2), \quad (2)$

Combined model $M_0$: $ScieAch_{ij} = \gamma_{00} + \mu_{0j} + \varepsilon_{ij}, \quad (3)$

Intraclass correlation coefficient: $ICC = \tau_{00}^2/(\sigma^2 + \tau_{00}^2), \quad (4)$

Design Effect Statistics $DE = 1 + (n_c - 1)/ICC, \quad (5)$

where $n_c$ denotes the number of students per school; $\gamma_{00}$ the grand mean ScieAch achievement; $ScieAch_{ij}$: Sciences achievement score of student i nested in school j; $\beta_{0j}$ : the Sciences achievement mean for school j; $\tau_{00}^2$: between-school variance and $\sigma^2$: within-school variance.

The results from the $M_0$ model (Eq: (3)) are summarized in (Table 2, column 2). The results showed that, the grand mean ScieAch achievement was 268.76. According to Raudenbush & Bryk [12], the range of plausible values = $\gamma_{00} \pm 1.96 \sqrt{\tau_{00}}$ that
is 95% CI [260.74, 276.78]. Between-school variance $\tau^2_{00}$ was 4146.07 and within-

school variance $\sigma^2$ was 8922.70. These results indicate that about 31.73% of the
total variance is attributable to school differences and the remaining 68.27% of the
total variance comes from individual differences among students within schools.

According to Hox [25], if the test $H_0 : \tau^2_{00} = 0$ vs $H_a : \tau^2_{00} > 0$ is significant for the
null hypothesis, the population is considered heterogeneous. The simulated p-value
(10,000 times) shows that this test is significant at the 0.01 level ($RLRT = 1705.62,$
$p < 0.001$). The test of homogeneous variance rejected the homogeneous population. The estimated values of $DE$ are greater than 2 ($n_c = 6820/275 = 24.8, DE = 76.02$). So the use of HLM is necessary to explain the variability between schools
[25, 26, 29].

3.1.2 Do schools make a difference in the home resources for learning index?

To study the variability of this index across the schools we used “One-way random


effect ANOVA” model [12]. The answer to this research question lets us know if

the Moroccan students are on an the same equal footing towards school

\begin{align}
\text{Level 1 model : } & HRL_{ij} = \beta_{0j} + \epsilon_{ij}; \epsilon_{ij} \sim N(0, \sigma^2), \\
\text{Level 2 model : } & \beta_{0j} = \gamma_{00} + \mu_{0j}; \mu_{0j} \sim N(0, \tau^2_{00}), \\
\text{Combined model } & M_1 : HRL_{ij} = \gamma_{00} + \mu_{0j} + \epsilon_{ij},
\end{align}

\(\gamma_{00}\): the grand mean of $HRL$ achievement; $HRL_{ij}$: $HRL$ achievement score of

student i nested in school j; $\beta_{0j}$: the $HRL$ achievement mean for school j ; $\tau^2_{00}$:

between-school variance and $\sigma^2$: within-school variance.

The results from the $M_1$ model (Eq: (8)) are summarized in (Table 2, column 3).

The results showed that, the grand mean $HRL$ achievement was $-0.02$. According
to Hox [25],the range of plausible values = $\gamma_{00} \pm 1.96 \sqrt{\tau_{00}}$ that is 95% $CI [-0.07,$

0.04]. Between-school variance $\tau^2_{00}$ was 0.1936 and within-school variance $\sigma^2$ was

0.4761. These results indicate that about 29% of the total variance is attributable
to school differences and the remaining 71% of the total variance comes from indi-

vidual differences among students within schools. According to Hox [25], if the
test $H_0 : \tau^2_{00} = 0$ vs $H_a : \tau^2_{00} > 0$ is significant for the null hypothesis, the popu-
lation is considered heterogeneous. The simulated p-value ( 10,000 times) shows that
this test is significant at the 0.01 level ($RLRT = 1641.31, p < 0.001$). The test of
homogeneous variance rejected the homogeneous population. Thus, on average the
availability of resources at home varies significantly across schools.

3.1.3 Schools with high $MEANHRL$ have also a high performance ?

To answer this research question, we used the “Means-as-outcomes” model. The
variable $MEANHRL$ will be centered by $HRL_{..} = \frac{1}{N} \sum_{j=1}^{N} \frac{1}{n_j} \sum_{i=1}^{n_j} HRL_{ij}$ (grand
mean centered).

\begin{align*}
\text{Level 1 model: } \text{Sci}e\text{Ach}_{ij} &= \beta_{0j} + \varepsilon_{ij}; \varepsilon_{ij} \sim N(0, \sigma^2), \\
\text{Level 2 model: } \beta_{0j} &= \gamma_{00} + \gamma_{1j}(\text{MEANHRL}_j - \overline{\text{HRL}_c}) + \mu_{0j}; \mu_{0j} \sim N(0, \tau^2_{00}), \\
\text{Level 2 model: } \gamma_{1j} &= \gamma_{10}, \\
\text{full model } M_2: \text{Sci}e\text{Ach}_{ij} &= \gamma_{00} + \gamma_{10}(\text{MEANHRL}_j - \overline{\text{HRL}_c}) + \mu_{0j} + \varepsilon_{ij},
\end{align*}

The term $\gamma_{00}$ represents the grand mean Sci Ach achievement given $\text{MEANHRL} = \overline{\text{HRL}_c}$; $\gamma_{10}$: the MEANHRL effect on the average performance in Sciences; $\mu_{0j}$: random intercepts and its variance $\tau^2_{00}$ measure the heterogeneity of schools in Sci Ach.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{relationship.png}
\caption{Relationship between MEANHRL index and SciAch: regression line according to the $M_2$ model (Eq: (12)).}
\end{figure}

The results of $M_2$ model Eq: (12) are represented in (Table 2 column 4). Thus, the MEANHRL explains $8\text{\textsuperscript{8}}$ 24.17\% of the total variance at school level. The MEANHRL is positively correlated with the student performance and is significant at the 0.01 level ($\gamma_{10} = 68.20, t(273) = 8.74, p < 0.001$). Moreover, when the MEANHRL increases by one unit, the students’ average score increases by 68.20

\textsuperscript{8} The proportion of variance explained is given by the formula: $R^2 = (\text{varNoPredictor} - \text{varPredictor})/\text{varNoPredictor}$ [12, 33].
units. The simulated p-value (10,000 times) shows that the test $H_0 : \tau^2_{00} = 0$ vs $H_a : \tau^2_{00} > 0$ is significant at the 0.01 level ($RLRT = 1256.67, p < 0.001$). Thus, the average performance of students in Sciences varies significantly across schools.

3.1.4 How varies the students’ achievement as a function of HRL: (a) at the student’s level and (b) across schools?

The Figure 2 represents 275 regression lines of $ScieAch$ on $HRL$ (weak lines) and the average line corresponding (solid line).

![Figure 2: Relationship between HRL index and ScieAch: 275 regression lines according to the $M_3$ model (Eq: (16)).](image)

From this Figure, the slopes and intercepts are found to be different, so to answer these questions we used the “Intercepts and Slopes as Outcomes” Model. The variable $HRL$ will be centered with the group-mean $\overline{HRL}_{j} = \frac{1}{n_j} \sum_{i=1}^{n_j} HRL_{ij}$ (group-mean centering) [12].

Level 1 model: $ScieAch_{ij} = \beta_{0j} + \beta_{1j}(HRL_{ij} - HRL_{i,j}) + \varepsilon_{ij}$; $\varepsilon_{ij} \sim N(0, \sigma^2)$, \hfill (13)

Level 2 model: $\beta_{0j} = \gamma_{00} + \mu_{0j}$; $\mu_{0j} \sim N(0, \tau^2_{00})$, \hfill (14)

Level 2 model: $\beta_{1j} = \gamma_{10} + \mu_{1j}$; $\mu_{1j} \sim N(0, \tau^2_{11})$, \hfill (15)

Full model $M_3$: $ScieAch_{ij} = \gamma_{00} + \gamma_{10}(HRL_{ij} - HRL_{i,j}) + \mu_{1j}(HRL_{ij} - HRL_{i,j}) + \mu_{0j} + \varepsilon_{ij}$, \hfill (16)
\( \beta_{0j} \): is the unadjusted mean for school \( j \), \( \mu_{0j} \): random effect that measures the difference between \( \gamma_{00} \) and the \( \beta_{0j} \) and its variance \( \tau^2_{00} \) measure the heterogeneity of schools in math achievement; \( \mu_{1j} \): random effect which measures the difference between the average slope \( \gamma_{10} \) and slope \( \beta_{1j} \); its variance \( \tau^2_{11} \) measure the heterogeneity of the effect of \( HRL \) index across schools.

The results of \( M_3 \) model (Eq: \((16)\)) are represented in (Table 2, column 5).

(a) At the student's level

The home resource for learning index \( HRL \) is positively correlated with the student achievement and is significant at the 0.01 level. When \( HRL \) increase by one unit the \( ScieAch \) increase by 5.58 units (\( \gamma_{10} = 5.58, t(6544) = 4.38, p < 0.001 \)). Thus, on average the availability of resources at home has a positive effect on student achievement in Sciences.

(b) Across schools

To answer this question we used the following two tests [34, 35]: (a) testing for random intercept in the presence of a random slope \( H_0: \tau^2_{00} = 0 \) vs \( H_a: \tau^2_{00} > 0 \) to know if the student’s average performance in Sciences (\( \beta_{0j} \)) varies across school in the presence of the random effect \( \mu_{1j} \) and (b) testing for a random slope in the presence of a random intercept \( H_0: \tau^2_{11} = 0 \) vs \( H_a: \tau^2_{11} > 0 \) to know if the effect of the \( HRL \) index is heterogeneous across schools. Note that, the functions \( exactRLRT \) accommodates only independent random effects. In against to the function \( simulation \) can accommodate correlated random effects, (non diagonal variance–covariance matrices of random effects \( D \)). But this function has limitations, for example in the case of an asymptotic test, under \( H_0 \) requires certain conditions, it has to say the value tested of the test \( \{ H_0 : \tau^2 = 0 \} \) must be located on the boundary space \( \mathbb{R}_+ \). For instance, they both only apply to conditional independence HLM.

Before answering these questions, examining the correlation between the two random effects \( \mu_{0j} \) and \( \mu_{1j} \). These effects are slightly positively correlated (\( \rho = cov(\mu_{0j}, \mu_{1j}) = 0.073 \), see Figure 3, but the variance–covariance matrices \( D \) is not diagonal. \( D = ( \begin{pmatrix} \tau^2_{00} & \rho \tau_{11} \tau^2_{00} \\ \rho \tau_{11} \tau^2_{00} & \tau^2_{11} \end{pmatrix} = ( \begin{pmatrix} 4155.9 & 69.9 \\ 69.9 & 223.41 \end{pmatrix} ; \ var(\hat{b}_j) = ( \begin{pmatrix} 16.74 & 0.26 \\ 0.26 & 3.83 \end{pmatrix} ; \hat{b}_j = ( \begin{pmatrix} \hat{\mu}_{1j} \\ \hat{\mu}_{0j} \end{pmatrix} .

For that, we propose to use the function \( simulation \) instead of the function \( exactRLRT \). The p-values of these tests are simulated by using the function \( simulate \), explicitly this function has been applied to objects \( m_0 \) (null model) and \( m_\text{A} \) (alternative model). Using the argument \( nsim \), the number of simulated values is fixed at 10,000, P-value plots are used to interpret these tests. The function \( plot \) traces this type of chart; this latter consists in representing the empirical p-values as a function of the nominal p-values of the \( (R)LR \) test; the nominal p-values are calculated using three distributions: \( \chi^2_1 \); \( \chi^2_2 \) and 50% - 50% mixture of \( \chi^2_1 \) and \( \chi^2_2 \).

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9 For the test \( H_0: \tau^2_{00} = 0 \) vs \( H_a: \tau^2_{00} > 0 \):

\( m_0 : ScieAch_{ij} = \gamma_{00} + \gamma_{10}(HRL_{ij} - HRL) + \mu_{1j}(HRL_{ij} - HRL) + \epsilon_{ij}, \)

\( m_\text{A} : ScieAch_{ij} = \gamma_{00} + \gamma_{10}(HRL_{ij} - HRL) + \mu_{1j}(HRL_{ij} - HRL) + \mu_{0j} + \epsilon_{ij}. \)

For the test \( H_0: \tau^2_{11} = 0 \) vs \( H_a: \tau^2_{11} > 0 \):

\( m_0 : ScieAch_{ij} = \gamma_{00} + \gamma_{10}(HRL_{ij} - HRL) + \mu_{0j} + \epsilon_{ij}, \)

\( m_\text{A} : ScieAch_{ij} = \gamma_{00} + \gamma_{10}(HRL_{ij} - HRL) + \mu_{1j}(HRL_{ij} - HRL) + \mu_{0j} + \epsilon_{ij}. \)
Figure 3: Scatterplot of the slope $\mu_{1j}$ as a function of the intercept $\mu_{0j}$.

Using the `ellipse()` function from the `ellipse` package [50], we added two ellipses representing the 95% confidence regions corresponding to the $2 \times 2$ matrix $\mathbf{D}$ (solid line) and $\text{var}(\hat{b}_j)$ (dashed) line, respectively.

(a) Testing for random intercept $H_0 : \tau_0^2 = 0$ vs $H_a : \tau_0^2 > 0$.

(b) Testing for a random slope $H_0 : \tau_{11}^2 = 0$ vs $H_a : \tau_{11}^2 > 0$.

Figure 4: Empirical p-values as a function of the nominal p-values.

According to the figure [4a], the nominal p-values obtained using the mixture distributions 50% – 50% of $\chi_1^2$ and $\chi_2^2$ are lower than those of the corresponding simulated values,
this shows that the test $H_0 : \tau_{00}^2 = 0$ vs $H_0 : \tau_{00}^2 > 0$ is significant at the 0.01 level; thus the average students’ performance vary significantly across schools. The figure 4b shows that the nominal p-values obtained using the mixture distributions 50%−50% of $\chi^2_1$ and $\chi^2_2$ are higher than the corresponding simulated values; this means that the test: $H_0 : \tau_{11}^2 = 0$ vs $H_a : \tau_{11}^2 > 0$ is not significant at the 0.01 level. Therefore, there is a homogeneity of the effect of $HRL$ across schools.

4 Discussion and conclusion

Before discussing the results of the models $M_0$, $M_1$, $M_2$ and $M_3$, let’s present some results of descriptive statistics.

According to Figure 3, the highest level of occupation of either parent (ASDHOCCP, Figure ??b); the highest level of education of either parent (ASDHEDUP, Figure ??c); the number of children’s books in the home (ASBH15A, Figure ??d) and the number of home study supports (ASDG05S, Figure ??e) are positively correlated with students’ performance ($ScieAch$). In contrast, the number of books in the home (ASBG04, Figure ??a) is negatively correlated with students’ performance. Thus, if the number of books at home increases, the students’ Sciences performance also decreases. This shows that the availability of home resources for learning has a positive effect on students’ performance, except the number of books in the home. Several studies reveal that learners resulting from low income socioeconomic communities face greater difficulty in school and manifest an academic lag more patent than those from more affluent backgrounds [2]. According to Astone & McLanahan [4], the cultural contribution of parents has become a basic factor for the students’ academic achievement. Studies conducted earlier by Balli et al. [6], show that students’ scholastic careers and the social status of their parents influence school evolution. The results of the $M_0$ model (Eq: (3), Table 2, column 2) show that the $HRL$ index varies significantly across schools and explains 29% of the total variance at the school level, these results can be explained by the diversity of the Moroccan society at the social, cultural, and economic level, which leads us to conclude that the Moroccan students are not on an the same equal footing towards school.

The results of the $M_1$ model (Eq: (8), Table 2, column 3) show that 31.73% of the total variance of the students’ achievement in Sciences is explained at the school level. Furthermore, this performance varies significantly across schools. Therefore the results of the $M_0$ and $M_1$ models reinforce the results obtained in the $M_2$ model (Eq: (12), Table 2, column 4), since we find that the increase of students’ performance depends on the increase of $MEANHRL$ significantly.

The results of the $M_3$ model (Eq: (16), Table 2, column 5) show that the availability of resources at home $HRL$ has a positive and significant effect on the performance of students in Sciences. However the result of the test $\{\tau_{11}^2 = 0\}$ shows that there is a homogeneity of this effect across schools; moreover, the $HRL$ effect does not vary significantly across schools. These results lead us to conclude that the $HRL$ index provides an improvement of students’ performance and affects positively their performance in Sciences, which confirms the results found by Coleman et al. [2]; Astone & McLanahan [4]; Balli et al. [6].

The analysis of the results of our study reveals that the family background remains one of
the major factors of influence on the performance of students in Sciences. However, a disad-
advantaged family environment does not automatically lead to poor student achievement. 
These results indicate that the proportion of students with an average greater than or equal
 to the international average estimated at 500 points is 3.02% of which 1.67% have a positive
index and 1.35% have a negative HRL index. Thus, the low level of achievement of some 
students does not necessarily mean that these students live in disadvantaged environments.

Figure 5: Relationship between ScieAch and the five variables of the HRL index: ASBG04, ASDG05S, ASBH15A, ASDHOCCP and ASDHEDUP.

Table 2: Parameter Estimates for Alternative Multilevel Sciences Achievement Models.

<table>
<thead>
<tr>
<th>Model</th>
<th>M0</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed effects</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>268.76 (4.09)$^*$</td>
<td>-0.02 (0.03)$^m$</td>
<td>269.17 (3.61)$^*$</td>
<td>268.76 (4.09)$^*$</td>
</tr>
<tr>
<td></td>
<td>[260.74, 276.78]</td>
<td>[-0.07, 0.04]</td>
<td>[262.08, 276.26]</td>
<td>[260.74, 276.78]</td>
</tr>
<tr>
<td>Slope (MEANHRL)</td>
<td>68.20 (7.80)$^*$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[52.83, 83.56]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope (HRL)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.58 (1.96)$^*$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[4.74, 12.42]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance components</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 1 ($\sigma$)</td>
<td>94.46</td>
<td>0.69</td>
<td>94.46</td>
<td>93.70</td>
</tr>
<tr>
<td>Level 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept ($\beta_0$)</td>
<td>64.39$^*$</td>
<td>0.44$^*$</td>
<td>56.07$^*$</td>
<td>64.47$^*$</td>
</tr>
<tr>
<td>Slope ($\beta_1$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(\mu_0, \mu_1)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Explained variance ($\tau_0^2$)</td>
<td>24.70%</td>
<td></td>
<td>0.25%</td>
<td></td>
</tr>
<tr>
<td>Explained variance ($\tau_1^2$)</td>
<td>0%</td>
<td></td>
<td>1.6%</td>
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</tr>
<tr>
<td>ICC</td>
<td>31.73%</td>
<td>29%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DE (N=6820, n=275)</td>
<td>76.02</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: $^*$: $p < 0.001$; $^m$: Test significant at the 0.01 level; $^n$: Test not significant at the 0.01 level; []: 95% confidence interval.
References


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