Abstract

The global financial crisis of 2007-2008 has highlighted the importance of a correct pricing of the so-called financial derivatives. Analyzing the methodology of pricing of non-listed derivatives by using the Monte Carlo Method for Pricing Complex Financial Derivatives: An Innovative Approach to the Control of Convergence.
Ilaria Bendato et al.

Carlo method, the Authors have realized that the determination of the sample size is not managed properly. This is because the research offices of banks rely on, as suggested by the literature of the field and technical manuals for practitioners, a standard number of simulation runs, by rules of thumb, between 1,000 and 10,000. The consequence is that financial institutions lead to financial statements fair values with no knowledge of its fluctuation band and the robustness of the result. Conscious of this practice, the Authors, dealing from a long time to the topic of output reliability in applications of discrete event simulation and Monte Carlo simulation, address the problem through the use of a methodology based on the control of Mean Pure Square Error (MSPE), already successfully tested in other contexts. Thanks to the proposed approach, applied for pricing complex derivatives, it is possible to determine the size of the experimental sample in order to ensure a pre-assigned degree of reliability of the output results.

Keywords: Complex derivatives, Pricing, Monte Carlo simulation, Experimental error, Mean Square Pure Error

1 Introduction

In a recent work, R. Mosca et al. [21] have sought to demonstrate the inadequacy of traditional approaches to control the convergence in the pricing of financial derivatives carried out by Monte Carlo simulation. For this aim it have been taken into consideration some derivatives whose price is, usually, estimated in a deterministic way, by means of the fundamental PDE of Black - Scholes - Merton.[12]

The mentioned study was particularly focused on the comparison between the traditional approaches and the Mean Square Pure Error (MSPE) technique proposed by R. Mosca et al. [1],[20],[21]. This technique for determining the size of the statistical sample in Monte Carlo simulations is a natural evolution of a methodology, designed by the authors, for establishing the optimal run length in discrete event simulation models time evolving [2],[3].

The traditional approaches to the sample size, suggested in research articles and technical manuals for practitioners, recommend a number of replicated runs between 1000 and 10000 but no attention is put on the control of the experimental error that affect the simulation output [4],[8],[9],[10].

In this article we propose some applications of the MSPE methodology to complex derivatives that do not have a closed formula for the determination of their price and therefore the integration by stochastic Monte Carlo simulation is the only way to obtain their price. The paper aims to demonstrate how the proposed technique can be an operational tool both for researchers and practitioners to determine the correct fair value of these complex derivatives.
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This is a fundamental activity for the Value at Risk (VaR) estimation and to determine the value of derivatives to be included in the financial institutions statements.

2 An application to pricing based on BS framework

Let us examine how financial instruments lacking a closed and precise appraisal formula can be priced. The purpose here is to determine the fair value of the stocks under consideration and perform an analysis of the error affecting the result.

The first algorithm developed using Matlab regards a two-asset Asian Spread Put Option, whose financial details are illustrated in Tab. 1.

<table>
<thead>
<tr>
<th>Start Date</th>
<th>March 30, 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>End Date</td>
<td>June 30, 2009</td>
</tr>
<tr>
<td>Market price of first asset</td>
<td>1.9</td>
</tr>
<tr>
<td>Market price of second asset</td>
<td>2.12</td>
</tr>
<tr>
<td>Strike</td>
<td>2</td>
</tr>
<tr>
<td>Free-risk rate</td>
<td>0.0425</td>
</tr>
<tr>
<td>Volatility of first asset</td>
<td>20%</td>
</tr>
<tr>
<td>Volatility of second asset</td>
<td>30%</td>
</tr>
<tr>
<td>Correlation</td>
<td>15%</td>
</tr>
</tbody>
</table>

Table 1: Financial parameters of the two-asset Asian Spread Put Option

The model takes into account how the two assets are correlated thanks to Cholesky factorization, a method by which random numbers \((\varepsilon_1, \varepsilon_2)\) are generated by a normal with a given correlation structure.

\[
\begin{vmatrix}
1 & 0 \\
\rho & \sqrt{1-\rho^2}
\end{vmatrix}
\begin{vmatrix}
x_1 \\
x_2
\end{vmatrix}
= 
\begin{vmatrix}
x_1 \\
x_2\sqrt{1-\rho^2}
\end{vmatrix}
\]

\[
\varepsilon_1 = x_1 \\
\varepsilon_2 = \rho x_1 + x_2\sqrt{1-\rho^2}
\]
where $\rho$ is the correlation coefficient between the two assets and $x_1, x_2$ is the pair of random variables with normal distribution.

Once the two processes are left to evolve for the option’s entire life, the pay-off value, given by $\max(-[\text{Average}_1 - \text{Average}_2 - K], 0)$ is estimated.

Once a significant number of simulations are performed on the possible final price level and just as many pay-off values are obtained (Fig. 1), their mean value is calculated by discounting these back to the pricing date using the formula (6). In order to determine the error stabilization zone on the model’s output, it was decided to chart out 25 $MSPE_{MED}$ curves as illustrated in Fig. 2. An analysis of the curves shows that the statistical stabilization of the error is achieved at an $nSim$ equal to $6 \cdot 10^4$. The red line in the graph in Fig. 2 shows the maximum experimental error that we can accept for pricing. Like the previous case, the error identified gives a maximum deviation in terms of confidence interval of $\pm 0.005$ with a degree of reliability of 95%.

In particular, using a $numSim$ of 60000, the confidence interval obtained for the price is equal to:

$$Price \geq 2.1975 - 3.182 \sqrt{\frac{7.8125 \cdot 10^{-7}}{4}}$$

$$Price \leq 2.1975 + 3.182 \sqrt{\frac{7.8125 \cdot 10^{-7}}{4}}$$

$$2.1961 \leq Price \leq 2.1989$$

This behaviour was assessed by replicating the $6 \cdot 10^4$ simulations for 300 times. It gave the price values in Fig. 3. In all cases the option’s value is equal to 2.20.

According to the literature, performing 10000 runs, the option’s price can be $2.19 - 2.20$, [8]. A result like this, though, is still affected by a large degree of instability. At 10000 runs there is still total instability of the experimental error whose impact will hence vary from one run to another preventing proper pricing as illustrated in Figure 4.

The other algorithm developed using Matlab regards the pricing of a two-asset basket option, whose financial details are illustrated in Tab. 2. The logical flow is the same as the previous. The only difference is the definition of the option’s payoff given by:

$$\max\left[\left(\frac{St_1 - K_1}{\text{numAssets}} + \frac{St_2 - K_2}{\text{numAssets}}\right), 0\right]$$

The graph for the $MSPE_{MED}$ curve illustrated in Fig. 5 shows a rather slow stabilization of simulator. Said stabilization is also very distant from an $nSim$
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<table>
<thead>
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<th>Start Date</th>
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</tr>
</thead>
<tbody>
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<tr>
<td>Market price of first asset</td>
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<td>Market price of second asset</td>
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<tr>
<td>Strike</td>
<td>42</td>
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<tr>
<td>Free-risk rate</td>
<td>0.0425</td>
</tr>
<tr>
<td>Dividend yield of first asset</td>
<td>0.0350</td>
</tr>
<tr>
<td>Dividend yield of second asset</td>
<td>0.04</td>
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<tr>
<td>Volatility of first asset</td>
<td>20%</td>
</tr>
<tr>
<td>Volatility of second asset</td>
<td>30%</td>
</tr>
<tr>
<td>Correlation</td>
<td>25%</td>
</tr>
</tbody>
</table>

Table 2: Financial parameters of the two-asset basket option

of 10000, a value, which in this case would give a significant error in the $y^*$ responses of:

$$3.182\sqrt{\frac{1.0624 \times 10^{-3}}{4}} = 0.0518$$

Only choosing to perform $5 \cdot 10^5$ simulations at the expense of a long simulation time would it be possible to obtain a confidence interval for the model output equal to:

$$1.2731 \leq \text{Price} \leq 1.2788$$

which, in terms of operating practice, is to be translated into:

$$1.27 \leq \text{Price} \leq 1.28$$

Performing 100 replications using an $nSim$ equal to $5 \cdot 10^5$ gives a result consistent with what was envisaged in the analysis of the graphs, namely a fair value error in the order of a cent (see Figure 6), and confirms the normality of the values arranged according to an $\mathcal{N}(1.2759, 0.0025)$.

Comparing the result obtained using the experimental method proposed by the authors with that suggested by the literature after the usual 10000 runs, namely a price for the option under consideration equal to 1.25, [8], it can be concluded that in this case the price difference becomes absolutely relevant and that the price at 10000 runs is affected by a quantity of experimental error, which can make it vary greatly from one campaign to another as illustrated in Fig. 7.
For instance, we can consider a Financial Institution which has a portfolio consisting of the three options evaluated previously: a plain-vanilla call [Appendix], a two-asset asian spread put and a two-asset basket call. The holdings (the quantity held or the number of contracts) of each instrument are respectively 100.000, 50.000 and 70.000. The value of this portfolio is given by the multiplication of the quantity by the estimated price.

As a result of an innacurate pricing methodology based on 10,000 runs, the portfolio could be either undervalued or overvalued:

\[
\text{Portfolio}_{\text{Under-Valued}} = 100000 \cdot 0.23 + 50000 \cdot 2.19 + 70000 \cdot 1.22 = 217.900$
\]

\[
\text{Portfolio}_{\text{Over-Valued}} = 100000 \cdot 0.25 + 50000 \cdot 2.21 + 70000 \cdot 1.33 = 228.600$
\]

\[
\text{Portfolio}_{\text{Fair-Valued}} = 100000 \cdot 0.24 + 50000 \cdot 2.20 + 70000 \cdot 1.28 = 223.600$
\]

This lack of precision causes serious mistakes for instance on VaR computing, hedging and accounting.

3 An application to pricing based on BS alternatives

The Geometric Brownian Motion (4) represents the stochastic dynamics proposed by Black-Scholes-Merton [12] to simulate the future behavior of an asset on which the option is referred. The stochastic differential equation (4) is in fact still the main benchmark for the development of many financial instruments. Most of the calculation modules of Bloomberg®, the market leader platform, adopt this model for Equity and Commodity Options.

Obviously, as each model, is based on a simplified representation of reality and is evaluated with certain assumptions among which there are two, in particular, that are strongly criticized by the specific literature:
1. The normal distributed returns assumption of the underlying asset which is translated in the simulation of a regular behavior in the future [13]

2. The assumption of a fixed volatility over time. [14]

Since 1973, several approaches have been proposed in the scientific literature to deal with these limitations, actually difficult to find in the turbulent behavior of the markets, especially after the recent financial crisis. Among the most popular alternatives in financial practice that tend to relax these constraints the Authors cite the models of Merton-Bates’s jump-diffusion model (JD model) and the stochastic volatility model (SABR model) [15].

1. JD model let prices make discrete jump from time to time: this feature allows the classical dynamics (4) to have not continuous levels over time. They represent more adequately the actual behavior of the asset. [16]

The equation can be implemented in a vectorialized way in Matlab [17] is given by:

\[
S(t + h) = S(t) \exp \left[ \left( \mu - \delta - \lambda k - \frac{\sigma^2}{2} \right) h + \sigma \sqrt{h} Z \right] \\
\exp \left[ m \left( \mu_J - \frac{\sigma_J^2}{2} \right) + \sigma_J \sum_{i=0}^{m} G_i \right]
\] (1)

where \( \mu_J \) and \( \sigma_J \) are the average and standard deviation of the jump’s magnitude; \( Z \) and \( G_i \) are Gaussian random variables. \( \mu \) represents the instantaneous expected rate of return of the asset, \( \delta \) is the dividend rate, \( \sigma \) is the instantaneous volatility of the asset’s return, \( \lambda \) is the yearly average of the number of jumps, \( m \) is a Poisson random variable with mean \( \lambda \) and \( h \) is the time step. The case without jumps can be obtained with \( m = 0, \mu_J = \sigma_J = \lambda = 0 \). Parameter \( k \) is given by \( k = \exp (-\mu_J) - 1 \) and it represents the negative value of the expected percentage jump.

2. SABR (Stochastic, \( \alpha, \beta, \rho \)) is a two-factor model that represents the dynamics of volatility according to the SDE [7]:

\[
dF = \alpha F^\beta dZ \\
d\alpha = \xi \alpha dW
\]

where \( F \) is the future/forward price, \( \beta \) is a constant deciding the distribution of the asset price, \( \alpha \) is the volatility of the forward price and \( \xi \) is the volatility
of the volatility. $dZ$ and $dW$ are two correlated Wiener processes.

The MSPE methodology proposed by the Authors, as a multi-purpose technique, allows its application to these stochastic financial models. The following are two applications of how the control of experimental error can be used in these alternatives to the classical BS model.

1. Let consider an European option whose underlying is regulated by (1), whose financial parameters are $\mu = 0.05$, $S(0) = 100$, $\delta = 0$, $\lambda = 3$, $\sigma = 0.25$, $\mu_J = 0.02$, $\sigma_J = 0.10$ and its pay-off is $\max[K - S(T), 0]$ where $K = 70$ is the strike price and $T = 0.5$ is the time to maturity.

The analysis of the stationary curves (Fig. 8) leads to trust a pricing conducted with the Monte Carlo technique employing at least 125000 simulations to obtain confidence bands on the output equal to:

$$
\text{Price} \in \left[0.3156 \pm 3.183 \sqrt{\frac{6.4895 \cdot 10^{-6}}{4}}\right]
$$

$$
0.3115 \leq \text{Price} \leq 0.3197
$$

Proceeding to simulate the number of standard run, the error on the price of the derivative increases in order to make the calculation of its fair value financially unacceptable, as the graph on the $[j, y^*]$ shows (Fig. 9). The abscissa axis represent the replications conducted ($j = 1, \ldots, 100$), obtained with a number of runs equal to:

- $i = 1, \ldots, 125000$ for the red line
- $i = 1, \ldots, 10000$ for the green line
- $i = 1, \ldots, 1000$ for the blue line

The output obtained in correspondence with these parameters is the option price ($y^*$).

2. Using a singular perturbation technique Hagan, Kumar, Lesniewski and Woodward obtain an analytical solution for an input volatility as a function of the current forward price [18]. The analytical input volatility can be plugged directly into BSM frameworks. The equivalent volatility, $\sigma_B$, is given by:

$$
\sigma_B = \nu \frac{z}{\chi^{(z)}} T + \nu \frac{z}{\chi^{(z)}} \frac{(1 - \beta)^2}{24} \frac{\alpha^2}{(F \cdot K)^{1-\beta} T} + 
+ \nu \frac{z}{\chi^{(z)}} \frac{\rho \beta \xi \alpha}{4(F \cdot K)^{1+\beta}} T + \nu \frac{z}{\chi^{(z)}} \frac{2 - 3 \rho^2}{24} \xi^2 T
$$

(2)
where:

\[
\nu = \frac{\alpha}{(F \cdot K)^{(1-\beta)2} \left[ 1 + \frac{(1-\beta)^2}{24} \ln (F/K)^2 + \frac{(1-\beta)^4}{1920} \ln (F/K)^4 \right]}
\]

\[
z = \frac{\xi}{\alpha} (F \cdot K)^{(1-\beta)/2} \ln \left( \frac{F}{K} \right)
\]

\[
\chi(z) = \ln \left( \frac{\sqrt{1 - 2\rho z + z^2} + z - \rho}{1 - \rho} \right)
\]

Thanks to the proposed approach this calibrated volatility may be used directly in the simulation of the motion, obtaining a greater consistency with market data.

The application of the MSPE is therefore also be extended to the alternative stochastic financial models to BSM.

4 Conclusions

The method proposed by the authors to determine the optimal number of simulation runs necessary to estimate the price of complex derivatives makes it possible to control the entity of the pure experimental error variance, typical of simulation experiments. Therefore, the number of runs chosen by the experimenter is no longer a subjective estimate, but an objective data item linked to the level of accuracy of the output that the experimenter wishes to achieve. Another problem solved by this method regards the robustness of the solution. The evolution curves of $MSPE_{MED}$ in the runs show that a set quantity of experimental error is such only when the curve is in a condition of stabilization. If this is not the case, the following series of runs may even provide extremely different price results. Finally, it should be noted that for reasons linked to the normality of the frequency distribution of the outputs the study was carried out only on the price without considering the effect of the $MSPE_{STDEV}$ and $VAR$ quantities, which, as a rule, also have a non-negligible impact on the response.

Obviously the cases studied show that the greater the stochasticity of the derivative, the greater the difference between the conventional method and the method proposed becomes relevant in terms of the impact on the price.
5 Appendix

The validity of the proposed methodology was verified by the pricing of a plain-vanilla Call Option. This derivative can be valued using the Black-Scholes Formula [11],[12] as illustrated below.

\[
c = SN(d_1) - K \exp(-rT)N(d_2)
\]

\[
d_1 = \left\{ \ln \left( \frac{S}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) T \right\} / \sigma \sqrt{T}
\]

\[
d_2 = d_1 - \sigma \sqrt{T}
\]

where

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(c)</td>
<td>Price of the European call</td>
</tr>
<tr>
<td>(S)</td>
<td>Stock price</td>
</tr>
<tr>
<td>(K)</td>
<td>Strike price of the Option</td>
</tr>
<tr>
<td>(r)</td>
<td>Risk-free interest-rate</td>
</tr>
<tr>
<td>(T)</td>
<td>Time to expiration in years</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>Annual volatility of the price change of the stock</td>
</tr>
<tr>
<td>(N(\cdot))</td>
<td>Cumulative normal distribution function</td>
</tr>
</tbody>
</table>

Particularizing the pricing of a European call option with the following characteristics: \(S = K = 2.438\), \(r = 0.05062\), \(T = 0.3863\) and \(\sigma = 0.35727\), the formula gives to infinity a value of \(c\) equal to 0.23787, which corresponds, in normal practice, to an estimated option price of 0.24.

The test case aims at assessing what result is obtained by operating in a stochastic rather than deterministic system. Therefore, a simulation model has been developed using Matlab numeric processing software for the pricing, [4], to describe the geometric Brownian motion of the price of the underlying asset \(S\):

\[
S + dS = S \exp \left[ (\mu - \frac{1}{2} \sigma^2)dt + \sigma dz \right]
\]

where \(dz\) is a Wiener process with mean zero and standard deviation one.

Performing the discretization of the time intervals and setting \(\mu\) equal to the free-risk rate gives a motion that can be implemented by the software:

\[
S + \Delta S = S \exp \left[ (r - \frac{1}{2} \sigma^2)\Delta t + \sigma \varepsilon_t \sqrt{\Delta t} \right]
\]

where \(\Delta S\) is a discrete variation of \(S\) in a given interval \(\Delta t\) and \(\varepsilon_t\) is the casual extraction from a standardized normal distribution.

Making the process evolve for the entire life of the option gives an estimate of payoff: \(\max(S_T - K, 0)\). Once a significant number of simulations are
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performed on the possible final price level and just as many payoff values are obtained, their mean value is calculated by discounting these back to the pricing date:

\[ \text{Price} = \frac{\sum_{i=1}^{n_{\text{Sim}}} \text{payoff}_i}{n_{\text{Sim}}} \exp(-rT) \] (6)

The arising problem lies in determining the right number of simulations \( n_{\text{Sim}} \) to be performed in order to give a correct price to the derivative along with an a-priori quantification of the experimental error made on the said price. The statistical behavior of the payoff quantity in the simulated runs was initially analyzed and it was seen that, as a result of the cut-off effect on the underlined asset caused by the Strike of the option, the distribution of the responses did not follow a Gaussian trend. Since the theory by which \( E(\sigma^2) = \text{MSPE} \), where \( \sigma^2 \) is the variance of the experimental error, is true if and only if the outputs of the model at hand have a normal distribution, it was decided to act directly on the price, a quantity, which does not pose this problem. As it will be verified afterwards, this hypothesis about the normality of Price distribution can be reliable when the outputs of the model pass a robust goodness-of-fit test. In the above mentioned cases, the experimenter is able to make inference on the confidence interval in which the responses could fall under a given significance level. When the Price distribution can’t be referable to a normal distribution, that is the values don’t pass the goodness-of-fit test, \( \text{MSPE} \) can no longer be considered a good estimator of \( \sigma^2 \). However, in this case, the methodology can be applied for determining the simulator’s stable area, as well as the optimal number of runs. It is emphasized that in all the experimental campaigns done, the failure of the normality test occurs in few theoretical cases, such as for the pricing of options deeply out-of-the-money \( (S_0 \ll K) \) or for values of \( S_0 \) very close to zero. By contrast, with this approach, quantities \( \text{VAR} \) and \( \text{MSPE}_{\text{STDEV}} \) are void of any relevance, as, considering the price’s structure, the only possible line of reasoning is that of sampling distribution of the mean values. Therefore, the mean response given by formula (7) is hence used as the price’s variability field with \( \pm 3\sqrt{\text{VAR} + \text{MSPE}_{\text{stdev}}} \rightarrow 0. \)

\[ y^* \geq \bar{y} - 3\sqrt{\text{MSPE}_{\text{med}}} - 3\sqrt{\text{VAR} + \text{MSPE}_{\text{stdev}}} \]

\[ y^* \leq \bar{y} + 3\sqrt{\text{MSPE}_{\text{med}}} + 3\sqrt{\text{VAR} + \text{MSPE}_{\text{stdev}}} \] (7)

Applying the steps of the procedure described above, a calculation of the statistical quantity \( \text{MSPE}_{\text{MED}}(i) \) with \( i = 1, \ldots, n_{\text{Sim}} \) performed for each \( i \)-th
run replication. These values transferred on the plane \((i, MSPE_{MED})\) give the chart in Fig.10.

If we were to proceed as suggested by the literature with a maximum number of replications equal to 10000, the simulator’s outputs - price - would range, with a 95% degree of reliability, between

\[
0.2397 - 3.182 \sqrt{\frac{3.42 \times 10^{-5}}{4}} \leq Price \leq 0.2397 + 3.182 \sqrt{\frac{3.42 \times 10^{-5}}{4}}
\]

\[
0.2304 \leq Price \leq 0.2490
\]

In terms of operating practice the interval becomes

\[
0.23 \leq Price \leq 0.25
\]

Performing 2000 simulation campaigns of 10000 runs each and placing the outputs of the single simulations in a graph \((j|j = 1, \ldots, nrepl; Price)\), what was stated above occurs (see green band, Fig. 11). Only 75% of the simulations gave an option price in line with 0.24 as their output (see red band, Fig. 11).

If the number of runs were increased to 30000, you would have a confidence interval for the price, with a degree of reliability of 95% equal to:

\[
0.2335 \leq Price \leq 0.2450
\]

Replicating the simulations 2000 times using a \(nSim\) equal to \(3 \cdot 10^4\) and saving the option price each time gives a graph as in Fig. 12, which shows, similarly to what was done above, the responses as a function of the \(j\)-th replications: this time 91.76% of the runs gives a fair value, which can be approximated to 0.24 (red band), while the green band is the confidence interval for the price as calculated above. In the case at hand, in order to have an accuracy of \(\pm 0.005\) for the option’s final price, an \(nSim\) of at least \(5 \cdot 10^4\) is needed as illustrated in the graph in Fig. 13. If the number of runs were 100000, you would have a confidence interval for the price, with a degree of reliability of 95%, equal to:

\[
0.2358 \leq Price \leq 0.2398
\]

Replicating the simulations 2000 times using an \(nSim\) equal to \(1 \cdot 10^5\) gives the graph as in Fig. 14, which shows how all of the responses give a value in line with 0.24.

Fitting the 2000 values obtained from the analysis above into a frequency histogram, as illustrated in Fig. 15, and performing a Chi-square goodness-of-fit test with a significance level of \(\alpha = 0.05\), it was possible to see that the values have a normal distribution according to a \(N(0.2379; 0.0024)\). In order to test
the method’s robustness when applied to the simulator, 8 MSPE_MED were calculated to determine whether the simulator’s stabilization zone actually converges in the designated interval (50000) for all the replications.

The graph in Fig. 16 shows the 8 resulting curves. As you can see, at a number of 50000 replications there is a level of confidence having an adequately stable error in the final result (0.001). However, it is recommended to perform 100000 replications to have a greater degree of robustness.

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Received: September 12, 2015; Published: October 10, 2015
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Figure 1: Resolution of the stochastic differential equations by using Monte Carlo method

Figure 2: 25 curves \((i, MSPE_{MED})\) with \(i = 1, \ldots, 9 \cdot 10^4\)
Figure 3: Graph \((j, y^*)\) with \(i = 1, \ldots, 9 \cdot 10^4\) and \(j = 1, \ldots, 300\)

Figure 4: Graph \((j, y^*)\) with \(i = 1, \ldots, 1 \cdot 10^4\) and \(j = 1 \ldots, 2000\)
Figure 5: Graph \((i, MSPE_{MED})\) with \(i = 1, \ldots, 5 \cdot 10^5\)

Figure 6: Graph \((j, y^*)\) with \(i = 1, \ldots, 5 \cdot 10^5\) and \(j = 1, \ldots, 100\)
Figure 7: Graph \((j, y^*)\) with \(i = 1, \ldots, 1 \cdot 10^4\) and \(j = 1, \ldots, 2000\)

Figure 8: Graph \((i, MSPE_{MED})\) with \(i = 1, \ldots, 125000\)
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Figure 9: Graph \((j, y^*)\) with \(i = 1, \ldots, 125000\) (red line), \(i = 1, \ldots, 10000\) (green line), \(i = 1, \ldots, 1000\) (blue line) and \(j = 1, \ldots, 100\)

Figure 10: Graph \((i, MSPE_{MED})\) with \(i = 1, \ldots, 5 \cdot 10^5\)
Figure 11: Graph $(j, \text{Price})$ with $i = 1, \ldots, 10000$ and $j = 1, \ldots, 2000$

Figure 12: Graph $(j, \text{Price})$ with $i = 1, \ldots, 30000$ and $j = 1, \ldots, 2000$
Figure 13: Graph \((i, MSPE_{MED})\) with \(i = 1, \ldots, 5 \cdot 10^5\) and an acceptable maximum error threshold (red line).

Figure 14: Graph \((j, Price)\) with \(i = 1, \ldots, 1 \cdot 10^5\) and \(j = 1, \ldots, 2000\).
Figure 15: Discrete $y^*$ distribution

Figure 16: Curves $(i, MSPE_{MED})$ with $i = 1, \ldots, 2 \cdot 10^5$