Enriched Uranium Market Portfolio Optimization

in Fuzzy Environment

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Abstract

In the present work forecasting future sales of a company involved in uranium enrichment is considered a problem of allocating company’s produce to global regional markets. Based on this concept optimization problem of company’s effective portfolio is formulated and solved. Relative total company share in the world is defined by the balance parity between probability distributions of global demand for enriched uranium and enriched uranium manufacture at competitive enterprises. Effective portfolios of company shares in the world regional markets take into account risk and uncertainty and are calculated from probability distribution of company’s relative world share, regional market quotas and restrictions, market risk measures and expected market efficiencies. Incorporating fuzzy and probability approaches to portfolio selection allows for accurate market information employment and exhaustive analysis of company potential and market opportunities.

Keywords: enriched uranium, regional market shares, effective portfolios, fuzzy sets, computational results
Introduction

Enriched uranium market is considerably steady which makes it practicable to forecast supply and demand [1-5]. Demand for uranium enrichment is determined by currently operating nuclear reactor capacities, which allows calculating the required fuel volumes and thus the demand for enriched uranium. Enriched uranium supply is also calculable as it consists of stable capacities of functioning uranium enrichment plants. Criterion taken into account in current work is expected market efficiency, calculated based on available information about global regional markets. Possibility of factual efficiency values deviating downwards and both upwards and downwards from their expected values is defined as risk and uncertainty respectively. Forecasting model presented below follows Markowitz’s approach to portfolio optimization [6] and Zadeh’s fuzzy set theory [7].

Fuzzy Portfolio Optimization Model

Let $n$ be the number of world regional markets, among which the considered company allocates its produce and $k$ be the number of years for which regional company sales need to be forecasted. Total company sales in $j$-th year $U_j^{OWN}$ are determined by the difference between probability distribution of the total produce of competing companies and the world demand probability distribution. Company’s portfolio is defined as vector $\bar{x}_j^T = (x_{1j}^T, ..., x_{nj}^T)^T$, where $x_{ij}$ is the proportion of the $i$-th regional market allocated to the considered company. Absolute company sales $U_j = (V_{1j} \cdot x_{1j}^T, ..., V_{nj} \cdot x_{nj}^T)^T$ are obtained from $V_{ij}$ – enriched uranium demand probability distributions in $i$-th world regional market ($i=1, ..., n$) in year $j$, ($j=1, ..., k$) subject to $\sum_{i=1}^n V_{ij} \cdot x_{ij} = U_j^{OWN}$.

Contingencies are considered to be contained within the variable demand and supply values, which are described within probability theory, while efficiencies measures are precisely unknown and are described as fuzzy sets [8]. Let $R_{ij}$ be efficiency of $i$-th regional market in year $j$, where $R_{ij}$ is a triangular fuzzy set defined by points $m_{ij}^-, m_{ij}, m_{ij}^+$:

$$R_{ij} = (m_{ij}^-, m_{ij}, m_{ij}^+), \quad m_{ij}^- < m_{ij} < m_{ij}^+, \quad i = 1, ..., n, \quad j = 1, ..., k. \quad (1)$$

Membership function of $R_{ij}$ is given by

$$\mu_{ij}(x) = \begin{cases} 0, x < m_{ij}^-, \\
\frac{x - m_{ij}^-}{m_{ij} - m_{ij}^-}, m_{ij}^- \leq x \leq m_{ij}, \\
\frac{m_{ij}^+ - x}{m_{ij}^+ - m_{ij}}, m_{ij} \leq x \leq m_{ij}^+, \\
0, x > m_{ij}^+, \end{cases} \quad (2)$$
where \( m^-_{ij}, m^+_{ij}, m^0_{ij} \) are estimates for minimal, expected and maximal values of efficiency of \( i \)-th regional market in year \( j \).

The difference \( m^+_{ij} - m^-_{ij} \) is a measure of uncertainty, as it penalizes both upside and downside deviations from the expected return, it is more appropriate to use \( m^+_{ij} - m^-_{ij} \) as a risk measure which only considers downside deviations.

Thus efficiency of company portfolio \( \bar{R}_j \) is given by

\[
\bar{R}_j = x_{i1} \cdot R_{i1} + \cdots + x_{n_j} \cdot R_{n_j} \quad \text{or} \quad \bar{R}_j = (\sum_{i=1}^{n} x_{ij} \cdot m^0_{ij}, \sum_{i=1}^{n} x_{ij} \cdot m^+_{ij}, \sum_{i=1}^{n} x_{ij} \cdot m^-_{ij}).
\]  

(3)

Expected portfolio efficiency in year \( j \) is given by \( \bar{m}_j = \sum_{i=1}^{n} x_{ij} \cdot m_{ij} \). \( \varphi(\bar{R}_j) = \sum_{i=1}^{n} x_{ij} \cdot (m^+_{ij} - m^-_{ij}) \) is a measure of portfolio uncertainty in year \( j \) \((j = 1,...,k)\). \( \varphi(\bar{R}_j) = \sum_{i=1}^{n} x_{ij} \cdot (m_{ij} - \bar{m}_{ij}) \) is a portfolio risk measure for year \( j \) \((j=1,2,...,k)\).

One way of acquiring the values \( R_{ij} \) is by using the information given by the regional demand probability distributions, assigning the values of regional demand probability distributions at 25%, 50% and 75% percentiles to \( m^-_{ij}, m^0_{ij}, m^+_{ij} \).

The company portfolio optimization model is formulated as follows:

\[
\begin{align*}
\max \bar{m}_j &= \sum_{i=1}^{n} x_{ij} \cdot m_{ij}, \\
\min \varphi(\bar{R}_j) &= \sum_{i=1}^{n} x_{ij} \cdot (m^+_{ij} - m^-_{ij}), \\
\min \omega(\bar{R}_j) &= \sum_{i=1}^{n} x_{ij} \cdot (m_{ij} - m^-_{ij}), \\
\text{subject to} \; \sum_{i=1}^{n} \delta_{ij} \cdot x_{ij} &= \delta_j, \\
\alpha_{ij} &\leq x_{ij} \leq \beta_{ij}, \\
\sum_{i=1}^{n} x_{ij} &= 1 \forall j = 1, ..., k, \\
\text{and} \; x_{ij} &\geq 0,
\end{align*}
\]

(4)

where \( \beta_{ij} \) - quotas for company in consideration \((0 \leq \beta_{ij} \leq 1)\) at \( i \)-th world regional market \((i=1,...,n)\) in year \( j \), \((j=1,...,k)\); \( \alpha_{ij} \) - provided minimal proportion of \( i \)-th regional market \((i=1,...,n)\) secured by company in year \( j \), \((j=1,...,k)\), \((0 \leq \alpha_{ij} \leq \beta_{ij})\); \( \delta_{ij} \) is relative regional market \( i \) size in year \( j \); \( \delta_j \) is the portion of world uranium enrichment produced by company in consideration.

**Computational results**

It is necessary to successively optimize each criterion of problem (4) to find its efficient frontier. Computational results of problem (4) partial criterion optimization are presented in Fig. 1-3.
Fig. 1. Partial criteria optimization results for company sales in Western Europe

Fig. 2. Partial criteria optimization results for company sales in Asia
Let $\bar{x}_j^{(1)} = (x_{1j}^{(1)}, ..., x_{nj}^{(1)})^T$ be company portfolio with highest total efficiency value, let $\bar{x}_j^{(2)} = (x_{1j}^{(2)}, ..., x_{nj}^{(2)})^T$ be company portfolio with lowest total risk measure, $\alpha_j$ – risk aversion factor in year $j$, $j = 1, ... k$.

Thus efficient portfolios with maximal expected efficiency among all portfolios with the same risk measure or alternatively portfolios with minimum risk measure for a set expected efficiency of problem (4) are defined by $\bar{x}^* = \alpha_j \cdot \bar{x}_j^{(1)} + (1 - \alpha_j) \cdot \bar{x}_j^{(2)}$, $0 \leq \alpha_j \leq 1, \forall j = 1, ... k$, where $\alpha_j$ is risk aversion factor [9-10].

Fig. 4-6 illustrate variation of the risk aversion factor in reference to partial criterion optimization solutions. Computational results presented below are derived from results shown in Fig. 1-3 and depict some of the efficient portfolios that correspond to varied risk aversion factor values.
Fig. 4. Company sales in Western Europe corresponding to various effective portfolios

Fig. 5. Company sales in Asia corresponding to various effective portfolios
Fig. 6. Company sales in China corresponding to various effective portfolios

Conclusion

In solving this problem, probability approach to portfolio optimization meets fuzzy approach, which allows for employment of both probability theory and concept of fuzzy sets. Optimization criteria in formulated problem are expressed in the form of fuzzy sets while the expected market volumes are given as probability distributions. The resulting problem is tractable and allows minimizing the portfolio risk and maximizing the portfolio efficiency using a specified risk aversion factor. Naturally, higher risk aversion factor value corresponds to lower portfolio efficiency, and vice versa, higher portfolio efficiency complies with higher risk. As the solutions to defined problem represent all possible outcomes to various management decisions, computational results presented above can be integrated into company marketing strategy. We are currently developing a scheme of formation of efficient portfolios based on fuzzy sets in the reinsurance markets.

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References


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